Shapiro steps in Josephson junctions with alternating critical current density

M. Moshe and R. G. Mints*

School of Physics and Astronomy, Raymond and Beverly Sackler Faculty of Exact Sciences, Tel Aviv University, Tel Aviv 69978, Israel
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We treat theoretically Shapiro steps in tunnel Josephson junctions with spatially alternating critical current density. Explicit analytical formulas for the width of the first integer (normal) and half-integer (anomalous) Shapiro steps are derived for short junctions. We develop a coarse-graining approach, which describes Shapiro steps in the voltage-current curves of the asymmetric grain boundaries in yttrium barium copper oxide thin films and different superconductor-ferromagnet-superconductor Josephson-type heterostructures.

I. INTRODUCTION

Series of resonances exist in Josephson tunneling junctions biased at an alternating voltage $V(t) = V_0 + V_1 \cos \omega_1 t$.[1–3] In junctions of conventional superconductors, these resonances appear at the frequencies $\omega_0 = 2eV_0/\hbar = n\omega_1$, where n is an integer. At the resonant values of the dc voltage $V_0 = n\hbar\omega_1/2e$, the supercurrent has a dc component. In the voltage-current curve, Shapiro resonances reveal themselves as a "ladder" of equidistant values of V_0 (*integer* Shapiro steps).[1–3]

The physical origin of Shapiro steps follows from the Josephson equations $j = j_c \sin \varphi$ and $\dot{\varphi} = 2eV/\hbar$, where j and j_c are the tunneling and critical current densities and φ is the phase difference across the junction. In order to find the current across the junction we integrate the voltage $V(t)$ and find the time-dependent phase $\varphi(t) = \varphi_0 + \omega_0 t + v_1 \sin \omega_1 t$. In this relation, $\varphi_0 = \varphi(0)$ is the initial value of the phase and $v_1 = 2eV_1/\hbar\omega_1$ is the dimensionless parameter of the problem. Knowing $\varphi(t)$, we obtain the tunneling current density in the form $j(t) = j_c \sin(\varphi_0 + \omega_0 t + v_1 \sin \omega_1 t)$. This formula demonstrates that $j(t)$ is a complex alternating function of time. Fortunately, $j(t)$ can be transferred into a series allowing for an easy qualitative and quantitative analysis,[2,3]

$$j = j_c \sum_{n=-\infty}^{\infty} (-1)^n J_n(v_1) \sin[\varphi_0 + (\omega_0 - n\omega_1)t], \quad (1)$$

where $J_n(x)$ is the first kind Bessel function of order n.

It is seen from Eq. (1) that for any resonant frequency $\omega_0 = n\omega_1$, the supercurrent density j has a dc component $\propto \sin \varphi_0$, which reaches its maximum j_m at $\varphi_0 = \pi/2$. As a result, the maximum value of the dc density is given by $j_m = j_c J_n(v_1)$.[2,3]

Anomalous Shapiro steps at the subharmonic resonant frequencies $\omega_0 = (n/q)\omega_1$ ($q > n$, where q and n are integers) were treated theoretically for short microbridges of conventional superconductors assuming that the current-phase relation includes high-order harmonic terms.[4] Qualitatively, the effect of terms $j_q \sin(q\varphi)$ on the supercurrent density j follows from Eq. (1). Indeed, substituting φ_0, ω_0, and v_1 by $q\varphi_0$, $q\omega_0$, and qv_1, we find that at any subharmonic frequency $\omega_0 = (n/q)\omega_1$, the supercurrent density j has a dc component $\propto \sin q\varphi_0$. In this case, the maximum value of j

corresponds to the phase $\varphi_0 = \pi/2q$ and is given by $j_m = j_q J_n(qv_1)$.

