Narrow Josephson junctions in thin films

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Abstract. We consider the edge-type Josephson junctions in thin films, for which the stray fields significantly affect the screening and tunneling currents. It is demonstrated that the spatial distribution of the phase difference φ across thin-film Josephson junctions is nonlocal. We find that in the limit of weak tunneling and short junctions the phase difference φ is a universal function. This function is proportional to the applied field H an depends only on the junction geometry. In the case of narrow thin strips we find this dependence analytically. Using this universal function we demonstrate that the maximum supercurrent across narrow junctions in thin films decays as $1/\sqrt{H}$, that is much slower than $1/H$ for bulk junctions.

1. Introduction

The interest in physics of edge-type Josephson junctions in high temperature superconducting thin films (see Fig. 1) is driven by experiments probing the basic features of the Josephson grain boundaries and, in particular, the symmetry of the order parameter [1-5]. Significant effect on the basic properties of these junctions is caused by stray fields outside the sample. These fields might result in nonlocal effects in the spatial distribution of phase difference φ across the junction [6-12]. In particular, non-local fluxons were observed [13] in junctions wide compared to the Pearl length $\Lambda = 2\lambda^2/d$ (λ is the London penetration depth, d is the film thickness). Under similar conditions, existence of non-local Josephson vortices carrying fractional flux has been predicted [10]. However, transport properties of nonlocal tunnel junctions were not discussed theoretically.

In this paper we present a model for evaluating maximum supercurrents in junction containing thin-film superconducting strips. The model enables us to evaluate the phase distribution in the junction for low tunneling rates and narrow strips, for which the phase distribution turns out to be universal.

2. The model

The edge-type junction in a thin strip of our interest is shown in Fig. 1. The system of coordinates is defined in the figure so that $0 < y < W$, $0 < z < d$. The London penetration depth $\lambda \gg d$, therefore, the thin film screening length is given by $\Lambda = 2\lambda^2/d \gg \lambda$. The Josephson tunneling sheet current is $\mathbf{g} = \mathbf{g}_c \sin\varphi$, where $\mathbf{g}_c$ is the critical sheet current. In the edge-type thin-film junctions the Josephson penetration depth ℓ takes the form [12]

$$\ell = c\phi_0/8\pi^2\Lambda g_c\,. \tag{1}$$

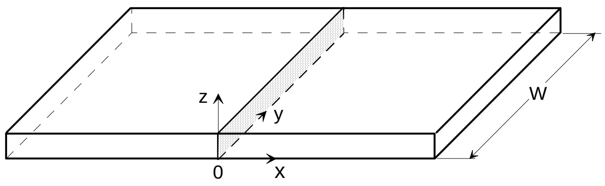

Figure 1. Sketch of a strip with an edge-type junction at $x = 0$.

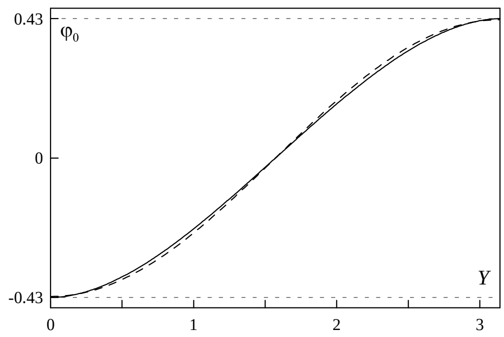

Figure 2. The solid line is $\varphi_0(Y)$, which is calculated by the finite element method. The dashed line shows $\varphi_0 = -0.43\,\cos Y$.

Next, we introduce the stream function $S(x, y)$ so that the sheet current $\mathbf{g} = (g_x, g_y)$ can be written as $\mathbf{g} = \mathrm{curl}\,(S\,\hat{\mathbf{z}}) = (\partial_y S, -\partial_x S)$. It follows from the Maxwell and London equations that the equilibrium current distribution in a thin-film narrow strip ($W \ll \Lambda$) is given by [12, 9]

$$-\frac{h_z}{\phi_0} + \frac{2\pi\Lambda}{c\phi_0}\nabla^2 S = -\frac{1}{2\pi}\delta(x)\partial_y\varphi(y)\,, \tag{2}$$

where h_z is the normal component of the field at the strip surface. One can see that for $W \ll \Lambda$ the self-fields of the currents in the strip can be discarded and h_z can be replaced with the applied field H. The boundary conditions for the stream function S are given in terms of the total current running through the strip,

$$I = \int_0^W \partial_y S dy = S(W) - S(0) = S(W)\,, \tag{3}$$

where we take $S(0) = 0$. The solution of the linear equation (2) is $S = S_\varphi + S_H$ with

$$S_\varphi = \frac{c\phi_0}{8\pi^3\Lambda}\int_0^W \varphi'(v)G(x, y; 0, v)dv, \qquad S_H = \frac{cH}{4\pi\Lambda}y(y - W) + I\frac{y}{W}\,, \tag{4}$$

where

$$G = \tanh^{-1}\frac{\sin V \sin Y}{\cosh X - \cos V \cos Y}. \tag{5}$$

is the Green's function for zero boundary conditions and $(Y, V) = (\pi y/W, \pi v/W)$ (see [14] for details). The stream function S_H describes the screening currents generated by the applied field and the uniform transport current, whereas S_φ describes the current perturbations due to the junction.

The current through the junction is $g_x(0, y) = g_c \sin\varphi(y) = \partial_y S(0, y)$. This results in the integral equation for the phase $\varphi(y)$:

$$\frac{W}{\ell}\sin\varphi = \int_0^\pi \frac{dV\,\varphi'(V)\,\sin V}{\cos Y - \cos V} + h\left(Y - \frac{\pi}{2}\right) + i\,, \tag{6}$$

where

$$\ell = \frac{c\phi_0}{8\pi^2\Lambda g_c}\,, \quad h = \frac{4W^2}{\phi_0}H\,, \quad i = \frac{8\pi^2\Lambda}{c\phi_0}I \tag{7}$$

are the characteristic length, reduced field and current.

