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Manipulating Josephson junctions in thin-films by nearby vortices

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ABSTRACT

It is shown that a vortex trapped in one of the banks of a planar edge-type Josephson junction in a narrow thin-film superconducting strip can change drastically the dependence of the junction critical current on the applied field, $I_c(H)$. When the vortex is placed at certain discrete positions in the strip middle, the pattern $I_c(H)$ has zero at $H=0$ instead of the traditional maximum of '0-type' junctions. The number of these positions is equal to the number of vortices trapped at the same location. When the junction–vortex separation exceeds $\sim W$, the strip width, $I_c(H)$ is no longer sensitive to the vortex presence. The same is true for any separation if the vortex approaches the strip edges.

1. Introduction

The very fact that Abrikosov vortices in the vicinity of Josephson junctions affect the junction properties is well documented and not surprising since the phase associated with vortex affects the junction phase difference [1–3]. Recent experiments with a vortex trapped in one of the banks of an edge-type planar junction in a thin-film superconducting strip showed that the vortex causes an extra phase difference on the junction that depends on the vortex position [4]. The effect is strong in particular when the vortex is close to the junction, the situation when the junction behavior can be changed from the conventional "0-type" to that of the π-junction.

Here we study how the field dependence of maximum critical tunneling currents $I_c(H)$ depends on position of a vortex trapped in one of the junction thin-film banks. This effect can be utilized for manipulating Josephson currents by controlling the vortex position.

2. Approach

Consider a thin-film strip of a width W with an edge-type Josephson junction across the strip that cuts the strip in two half-strips, Fig. 1. The strip is narrow: $W \ll \Lambda = 2\lambda^2/d$ where λ is the London penetration depth of the film material and d is the film thickness. Choose x along the strip and y across so that $0<y<W$ and the junction is at $x=0$. Let a vortex be trapped at some point $\boldsymbol{r}_0=(x_0,y_0)$ in the right half-strip ($x_0>0$).

The London equation integrated over the film thickness for the half-strip with vortex (shown by a thick line in Fig. 1) is:

$$h_z+\frac{2\pi\Lambda}{c}(\mathrm{curl}\,\boldsymbol{g})_z=\phi_0\delta(\boldsymbol{r}-\boldsymbol{r}_0). \tag{1}$$

Here $\boldsymbol{g}$ is the sheet current density and h_z consists of the applied field H and the self-field of the current $\boldsymbol{g}$.

The self-field of the current $\boldsymbol{g}$ is of the order g/c, whereas the second term on the left-hand side of Eq. (1) is of the order $g\Lambda/cW \gg g/c$. Hence, in narrow strips with $W\ll\Lambda$, the self-field can be discarded, unlike the *applied* field $H\hat{\boldsymbol{z}}$. Introducing the scalar stream function S via $\boldsymbol{g}=\mathrm{curl}[S(x,y)\hat{\boldsymbol{z}}]$, we obtain instead of Eq. (1):

$$\nabla^2 S=\frac{cH}{2\pi\Lambda}-\frac{c\phi_0}{2\pi\Lambda}\delta(\boldsymbol{r}-\boldsymbol{r}_0)\,. \tag{2}$$

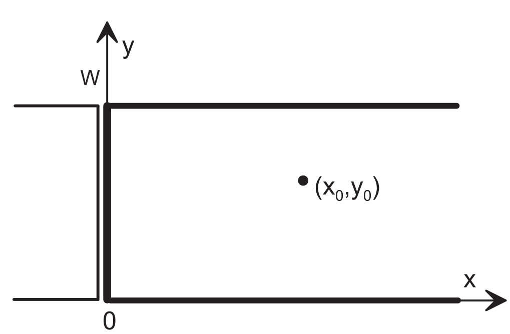

Fig. 1. The superconducting thin-film strip with a Josephson junction at $x=0$ and a vortex at (x_0,y_0). The half-strip containing the vortex is outlined by thick lines.