Observation of anomalous half-integer Shapiro steps has been reported recently for asymmetric grain boundaries in yttrium barium copper oxide (YBCO) films[5] and superconductor-ferromagnet-superconductor (SFS) heterostructures.[6–8]

Qualitative explanations of the origin of the anomalous Shapiro steps were proposed to understand the results of these experiments mainly by revealing the existence of a second harmonic term, $\sin 2\varphi$, in the current-phase relation (CPR). It was shown that data measured for the asymmetric grain boundaries in YBCO are indeed consistent with the existence of the term $\sin 2\varphi$ in the CPR.[9–11] In the case of SFS junctions, it was assumed that there is doubling of the Josephson frequency, which leads to a $\sin 2\varphi$ term in the CPR. It was suggested that this frequency doubling is caused by the splitting of energy levels in the ferromagnetic exchange field.[6] An alternative explanation[8] assumes the existence of a resonance between the spontaneous currents and rf modulation.[12]

Asymmetric grain boundary junctions in YBCO thin films and SFS heterostructures are arranged in sequences of interchanging 0- and π-shift tunneling junctions, as shown in Fig. 1. π shifts spontaneously appearing across superconducting banks of tunneling junctions were first considered for SFS heterostructures.[13,14] These π shifts lead to negatively biased critical current density j_c and anomalous Josephson proper-
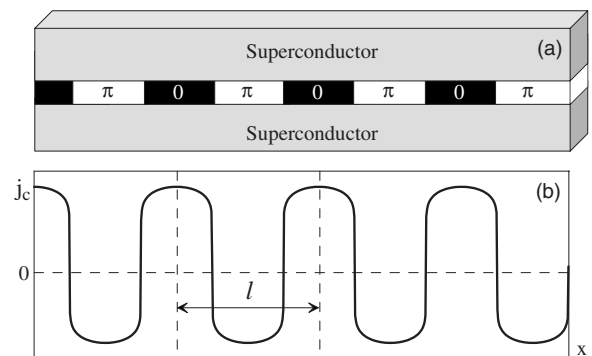

FIG. 1. Schematic diagrams: (a) Josephson junction made of periodically interchanging 0- and π-biased fragments and (b) alternating critical current density distribution.

ties of sequences of 0-π-biased tunneling junctions.[15–20] The observed anomalies are especially significant if the lengths of 0 and π fragments are much less than the Josephson penetration depth λ_J defined by the spatial average of $|j_c|$.[21–24]

Thus, several qualitative explanations have been put forward to understand the physics of fractional Shapiro steps. However, to the best of our knowledge, there is, at present, no theoretical model describing the anomalous Shapiro steps and their basic features in Josephson junctions with alternating critical current density.

In this paper, we treat Shapiro steps in Josephson junctions arranged in periodic or almost periodic sequences of interchanging 0- and π-biased fragments with $l \ll \lambda_J$. We develop a coarse-graining approach which is applicable to treat time-dependent phenomena.

In our theoretical model, we assume that the tunneling current density is given by the *standard* CPR dependence $j = j_c(x) \sin \varphi(x,t)$, but with a critical current density $j_c(x)$, spatially alternating along the Josephson junction. As an illustration, we treat the case of a short junction that allows us to obtain explicit analytical results. We demonstrate that in Josephson junctions with spatially alternating critical current density, the half-integer (anomalous) Shapiro steps exist in addition to the integer (normal) Shapiro steps. We calculate the dependence of the width of these steps on the flux inside the junction and the voltage across the junction.

The organization of this paper is as follows. In Sec. II, we review briefly the coarse-graining approach to Josephson junctions with spatially alternating critical density. In Sec. III, this general approach is applied to calculate explicit formulas for the width of the integer (normal) and half-integer (anomalous) Shapiro steps in short junctions. Section IV discusses these results.

