Research statement
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My research interests lie in applied mathematics. In particular, using a combination of tools such as mathematical modeling, asymptotic analysis, numerical methods, rigorous analysis, and collaboration with experimentalists in order to solve problems that arise from exciting applications.

My main research topic is the theory of singular solutions of the nonlinear Schrödinger equation (NLS) with applications to nonlinear Optics. In nonlinear Optics, the NLS models the propagation of intense laser beams in a medium such as air, water or glass. Such intense laser beams undergo catastrophic self-focusing up to a point of collapse where, mathematically, the solution becomes singular. My main results in this field include finding new singular solutions of the NLS and one of my papers in this subject [2] has been selected as **one of the winners of the 2007 SIAM student paper competition**. Following this theoretical work, the new singular solutions were later **observed experimentally** by the nonlinear Optics laboratory in Cornell University, in collaboration with us [5]. Some of the questions that arised from this collaboration then motivated us to a develop a nonlinear Geometrical Optics method for predicting the initial propagation dynamics of high-power beams and pulses [6]. Our research also included a comprehensive numerical study of NLS solutions as they approach singularity. Accordingly, my work also included the **development of a novel numerical method** for solving such singular problems [7]. Finally, recently I have also branched out of those fields into auction theory.

1 Current research

The nonlinear Schrödinger equation (NLS) is one of the canonical nonlinear equations in physics, arising in various fields such as nonlinear Optics, plasma physics, Bose-Einstein condensates (BEC), and water waves. In nonlinear Optics it models the propagation of high power laser beams in media such as air, water or glass. Experiments conducted in the early 60's showed that, in such cases, laser beams undergo catastrophic self-focusing up to a point of collapse. That is the laser power focuses into a smaller and smaller spot until the material breaks-up.

1.1 Mathematical analysis: New singular solutions of the NLS

Mathematically, NLS solutions become singular at the point of collapse. One of the open questions in this field for more than 40 years is what is the behavior of NLS solutions as they collapse. Since the 80s, the common perception is that all singular solutions of the NLS collapse with a single profile known as the Townes profile. This result served as a basis for essentially all the asymptotic theory of the NLS and of the perturbed NLS. However, for many years this result was based only on numerical simulations rather than on a full rigorous proof. In fact, only recently (2003) Merle and Raphael proved this result. For this proof, in part, Merle was awarded the prestigious 2005 Bôcher Prize.

In my research I show that this common perception is false, by showing that there are also NLS solutions that do not collapse with the Townes profile, but rather collapse with a ring profile [1], see Figure 1. On the basis of these results, I was **awarded the Eshkol scholarship for the years 2006-2009**.

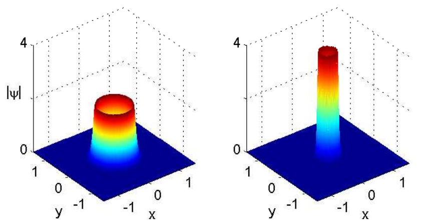

Figure 1: Ring solution of the NLS at initial stages of collapse.

In addition, we extended these results to a broader family of NLS equations (known as super-critical) [2]. Even though this family of equations models relevant physical settings, very little is known on the behavior of singular solutions of these equations. Finding singular ring solutions for this family of equations involved constructing novel asymptotic structures. The new ring solutions have a new and surprising behavior, which is very different than the behavior of all previously known singular NLS solutions. For example, they can have any blowup rate between half and one, while for all previously known solutions the blowup rate is either half and slightly faster than half. This work was chosen as one of the **winners of the 2007 SIAM student paper award**.

In a subsequent work, we studied yet another family of singular ring solutions of the NLS which are singular vortex solutions [3,4]. Vortices have been intensively studied, both theoretically and experimentally, in the nonlinear optics literature as well as in BEC. However, almost all of this research effort has been on non-collapsing vortices. Therefore, there is a huge gap between the available theory on non-vortex and vortex singular NLS solutions. The goal of our study was to close this gap. In the development of the rigorous theory for vortices, we found cases in which stronger results can be obtained for collapsing vortices (e.g., the critical power for collapse). However, the most surprising result of this study was that some of the non-rigorous results for singular non-vortex solutions (e.g., stability of blowup profiles) completely change for vortex solutions.

[1] G. Fibich, N. Gavish, and X.P. Wang, *New singular solutions of the nonlinear Schrödinger equation*, Physica D, **211** (2005), 193–220.

[2] G. Fibich, N. Gavish, and X.P. Wang, *Singular ring solutions of critical and supercritical nonlinear Schrödinger equations*, Physica D **231** (2007), 55–86.

[3] G. Fibich and N. Gavish, *Critical power of collapsing vortices*, Physical Review A **77** (2008), 045803.

[4] G. Fibich and N. Gavish, *Theory of singular vortex solutions of the nonlinear Schrödinger equation*, Physica D **237** (2008), 2696. University.