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This is a *linear* Poisson equation, that formally simplifies the problem as compared to Eq. (1). Physically, this simplification comes about since in narrow films the major contribution to the system energy is the kinetic energy of supercurrents, while their magnetic energy can be discarded.

The boundary condition $g_y = 0$ at the strip edges translates to $S = 0$ at the edges $y = 0, W$ (in the absence of transport current). Besides, one can disregard Josephson tunneling currents relative to those of the vortex, i.e. to set $g_x(0, y) = 0$ as well. The Green's function $G(\boldsymbol{r}, \boldsymbol{r}')$ which satisfies $\nabla^2 G = -4\pi\delta(\boldsymbol{r} - \boldsymbol{r}')$ (as the electrostatic potential of a unit linear charge at $\boldsymbol{r}'$) with zero boundary conditions at the edges of the half-srtip (delineated in Fig. 1 by thick lines) is found by conformal mapping [5–7]:

$$u + iv = -i\cosh\pi(x + iy), \tag{3}$$

transforms the half-plane $u > 0$ to the half-strip of a width 1 (hereafter we use W as a unit length). Explicitly, this transformation reads:

$$u = \sinh\pi x \sin\pi y, \qquad v = -\cosh\pi x \cos\pi y. \tag{4}$$

The complex potential $G(w, w')$ for a linear unit charge at $w' = u' + iv'$ at the half plane $u > 0$ is [8]:

$$G = -2\ln\frac{w - w'}{w - \tilde{w}'} = -2\left[\ln\frac{r_1}{r_2} + i(\theta_1 - \theta_2)\right], \tag{5}$$

where $w = u + iv, \tilde{w}' = -u' + i\,v'$ is the position of fictitious image source on the opposite side of the grounded plane $u = 0$. The corresponding moduli and phases are:

$$\begin{aligned} r_1 &= \sqrt{(u - u')^2 + (v - v')^2}, \\ r_2 &= \sqrt{(u + u')^2 + (v - v')^2}, \\ \theta_1 - \theta_2 &= \tan^{-1}\frac{v - v'}{u - u'} - \tan^{-1}\frac{v - v'}{u + u'}. \end{aligned} \tag{6}$$

Below we evaluate the phase at the junction bank $x = +0$ and make use of

$$\mathrm{Im}G(0, y; \boldsymbol{r}') = -4\tan^{-1}\frac{\cos\pi y - \cosh\pi x' \cos\pi y'}{\sinh\pi x' \sin\pi y'}, \tag{7}$$

which follows from Eqs. (5) and (6).

We now note that the sheet current is expressed either in terms of the gauge invariant phase φ or via the stream function S: $\boldsymbol{g} = -(c\phi_0/4\pi^2\Lambda)\nabla\varphi = \mathrm{curl}S\boldsymbol{z}$. (This relation written in components shows that $(4\pi^2\Lambda/c\phi_0)S(\boldsymbol{r})$ and $\varphi(\boldsymbol{r})$ are the real and imaginary parts of an analytic function.) In particular, we have

$$\frac{\partial\varphi}{\partial y} = -\frac{4\pi^2\Lambda}{c\phi_0}g_y. \tag{8}$$

On the other hand, the sheet current $\boldsymbol{g}_0$ for a *unit* δ-function source can be expressed either via real or imaginary parts of G [8]. In particular, we have:

$$g_{0y} = -\frac{\partial\mathrm{Re}G}{\partial x} = -\frac{\partial\mathrm{Im}G}{\partial y}. \tag{9}$$

*2.1. Contribution of the field **H** at the right bank to the phase difference at the junction*