II. JOSEPHSON JUNCTION WITH SPATIALLY ALTERNATING CRITICAL CURRENT DENSITY

Consider a tunneling junction of length L ($0 \leq x \leq L$) with a critical current density $j_c(x)$ alternating with a length scale $l \gg \lambda$, where λ is the London penetration depth (see Fig. 1 for the geometry of the problem). It is convenient for the following calculations to write

$$j_c(x) = \langle j_c \rangle [1 + g(x)], \qquad (2)$$

where $\langle j_c \rangle$ is the average value of $j_c(x)$ defined as

$$\langle f \rangle = \frac{1}{L} \int_0^L dx\, f(x). \qquad (3)$$

The function $g(x)$ describes spatial variations of $j_c(x)$ and has a zero average value, $\langle g(x) \rangle = 0$. In what follows, we are interested in the case of small average values of the critical current density, i.e., we assume that $\langle j_c(x) \rangle \ll \langle |j_c(x)| \rangle$. This condition means that the typical value of the dimensionless function $g(x)$ is big compared to unity or in other words $\langle |g(x)| \rangle \gg 1$.

In the above notations the equation for the phase difference $\varphi(x,t)$ across the junction reads

$$\tau^2 \ddot{\varphi} - \Lambda^2 \varphi'' + [1 + g(x)] \sin \varphi = 0, \qquad (4)$$

where $1/\tau$ is the Josephson frequency and Λ is the effective Josephson penetration depth,

$$\Lambda^2 = \frac{c \phi_0}{16 \pi^2 \lambda \langle j_c \rangle}. \qquad (5)$$

We now treat a sample subjected to magnetic field H_a and alternating voltage $V(t) = V_0 + V_1 \cos \omega_1 t$ applied across the junctions. It is assumed that $2eV_1$ is small compared to $\hbar \omega_1$, i.e., $v_1 = 2eV_1/\hbar \omega_1 \ll 1$. In this case, coarse graining can be applied to solve Eq. (4).[23]

Two types of terms appear in Eq. (4): terms alternating over the length l and smooth terms varying over the length $\Lambda \gg l$. The fast alternating terms cancel each other independently of the smooth terms, which also cancel each other. Therefore, we write[23]

$$\varphi(x) = \psi(x) + \xi(x), \qquad (6)$$

where $\psi(x)$ is a smooth function with the length scale of order Λ and $\xi(x)$ alternates with the length scale of order l. Under the above assumptions, the average value of $\xi(x)$ is zero and the typical amplitude of variations of $\xi(x)$ is small, i.e., $\langle \xi(x) \rangle = 0$ and $\langle |\xi(x)| \rangle \ll 1$.[23–26]

Substituting Eq. (6) into Eq. (4) and keeping terms up to first order in $\xi(x)$, we find[23]

$$\Lambda^2 \psi'' - \frac{j_\psi(x)}{\langle j_c \rangle} = 0, \qquad (7)$$

$$\Lambda^2 \xi'' - \frac{j_\xi(x)}{\langle j_c \rangle} = 0, \qquad (8)$$

where the smooth $j_\psi(x)$ and alternating $j_\xi(x)$ components of the tunneling current density $j = j_\psi + j_\xi$ are

$$j_\psi = \langle j_c \rangle (\sin \psi - \gamma \sin \psi \cos \psi), \qquad (9)$$

$$j_\xi = \langle j_c \rangle g(x) \sin \psi. \qquad (10)$$

The dimensionless constant γ is equal to

$$\gamma = \langle g(x) \xi_g(x) \rangle = \Lambda^2 \langle \xi_g'^2(x) \rangle > 0, \qquad (11)$$

and the rapidly alternating phase $\xi_g(x)$ is defined by

$$\xi(x) = -\xi_g(x) \sin \psi. \qquad (12)$$

It follows from Eqs. (8), (10), and (12) that

$$\Lambda^2 \xi_g'' + g(x) = 0, \qquad (13)$$

i.e., the rapidly alternating phase shift $\xi_g(x)$ is a characteristic of the sample.