The boundary conditions for the integral equation (7) are

$$\varphi'(0) = \varphi'(W) = 0\,; \tag{8}$$

they follow from the fact that $\varphi' \propto g_y(0, y)$, which vanish at the edges [12].

For narrow strips with $W \ll \ell$, the tunneling current and the total current in Eq. (7) can be neglected in the lowest approximation. The phase is then given by

$$\int_0^\pi \frac{dV\,\varphi'(V)\,\sin V}{\cos Y - \cos V} = h\left(\frac{\pi}{2} - Y\right)\,. \tag{9}$$

Hence, the function $f(Y) = \varphi(Y)/h$ depends on the geometry of the problem only. Eq. (9) can be inverted, see [14, 15]. Numerically, $f(Y)$ can be found by minimizing the functional

$$\mathcal{F} = \int_0^\pi \int_0^\pi f'(V)G(0,Y;0,V)f'(Y)dVdY + \int_0^\pi Y(Y-\pi/2)f'(Y)dY. \tag{10}$$

This can be done using the *finite element* method [16]. The universal phase $\varphi(Y)$ calculated using this approach is shown in Fig. 2.

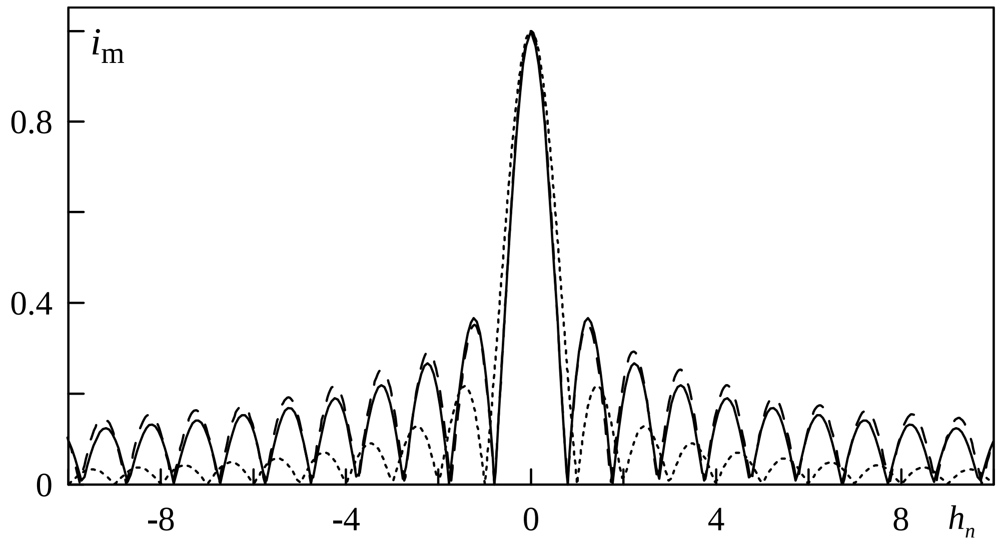

Figure 3. The maximum current $i_m = I_m/g_cW$ as a function of the normalized applied field $h_n = 4a_0W^2H/\pi\phi_0$. The dashed line is the approximation (12) and the dotted line is the maximum current for bulk junctions.

3. The maximum current

The maximum supercurrent across Josephson junction is

$$I_m(h) = \max\left\{\int g_c(y)\sin\left[hf_0(y) + \theta\right]dy\right\}. \tag{11}$$

The result of this calculation for a constant critical sheet current g_c is shown in Fig. 3.

There are two major differences between the maximal supercurrent of edge-type thin-film junctions and junctions in the bulk. First, the zeros $I_m(H)$ for bulk junctions are periodic with the period $\phi_0/2\lambda W$. In the edge-type thin-film junctions the zeros are equidistant only in large fields where the period is $\approx 1.8\phi_0/W^2$. Second, maxima of $I_m(H)$ for bulk junctions decay as $1/H$. For edge-type thin-film junctions, the maximal current decays proportionally to $1/\sqrt{H}$.

Our numerical study shows that with the accuracy better than 2% the universal phase can be approximated by the function

$$\varphi(Y) \simeq -a_0 h \cos Y, \quad a_0 = 0.43. \tag{12}$$

In this case the maximal current I_m for junctions with constant g_c has a simple form

$$I_m(h) = J_0(a_0 h)\,, \tag{13}$$

where J_0 is the zero order Bessel function. The current (13) is shown in Fig. 3. It is seen that the approximate analytical and numerical results nearly coincide.

4. Conclusions

We study the field dependence of the maximal supercurrent $I_m(H)$ in the edge-type Josephson junctions in thin-film strips. The strip width W is supposed to be much less than the Pearl length Λ and the thin-film Josephson length ℓ. We find that due to the zero slope of the phase at the edges of the junction, the maxima of $I_m(H)$ decay asymptotically as $1/\sqrt{H}$. This behavior is a signature of nonlocality caused by the stray fields and is very different from the standard Fraunhofer pattern $I_m(H) \propto 1/H$ observed in Josephson tunnel junctions in the bulk. In addition, we find that the nonlocality of the problem affects the distance between the zeroes of $I_m(H)$: in the edge-type thin-film junctions zeros are spaced by $\Delta H \sim \phi_0/W^2$, which is very different from standard Fraunhofer patterns for bulk junctions.

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