The solution of Eq. (2) without a vortex, $\nabla^2 S = -4\pi(-cH/8\pi^2\Lambda)$, is

$$\begin{aligned} \mathrm{Im}S(0, y) &= -W^2\int d\boldsymbol{r}'\frac{cH}{8\pi^2\Lambda}\mathrm{Im}G(0, y; \boldsymbol{r}') \\ &= \frac{cHW^2}{2\pi^2\Lambda}\int d\boldsymbol{r}'\tan^{-1}\frac{\cos\pi y - \cosh\pi x' \cos\pi y'}{\sinh\pi x' \sin\pi y'}, \end{aligned} \tag{10}$$

where the integrals are extended over the half-strip: $0 < x' < \infty, 0 < y' < 1$. The last integral $\mathcal{I}$ can be evaluated in terms of Lerch transcendents [6], which are not particularly illuminating. Hence, for each y we do the integration numerically. The function $\mathcal{I}(y)$ can be approximated as

$$\mathcal{I} \approx 0.43\cos\pi y - 0.03\sin 2\pi y, \tag{11}$$

with accuracy less than 0.5%. The quantity $\mathcal{I}$ has been calculated employing a different method (see Ref. [9]). We use the approximation Eq. (11) in the numerical work below.

At the junction bank $x = +0, g_y(0, y) = -\partial_y\mathrm{Im}S(0, y)$, and we obtain with the help of Eqs. (8), (10), and (11):

$$\frac{\partial\varphi}{\partial y} = -\frac{h}{2}\frac{\partial\mathcal{I}}{\partial y}. \tag{12}$$

or after integration over y:

$$\varphi_H(+0, y) = -\frac{h}{2}\mathcal{I}(y) + \varphi_0, \qquad h = \frac{4W^2H}{\phi_0}. \tag{13}$$

The subscript H here is to indicate that this contribution to the phase is due to the applied field; φ_0 is an arbitrary constant [9].

2.2. Contribution of a vortex at $\boldsymbol{r}_0$ to the phase difference at the junction

To find this contribution, we use the relation Eq. (8) along with

$$g_y(+0, y) = -\frac{\partial\mathrm{Im}S}{\partial y} = -\frac{c\phi_0}{8\pi^2\Lambda}\frac{\partial\mathrm{Im}G(0, y; \boldsymbol{r}_0)}{\partial y}. \tag{14}$$

We obtain after integration over y:

$$\begin{aligned} \varphi_v(y; \boldsymbol{r}_0) &= \frac{1}{2}\mathrm{Im}G(0, y; \boldsymbol{r}_0) \\ &= -2\tan^{-1}\frac{\cos\pi y - \cosh\pi x_0 \cos\pi y_0}{\sinh\pi x_0 \sin\pi y_0}, \end{aligned} \tag{15}$$

where an arbitrary constant is omitted.

It is worth observing that at large vortex–junction separations, $x_0 \gg 1$, this contribution is a constant which does not depend on x_0:

$$\varphi_v(y; \boldsymbol{r}_0) = \pi(2y_0 - 1) + \mathcal{O}(e^{-\pi x_0}); \tag{16}$$

in other words, corrections to this constant are exponentially small with the length scale W/π.

3. The critical current $I_c(H, \boldsymbol{r}_0)$

The total phase difference at the junction is

$$\varphi(y; H, \boldsymbol{r}_0) = \varphi_H(y) + \varphi_v(y; \boldsymbol{r}_0) + \varphi_0. \tag{17}$$

The field induced phase difference $\varphi_H(y)$ is twice as large as $\varphi_H(+0, y)$ which was evaluated for a half-strip in Eq. (13) because both right and left half-strips contribute equally.

The Josephson current density $g_c\sin\varphi(y)$ integrated over the junction length gives the total current I:

$$\frac{I(H, \boldsymbol{r}_0)}{g_cW} = A\cos\varphi_0 + B\sin\varphi_0, \tag{18}$$

$$A = \int_0^1 \sin(\varphi_H + \varphi_v)dy, \quad B = \int_0^1 \cos(\varphi_H + \varphi_v)dy. \tag{19}$$

The right-hand side of Eq. (19) is easily transformed to

$$\sqrt{A^2 + B^2}\cos(\varphi_0 - \psi), \quad \psi = \sin^{-1}\frac{B}{\sqrt{A^2 + B^2}}. \tag{20}$$

Maximizing this relative to the free parameter φ_0 one obtains the normalized critical current:

$$J_c = \frac{I_c(H, \mathbf{r}_0)}{g_c W} = \sqrt{A^2 + B^2}. \tag{21}$$

It is worth noting that $\varphi_H(y)$ is an odd function relative to the strip middle, whereas for a general vortex position $\varphi_v(y)$ is neither odd nor even unless $y_0 = 1/2$. In the latter case $\varphi_v(y)$ of Eq. (15) is also odd relative to the strip middle; as a result $A = 0$ and $J_c = |B|$.