To summarize the coarse-graining approach it is worth noting that the typical values of $\xi_g(x)$ are small, but at the same time the typical values of $g(x)$ are big, i.e., $\langle |\xi_g(x)| \rangle \ll 1$ and $\langle |g(x)| \rangle \gg 1$. As a result, the dimensionless parameter γ, which is proportional to the average of the product of the two rapidly alternating functions $\xi_g(x)$ and $g(x)$, might be of the order of unity.[23,24] The value of ξ_g' can be estimated as

$\xi'_g \sim \xi_g/l$. It follows then from Eq. (11) that $\langle \xi'^2_g \rangle \sim \gamma l^2/\Lambda^2 \ll 1$. Therefore, for ξ'_g, we have the estimate $\Lambda \langle |\xi'_g| \rangle \sim \sqrt{\gamma} \sim 1$.

III. SHAPIRO STEPS IN SHORT JUNCTIONS

In a short junction ($L \ll \Lambda$), $\psi(x,t)$ is almost linear in x, the time dependence of $\psi(x,t)$ is given by $V(t)$, and

$$\psi'(x,t) = \frac{4\pi\lambda}{\phi_0} H_i(t), \qquad (14)$$

$$\dot{\psi}(L/2,t) = \frac{2e}{\hbar} V(t), \qquad (15)$$

where $H_i(t)$ is the time-dependent field in the junction.

At the sample edges, the derivative $\varphi' = \psi' + \xi'$ is proportional to the field $H|_{0,L}$ including the self-field generated by the total current I.[27] In the case of a short junction, we obtain

$$\left(\frac{H_a}{H_s} \mp \frac{1}{2} \frac{I}{I_s} \right)_{0,L} = \Lambda \varphi'|_{0,L} = \Lambda \psi'|_{0,L} + \Lambda \xi'|_{0,L}, \quad (16)$$

where $H_s = \phi_0/4\pi\lambda\Lambda$ and $I_s = \langle j_c \rangle \Lambda$. It follows from Eq. (12) that the derivative $\xi' = -\xi'_g \sin\psi$. In order to find ξ'_g, we integrate Eq. (13) from 0 to L and arrive at the relation $\xi'_g(0) = \xi'_g(L)$, which allows us to rewrite the boundary condition [Eq. (16)] in the final form,

$$\left(\frac{H_a}{H_s} \mp \frac{1}{2} \frac{I}{I_s} \right)_{0,L} = \Lambda \psi'|_{0,L} - \alpha \sin\psi|_{0,L}, \qquad (17)$$

where the terms $\alpha \sin\psi|_{0,L}$ are caused by the high-density edge currents and α is a constant. It is worth noting here that α is a characteristic of the sample. Next, using the above estimate for ξ'_g, we obtain

$$\alpha = \Lambda \xi'_g|_{0,L} \sim \sqrt{\gamma} \sim 1. \qquad (18)$$

To summarize the above analysis, we find that in the coarse-graining approach, the Josephson junctions with spatially alternating critical current density and $l \ll \lambda_J$ are characterized by two dimensionless parameters γ (for the inner part of the junction) and α (for the edges of the junction), where $\lambda_J = \sqrt{c\phi_0/16\pi^2\lambda\langle|j_c|\rangle} \ll \Lambda$ is the local Josephson length.

Next, we find the derivative of $\psi'|_{0,L}$ using the first integral of Eq. (7), which can be written as

$$\frac{\Lambda^2}{2} \psi'^2 + \cos\psi - \frac{\gamma}{4} \cos 2\psi = \text{const}. \qquad (19)$$

Finally, we combine Eqs. (17) and (19) relating the total current I, applied field H_a, and phase ψ at the edges. In the case of a short junction, we find

$$I = 2I_s \sin\left(\pi \frac{\phi_i}{\phi_0} \right) \left\{ \alpha \cos\psi_m \right.$$
$$\left. - \frac{L}{2\pi\Lambda} \frac{\phi_0}{\phi_a} \left[1 - \gamma \cos\psi_m \cos\left(\pi\frac{\phi_a}{\phi_0} \right) \right] \sin\psi_m \right\}, \quad (20)$$

$$\phi_i = \phi_a + \frac{\alpha}{2\pi} \frac{L}{\Lambda} \phi_0 \cos\left(\pi\frac{\phi_a}{\phi_0} \right) \sin\psi_m, \qquad (21)$$

$$\psi_m = \psi(L/2,t) = \psi_0 + \omega_0 t + v_1 \sin\omega_1 t, \qquad (22)$$

where the "applied", ϕ_a, and "internal", ϕ_i, fluxes are defined as $\phi_{a,i} = 2L\lambda H_{a,i}$, $\omega_0 = 2eV_0/\hbar$, and ψ_0 is a constant, which is used to maximize the total current I.