1.2 Nonlinear Optics: The nonlinear Geometrical Optics method

Following our theoretical study, our prediction of the new collapsing ring profile was confirmed experimentally by the nonlinear Optics laboratory in Cornell University, in collaboration with us [5]. In particular, this experiment showed that it is possible to control the collapse profile, i.e., whether the laser would collapse with a Townes profile or with a ring profile. This motivated a more comprehensive study of how to control the initial propagation dynamics of high-power lasers which has important applications for laser propagating in air.

Indeed, the propagation of high-power lasers through the atmosphere is currently one of the most active areas of research in nonlinear optics, with potential military and civil applications. Typically, a single high-power beam propagating in air breaks into several long and narrow filaments, a phenomenon known as multiple filamentation. The control of the filamentation pattern is crucial for applications. Various methods for deterministic multiple filamentation were developed, but each of them handle only an isolated case. We present **the first analytic method**, known as the nonlinear Geometrical Optics method, for predicting the initial filamentation dynamics of high-power beams and pulses [6], see Figure 2. Using this method we study the filamentation pattern of a variety of input profiles and pulses in 1D, 2D and 3D, without solving partial differential equations. Moreover, this method predicts a new type of temporal split. This method is now used as a **basis for two different sets of experiments**, one conducted in Cornell and the second in the Hebrew University.

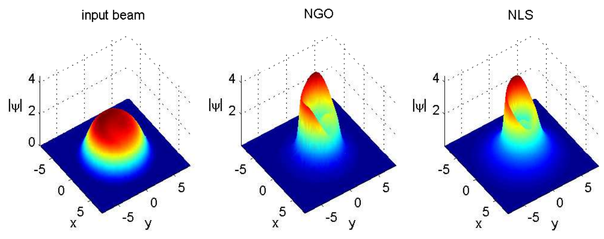

Figure 2: NGO method applied to an irregular input beam.

[5] T. D. Grow, A. A. Ishaaya, L. T. Vuong, A. L. Gaeta, N. Gavish, and G. Fibich, *Collapse dynamics of super-gaussian beams*, Optics Express **14** (2006), 5468–5475.

[6] N. Gavish, G. Fibich, L. T. Vuong, and A. L. Gaeta, *Predicting the filamentation of high-power beams and pulses without numerical integration: A nonlinear geometrical optics method*, Phys. Rev. A **78** (2008), 043807.

1.3 Scientific computing: The SGR method for simulations of collapsing solutions

The research of the new singular solutions included a comprehensive numerical study which gave important information on the stability and dynamics of these solutions. In general, numerical simulations of collapsing solutions are very demanding and several specialized methods were developed for this type of problems (e.g., the dynamical rescaling method). For the numerical simulations required for my research I initially studied and implemented the IGR method, developed by Ren and Wang, which was the most advanced method available at that time for simulations of collapsing NLS solutions.

I used this method successfully for our first paper [1]. However, as we considered more sophisticated families of solutions, e.g., rotating (vortex) solutions [3], it turned out that the simulations were too demanding even for this state-of-the-art method. Therefore, I developed, in collaboration with Dr. Adi Ditkowsky, a **new numerical method** for simulations of such solutions [7], which ensures that singular regions of the solution remain well resolved, see Figure 3. This method not only worked in cases where previous methods failed, but also dramatically improved the runtime needed for such simulations by a factor of 60, e.g., **from 30 minutes to 30 seconds!**

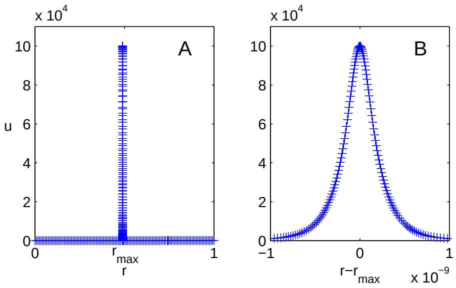

Figure 3: A: Solution of the NLS near singularity. Blue markers correspond to grid points. B: Same data as in A in the region $[r_{max} - 10^{-9}, r_{max} + 10^{-9}]$

[7] A. Ditkowsky and N. Gavish, *A grid redistribution method for singular problems*, Submitted to JCP.

1.4 Auction theory

Auctions are important monetary tools, central in the back bone of the economy. For example, during October 2008, the US federal reserve auctioned short terms loans in a total sum of 300 billion dollars! Most of the literature on mathematical auction theory assume that all players behave the same. Although this assumption is restrictive and often unrealistic, the symmetric case has the advantage that the optimal bidding strategy is given by a single ODE and hence relatively easy to analyze.

I have now begun to study a class of auctions in which no symmetry between the players is assumed. In this case, the optimal bidding strategy is a given by a set of nonlinear ODEs which on one side suffer from pathologies around the origin that give rise to non-unique solutions and on the other side have an open boundary, i.e., it is not a-priori known where the boundary is. I am attacking this problem using rigorous analysis, asymptotic analysis and numerics.

2 Future plans

I am looking for a field where an applied math approach, i.e., a combination of tools such as mathematical modeling, asymptotic analysis, numerical methods, rigorous analysis, and collaboration with experimentalists, can be used to solve interesting problems that arise from applications. Possible fields include biology, medicine, economy or physics.