It is readily seen that the critical current Eq. (21) can also be written as

$$J_c = \left| \int_0^1 e^{i(\varphi_H + \varphi_v)} dy \right|. \tag{22}$$

In some situations this form of J_c is more convenient.

Below, we consider a few cases of interest.

3.1. No vortex is present

The normalized critical current $J_c = I_c/g_c W$ evaluated with the help of Eqs. (19) and (21) is shown in Fig. 2 versus reduced field $h = 4HW^2/\phi_0$. As expected, in vortex absence, $J_c(h)$ is symmetric with respect to $h = 0$ at which J_c reaches maximum, the behavior characteristic of 0-type junctions.

It is worth mentioning that $J_c(h)$ of planar thin film junctions differs from the common Fraunhofer pattern. Keeping only the term with $\cos \pi y$ in Eq. (11) for $\mathcal{I}$ one has $J_c(h) = J_0(0.43\, h)$, where J_0 is the Bessel function of the first kind. In particular, the pattern maxima go as $1/\sqrt{h}$ at large h, whereas zeros have periodicity $\Delta h = \pi/0.43$ which corresponds to $\Delta H = \phi_0/A_{\text{eff}}$ with the effective area $A_{\text{eff}} \approx 0.55 W^2$ [9,10].

3.2. Vortex is near the strip edges

If the vortex is near $y_0 = 0, 1$, its contribution Eq. (15) to the phase difference is $\varphi_v = \pi$ and is independent of the junction–vortex separation. Physical reasons for this are discussed in [7]. Clearly, the tunneling current is not affected, and $J_c(h)$ is the same as in the vortex absence.

3.3. Vortex is far from the junction, $x_0 > 2$

In this situation the vortex contribution to the phase difference at the junction is a y independent constant given in Eq. (16). Then Eq. (22) shows that the vortex has no effect on the pattern $J_c(h)$. We thus conclude that the vortex at a distance $x_0 > 2W$ does not affect the pattern $J_c(h)$ of the junction.

3.4. Zero-field $J_c(0, \mathbf{r}_0)$

One of the relevant characteristics of the pattern $J_c(h)$ is the value of zero-field critical current $J_c(0)$. In particular, $J_c(0) = 0$ signals a qualitative difference of the junction from the '0-type'. To find $J_c(0)$ we start with Eq. (22) with $\varphi_H = 0$. The vortex factor $e^{i\varphi_v}$ can be transformed using the logarithmic form of the inverse tangent in Eq. (15) for φ_v:

$$\varphi_v = -2\tan^{-1} u = -i \ln \frac{i+u}{i-u}, \tag{23}$$

$$u = \frac{\cos \pi y - \cosh \pi x_0 \cos \pi y_0}{\sinh \pi x_0 \sin \pi y_0}. \tag{24}$$

We thus obtain

$$J_c(0) = \left| \int_0^1 \frac{i+u}{i-u} dy \right|. \tag{25}$$

One can go here to integration over u:

$$u = C\cos \pi y - D,$$
$$C = \frac{1}{\sinh \pi x_0 \sin \pi y_0}, \quad D = \frac{\cosh \pi x_0 \cos \pi y_0}{\sinh \pi x_0 \sin \pi y_0}, \tag{26}$$

and obtain

$$J_c(0, \mathbf{r}_0) = \frac{1}{\pi} \left| \int_{u_1}^{u_2} \frac{du}{\sqrt{(u-u_1)(u-u_2)}} \frac{i+u}{i-u} \right|,$$
$$u_1 = -C - D, \qquad u_2 = C - D. \tag{27}$$

3.5. The vortex in the strip middle

It is shown in this section that a vortex at some positions at the strip middle has an exclusive property to cause a shift in the pattern $J_c(h)$ so that instead of maximum at $h = 0$, $J_c(0)$ is zero, Fig. 3, the feature commonly ascribed to $(0, \pi)$ junctions.