Calculation of the width of Shapiro steps similar to the one given by Tinkham[3] leads to the following results: two series of steps appear at frequencies

$$\omega_0 = (n + 1/2)\omega_1 \quad \text{and} \quad \omega_0 = n\omega_1, \qquad (23)$$

where n is an integer.

In the case of low oscillating voltage ($v_1 \ll 1$), we obtain the widths of the first *half-integer*, $I_{1/2}(\phi_a)$, and the first integer, $I_1(\phi_a)$, Shapiro steps in the form

$$I_{1/2} = v_1 I_c \left| \alpha^2 \cos^2\left(\pi\frac{\phi_a}{\phi_0} \right) + \frac{\gamma\phi_0}{2\pi\phi_a} \sin\left(2\pi\frac{\phi_a}{\phi_0} \right) \right|, \quad (24)$$

$$I_1 = v_1 I_c \sqrt{\left(\frac{\alpha\Lambda}{L} \right)^2 + \left(\frac{\phi_0}{\pi\phi_a} \right)^2} \left| \sin\left(\pi\frac{\phi_a}{\phi_0} \right) \right|, \qquad (25)$$

where $I_c = \langle j_c \rangle L$. Explicit formulas (24) and (25) reveal quite a few remarkable features of $I_{1/2}(\phi_a)$ and $I_1(\phi_a)$. In what follows, we discuss them in detail.

IV. RESULTS AND DISCUSSION

The function $I_{1/2}(\phi_a)$ is equal to zero for the *two* series: $\phi_a = (n+1/2)\phi_0$ and $\phi_a = (n - \psi_\theta/\pi)\phi_0$, where n is an integer and ψ_θ is defined by $\tan\psi_\theta = \pi\alpha^2\phi_a/\gamma\phi_0$. The angle ψ_θ depends on ϕ_a; i.e., in general, the roots of equation $I_{1/2}(\phi_a) = 0$ are not equidistant, and the *actually observed* dependence $I_{1/2}(\phi_a)$ is defined by the dimensionless ratio $\psi_a = \gamma/2\pi\alpha^2$.

Next, it follows from Eq. (24) that for $\phi_a = 0$, the width of the first anomalous half-integer step $I_{1/2}(0)$ equals $v_1 I_c(\gamma + \alpha^2)$. In the *low-flux* region ($\phi_a/\phi_0 \ll \psi_a$), the main contribution to $I_{1/2}(\phi_a)$ comes from the second term in Eq. (24). This term originates from the alternating currents flowing across *all* junctions. As a result, in the low-flux region, we have $I_{1/2} \propto (\phi_0/2\pi\phi_a)|\sin(2\pi\phi_a/\phi_0)|$, i.e., $I_{1/2}(\phi_a)$ is described by the Fraunhofer pattern but with a double frequency. In Fig. 2(a), we show $I_{1/2}(\phi_a)$ in the low-flux region ($\phi_a/\phi_0 \ll 250$) for the data $\alpha = 0.03$ and $\gamma = 1.5$, which lead to $\psi_a \approx 250$.

In the *high-flux* region ($\phi_a/\phi_0 \gg \psi_a$), the main contribution to $I_{1/2}$ comes from the first term in Eq. (24). This term originates from the high-density currents flowing across the *edge* junctions. As a result, in the high-flux region, we have $I_{1/2} \propto \cos^2(\pi\phi_a/\phi_0)$, i.e., $I_{1/2}(\phi_a)$ is a periodic function. In Fig. 2(c), we show $I_{1/2}(\phi_a)$ in the high-flux region ($\phi_a/\phi_0 \gg 0.25$) for the data $\alpha = 1$ and $\gamma = 1.5$, which lead to $\psi_a \approx 0.25$.