To find these positions we note that for $y_0 = 1/2$, $C = 1/\sinh \pi x_0$ and $D = 0$. The integral in Eq. (27) then takes the form

$$\mathcal{J} = \int_{-C}^{C} \frac{du}{\sqrt{u^2 - C^2}} \frac{i+u}{i-u} = i \int_0^{\pi} \frac{i + C\cos v}{i - C\cos v} dv, \tag{28}$$

where the substitution $u = C\cos v$ has been used. The last integral here can be written as $\int_0^{2\pi} dv/2$ since only $\cos v$ enters the integrand. Further substitution $z = e^{iv}$ transforms the integral to a contour integral over the unit circle in the complex plane z:

$$\mathcal{J} = \frac{1}{2} \oint \frac{dz}{z} \frac{z^2 + 2iz/C + 1}{z^2 - 2iz/C + 1}. \tag{29}$$

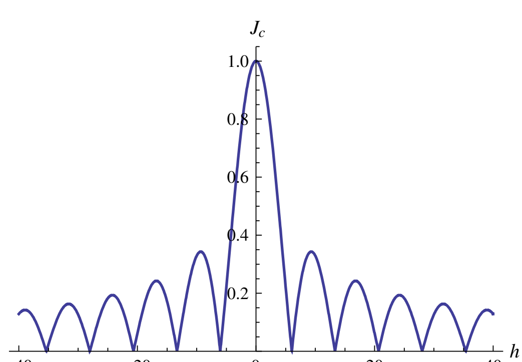

Fig. 2. The normalized Josephson critical current $J_c = I_c/g_c W$ vs $h = 4HW^2/\phi_0$ in the vortex absence.

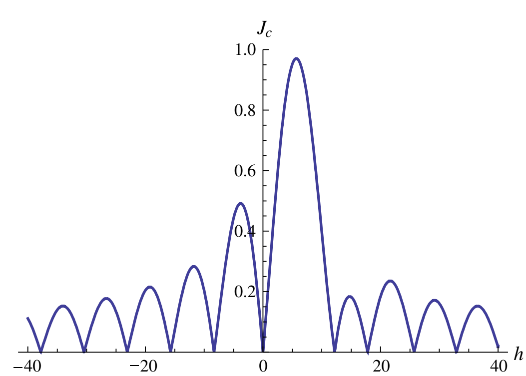

Fig. 3. The normalized Josephson critical current $J_c(h)$ for the vortex situated close to the junction in the strip middle: $x_0 = 0.175, y_0 = 0.5$.

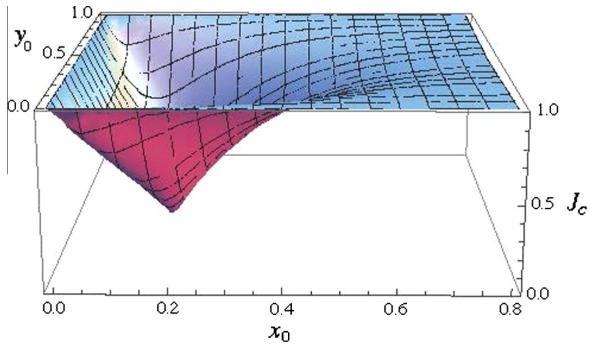

Fig. 4. The normalized zero-field Josephson critical currents $J_c(0, x_0, y_0)$. The sharp minimum corresponds to $J_c(0, 0.175, 0.5) = 0$. It is seen that this zero is isolated and no other zeros of $J_c(0)$ exist for a single vortex at this point.

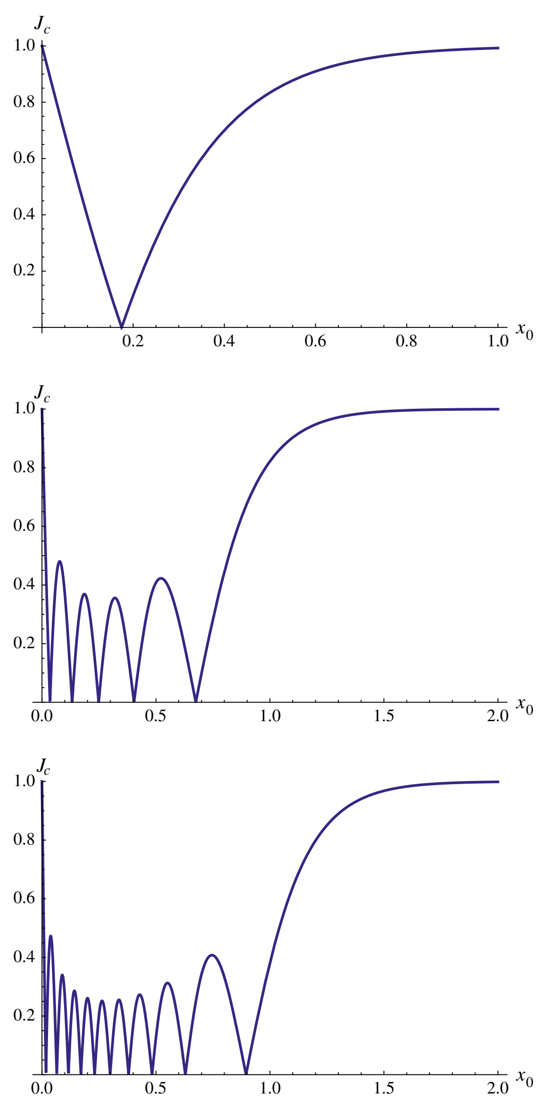

Fig. 5. The normalized zero-field Josephson critical currents $J_c(0)$ for vortices in the strip middle $y_0 = 0.5$ as a function of x_0: the upper panel is for $N = 1$, the middle panel $N = 5$, and the lowest panel $N = 10$. Roughly, the intervals $\Delta x_0 \propto 1/(N + 1 - n)$ where n is the number of the zero counted from $x_0 = 0$.

The product of the roots of $z^2 - 2iz/C + 1 = 0$ is unity, hence only one of them is inside the unit circle. Then one readily obtains

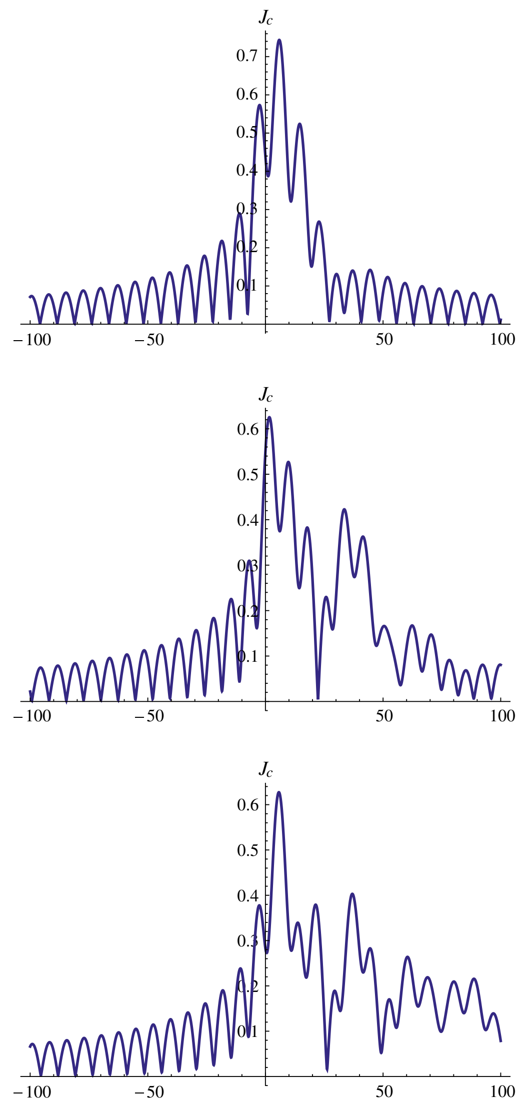

Fig. 6. The upper panel: normalized Josephson critical currents $J_c(h)$ for a vortex at $x_0 = 0.1, y_0 = 0.3$. The middle panel: the same for 3 vortices trapped at the same location. The lowest panel: the same for 5 vortices trapped at the same location.