In Fig. 2(b), we show the function $I_{1/2}(\phi_a)$ for the intermediate values of the applied flux ($\phi_a/\phi_0 \sim \psi_a$). Using the data $\gamma = 1.5$ and $\alpha = 0.3$, we find that $\psi_a \approx 2.5$. In this case, ψ_θ
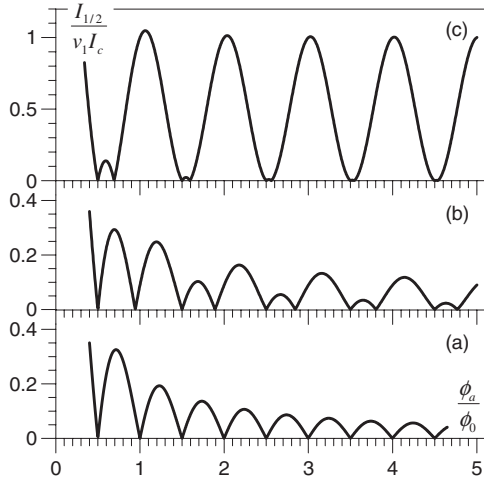
FIG. 2. Dependence of the width of the first half-integer Shapiro step $I_{1/2}$ on applied flux ϕ_a given by Eq. (24) for $\gamma = 1.5$ and (a) $\alpha = 0.03$, (b) $\alpha = 0.3$, and (c) $\alpha = 1$.

is strongly flux dependent. As a result, the function $I_{1/2}(\phi_a/\phi_0)$ is manifestly aperiodic.

We now discuss the width of the first integer Shapiro step I_1. It follows from Eq. (25) that $I_1(\phi_a)$ is equal to zero for the series $\phi_a = n\phi_0$, where n is an integer.

In the high-flux region ($\phi_a/\phi_0 \gg L/\pi\alpha\Lambda$), the main contribution to I_1 comes from the first term in Eq. (25). This term originates from the high-density currents flowing across the edge junctions. As a result, in the high-flux region, we obtain $I_1(\phi_a) \propto |\sin(\pi\phi_a/\phi_0)|$, i.e., $I_1(\phi_a)$ is a periodic function. In Fig. 3(a), we show $I_1(\phi_a)$ in the high-flux region ($\phi_a/\phi_0 \gtrsim 0.2$) for the data $\alpha = 0.3$ and $L/\Lambda = 0.2$, which lead to $L/\pi\alpha\Lambda \approx 0.2$.

Next, it follows from Eq. (25) that the width of the first integer step $I_1(0)$ equals $v_1 I_c$. In the low-flux region ($\phi_a/\phi_0 \ll L/\pi\alpha\Lambda$), the main contribution to I_1 comes from the second term in Eq. (25). This term originates from the alternating currents flowing across all junctions. As a result, in the low-flux region, we obtain $I_{1/2} \propto (\phi_0/\pi\phi_a)\sin(\pi\phi_a/\phi_0)$; i.e., $I_1(\phi_a)$ is described by the Fraunhofer pattern. In Fig. 3(b), we show $I_1(\phi_a)$ in the low-flux region ($\phi_a/\phi_0 \lesssim 20$) for the data $\alpha = 0.003$ and $L/\Lambda = 0.2$, which lead to $L/\pi\alpha\Lambda \approx 20$.
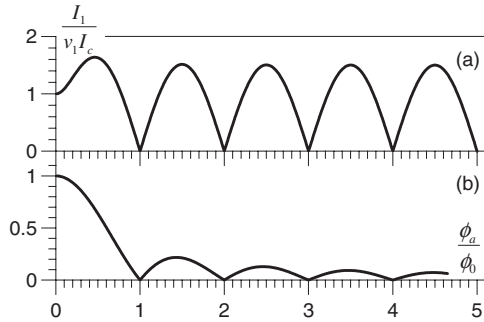

FIG. 3. Dependence of the width of the first integer Shapiro step I_1 on applied flux ϕ_a given by Eq. (25) for $L/\Lambda = 0.2$ and (a) $\alpha = 0.3$ and (b) $\alpha = 0.003$.