$$\mathcal{J} = -i\pi\left(1 - \frac{2}{\sqrt{1 + C^2}}\right) = -i\pi(1 - 2\tanh \pi x_0). \tag{30}$$

Thus, the zero-field critical current for a vortex at $(x_0, 1/2)$ is [6]:

$$J_c(0, x_0, 1/2) = |1 - 2\tanh(\pi x_0)|. \tag{31}$$

It is seen that $J_c(0, x_0, 1/2)$ has only one root $x_0 \approx 0.175$. At $x_0 = 0$ and approximately for $x_0 > 2$ $J_c(0, x_0, 1/2) = 1$ in agreement with the earlier conclusion that the far-away vortex does not matter for $J_c(h)$. Moreover, the point $(0.175, 0.5)$ of the plane (x_0, y_0) is the only one (for a single vortex) where $J_c(0, x_0, y_0) = 0$. This is seen in Fig. 4 where $J_c(0, x_0, y_0)$ is evaluated numerically using Eq. (27).

If N vortices are trapped at the same point $\boldsymbol{r}_0 = (x_0, y_0)$, the vortex phase Eq. (23) acquires a factor N. As a result one has to replace the factor $(i + u)/(i - u)$ in Eqs. (25), (27), (28) with $(i + u)^N/(i - u)^N$. In turn, this leads to a pole of the order N inside the unit circle in integration over z. In principle, one can proceed with analytical evaluation, but the result is increasingly cumbersome with increasing N. We resort then to numerical evaluation of $J_c(0, x_0, 1/2)$ examples of which are shown in Fig. 5. It is seen that the number of positions x_0 for which the pattern $J_c(h)$ has zero at

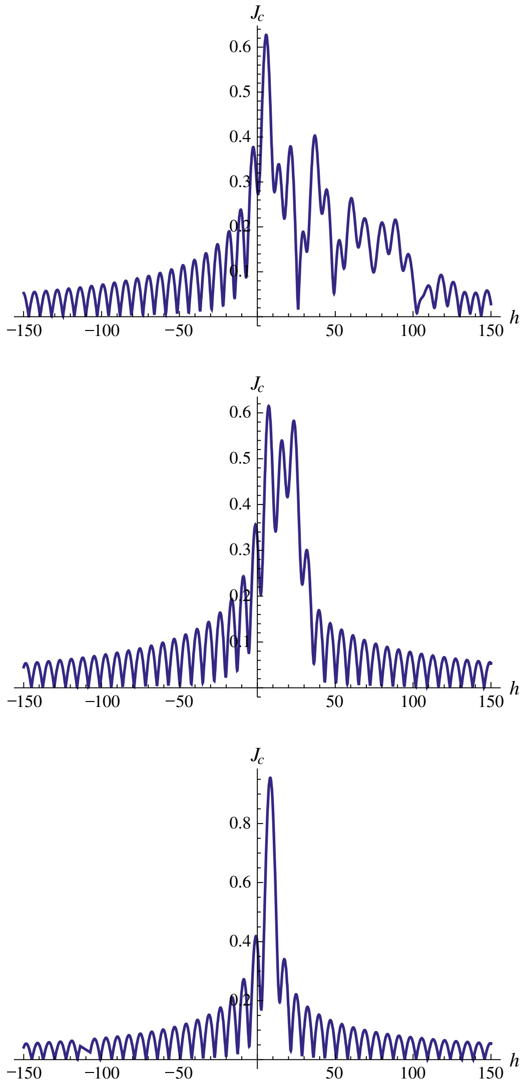

Fig. 7. The upper panel: normalized Josephson critical currents $J_c(h)$ for five vortices $x_0 = 0.1, y_0 = 0.3$. The middle panel: the same for $\boldsymbol{r}_0 = (0.3, 0.3)$. The lowest panel: 5 vortices trapped at $\boldsymbol{r}_0 = (0.5, 0.3)$.

$h = 0$ is equal to the number of vortices trapped at x_0. The density of these points also increases with N, so that for large number of vortices trapped, nearly any place x_0 of the trap in the interval $0 < x_0 \lesssim 2$ will make the pattern $J_c(h, x_0, 1/2)$ to have near-zero at $h = 0$. The upper bound of this interval is related to the fact that for $x_0 \gtrsim 2$ the vortex effects upon the pattern $J_c(h)$ vanish and $J_c(0)$approaches unity exponentially as is seen from Eq. (31).

3.6. Arbitrary position of a nearby vortex

The upper panel of Fig. 6 shows $J_c(h)$ for a sigle vortex at $x_0 = 0.1, y_0 = 0.3$. Characteristic features of this $J_c(h)$ are the presence of non-zero minima and a strong asymmetry of the pattern relative to $h \to -h$, the manifestation of different possibilities for superpositions of the screening and vortex currents in the junction vicinity. The effect of a vortex is strongest on the side of positive h. This is seen better yet if two vortices are trapped at the same position $\boldsymbol{r}_0 = (0.1, 0.3)$, the middle panel, or five shown in the lowest panel. Note that the pattern at $h < 0$ is well ordered with a repetition step $\Delta h \approx 7.1$ which corresponds to $\Delta H \approx 1.8\phi_0/W^2$ as should be for a pattern caused exclusively by the applied field H [10,9,6].

One can see in Fig. 7 an example of $J_c(h)$ for 5 vortices trapped at the same transverse coordinate $y_0 = 0.3$ but at increasing distances from the junction $x_0 = 0.1$, 0.3, 0.5. We have chosen a broader domain $|h| < 150$ to show that vortex effects on the right side of the pattern persist up to a large value of h, which however decreases with increasing separation.

If the vortex approaches the strip edges $y_0 = 0$ or 1, $J_c(h)$ approaches the pattern shown in Fig. 2 for no vortices. As argued in [7], in this case the vortex causes the junction phase difference to acquire an extra π, which does not change the tunneling current, but affects the junction energy.

4. Discussion

We have shown that a vortex at one of the banks of the plane thin-film Josephson junction distorts the pattern of the field dependent critical current $J_c(h)$ in a strongly asymmetric way: as is seen in Figs. 3, 6, 7, the distortion at the side $h > 0$ for a vortex is strong, whereas for $h < 0$ it is weak and more regular (for anti-vortex the picture flips relative to $h \to -h$). Actually, this asymmetry is seen in experiment [4].

We also show that the vortex effect upon the pattern $J_c(h)$ disappears exponentially when the junction–vortex separation $x_0 \gtrsim 2W$ with the length scale W/π. This, however, does not mean that the junction "does not feel" the far-away vortex; as Eq. (16) shows, the junction phase difference acquires a constant addition dependent on the transverse vortex coordinate y_0 [7]. Hence, the junction energy influenced by the vortex for all junction–vortex separations.

In principle, effects discussed here open possibilities to manipulate properties of Josephson junctions by trapping vortices in junction banks. We identified a number of vortex positions $(x_0, 1/2)$ for which the zero-field critical current $J_c(0)$ turns zero. Hence, by measuring $J_c(0)$ one can say whether or not one of these positions $(x_0, 1/2)$ is occupied by a vortex, an interesting possibility for applications.

Our calculations are valid for sufficiently thin and narrow superconducting strips for which the condition $W \ll \Lambda$, the Pearl length, is satisfied. This condition allows us to disregard the magnetic energy of supercurrents relative to their kinetic energy. For other types of junctions (e.g., made of thick overlapping films) our solutions *per se* do not apply.

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