Let us now illustrate the above calculations by using a model dependence for alternating critical current density,

$$j_c(x) = j_0 + j_1 \sin\left(\frac{2\pi}{l}x + \theta\right), \tag{26}$$

where $j_0 = \langle j_c(x)\rangle$, j_1 are constants, θ is an angle from the interval $0 \leq \theta \leq \pi$, and $L/l = N$ is an integer ($N \gg 1$). It follows from Eqs. (2) and (13) that

$$g(x) = \frac{j_1}{j_0} \sin\left(\frac{2\pi}{l}x + \theta\right), \tag{27}$$

$$\xi_g(x) = \frac{4\lambda l^2 j_0}{c\phi_0} g(x). \tag{28}$$

Next, knowing $g(x)$ and $\xi_g(x)$, we use Eqs. (11) and (20) and find the parameters

$$\gamma = \frac{2\lambda l^2 j_1^2}{c\phi_0 j_0}, \tag{29}$$

$$\alpha = \sqrt{2\gamma} \cos\theta. \tag{30}$$

It follows from Eqs. (24) and (25) that the widths of the first half-integer $I_{1/2}$ and integer I_1 Shapiro steps are

$$I_{1/2} = \frac{v_1 j_1 L}{\cos\psi_\theta} \frac{l_1^2}{\Lambda_1^2} \left| \cos\left(\frac{\pi\phi_a}{\phi_0}\right)\cos\left(\frac{\pi\phi_a}{\phi_0} - \psi_\theta\right) \right|, \tag{31}$$

$$I_1 = v_1 \sqrt{\left(j_0 L \frac{\phi_0}{\pi\phi_a}\right)^2 + \left(\frac{j_1 l_1}{4\pi}\right)^2} \left| \sin\left(\frac{\pi\phi_a}{\phi_0}\right) \right|, \tag{32}$$

where we define $\tan\psi_\theta = (\phi_0/2\pi\phi_a)/\cos^2\theta$, $l_1 = l\cos\theta$, and $\Lambda_1^2 = c\phi_0/4\lambda j_1$.

An interesting feature follows from Eq. (32) for the limiting case $j_0 = 0$. Using Eq. (32), we find that if the average value of the critical current density $j_0 = 0$, then the width of the first integer Shapiro step equals

$$\tilde{I}_1(\phi_a) = \frac{2eV_1}{\hbar\omega_1} \frac{j_1 l}{2\pi} \cos\theta \left| \sin\left(\frac{\pi\phi_a}{\phi_0}\right) \right|. \tag{33}$$

This current is the contribution coming from the edge fragments, and therefore, the value of $\tilde{I}_1$ does not depend on the total length of the junction L.

V. SUMMARY

The anomalous Shapiro steps in our model exist due to (a) successful interference between the spatial alternations of the critical current density $j_c(x)$ and phase factor $\sin\varphi(x,t)$, leading to generation of the second harmonic in the Josephson current density and (b) high current density at the edges of the junction, resulting in anomalous dependencies $I_{1/2}(H_a)$ in terms of periodicity at low fields and asymptotic behavior at high fields.

To summarize, we demonstrate the existence of anomalous half-integer Shapiro steps for short Josephson junctions

with spatially alternating critical current density $j_c(x)$. We derive explicit formulas given by Eqs. (24) and (25) for the width of the first integer and half-integer Shapiro steps. The general approach is applied to the case of a simple model for the critical current density dependence on the coordinate along the junction [see Eq. (26)]. The results obtained in the framework of this model might be useful for analysis of the experimental data for the asymmetric grain boundaries in YBCO thin films and different superconductor-ferromagnet-superconductor Josephson-type heterostructures.

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*mints@post.tau.ac.il

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