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Josephson torque on junctions between anisotropic superconductors

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Abstract

Properties of Josephson junctions between anisotropic superconductors are reviewed briefly. We focus on the mechanical torque experienced by a tunnel junction and estimate the torque as a small but measurable quantity.

Keywords: Anisotropic superconductors; Josephson vortices; Torque measurement

1. Introduction

The initial stage of flux penetration into a twinned high-T_c superconductor is determined by the vortices located at the twin planes. As a rule the twin planes have Josephson properties. Hence Josephson junctions between anisotropic superconductors are subject of considerable interest.

A tunnel junction between two anisotropic superconductors has been treated by one of us for the case of perfectly aligned anisotropic superconductors [1,2]. It has been shown that for uniaxial superconductors forming a planar tunnel contact the flux penetration starts at a critical field

$$H_{c1} = \frac{\Phi_0}{\pi^2 \lambda_c \Lambda}\sqrt{\frac{k}{\cos^2\alpha + k\sin^2\alpha}}. \tag{1}$$

It is assumed here that the junction plane is parallel to the *ac*-plane, $\Lambda^2 = \Phi_0 c/16\pi^2 j_c \lambda_c$, j_c is the Josephson critical current density, λ_c is the London penetration depth, $k = \lambda_c/\lambda_a \gg 1$ is the anisotropy parameter, and α is the angle between the c-axis and the external magnetic field $\boldsymbol{H}_e$ which is parallel to the junction plane.

It has also been shown in Refs. [1,2] that a single Josephson vortex directed at an angle θ to the c-axis is described by the sine-Gordon equation for the phase difference φ across the junction $\Lambda_J^2\varphi'' - \sin\varphi = 0$, where

$$\Lambda_J^2(\theta) = \Lambda^2(\sin^2\theta + k\cos^2\theta). \tag{2}$$

In the equilibrium, in the vicinity of the critical field H_{c1}, i.e., for $|H_e - H_{c1}| \ll H_e$ the angles θ and α are related by $\tan\theta = k\tan\alpha$ and the line energy of a Josephson vortex ε is [1]

$$\varepsilon = \frac{\Phi_0^2}{4\pi^3\lambda_c\Lambda}\sqrt{\frac{k + k^2\tan^2\alpha}{1 + k^2\tan^2\alpha}}. \tag{3}$$

Thus, in general, the vortices are not parallel to the external magnetic field and the angular dependence of $\varepsilon(\alpha)$ results in a Josephson torque.

2. Josephson torque

The Josephson torque has been calculated for an infinite thick slab with an infinite set of parallel tunnel junctions separated by a distance d [3]. In this case the Gibbs potential is given by

$$G = \frac{\sqrt{k}}{4\pi}H_e\cos\alpha\sqrt{H_0^2 - H_e^2\sin^2\alpha}, \tag{4}$$

where

$$H_0 = \frac{4}{\pi^2 k^{1/3}}\frac{\Phi_0}{\lambda_c\Lambda}. \tag{5}$$

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The Josephson torque $\tau_y(\alpha)$ is then equal to [3]

$$\tau_y = -\frac{\mathrm{d}G}{\mathrm{d}\alpha} = \frac{\sqrt{k}}{4\pi} H_\mathrm{e} \sin\alpha \frac{H_0^2 + H_\mathrm{e}^2 \cos 2\alpha}{\sqrt{H_0^2 - H_\mathrm{e}^2 \sin^2\alpha}}. \quad (6)$$

Note an unusual feature of Eq. (6): when the angle $\alpha \to \pi/2$, the torque tends to a finite value

$$\tau_y(\pi/2) = \frac{H_\mathrm{e}\sqrt{k}}{4\pi}\sqrt{H_0^2 - H_\mathrm{e}^2}. \quad (7)$$

For $\alpha > \pi/2$ (or $H_{ez} < 0$), one has to replace H_{ez} with $|H_{ez}|$, i.e., the potential $G(\alpha) \propto |\pi/2 - \alpha|$ in the vicinity of $\alpha = \pi/2$. This non-analyticity of G and τ at $\alpha = \pi/2$ is an artefact of the infinite slab geometry. To treat the vicinity of $\alpha = \pi/2$ accurately one has to consider the sample as an oblate ellipsoid with z-axis as the axis of rotation.

In this case the thermodynamic potential which is minimum in equilibrium is [4]

$$\tilde{F} = F - \frac{\boldsymbol{H}\cdot\boldsymbol{B}}{4\pi} - \frac{1}{2}\boldsymbol{M}\cdot\boldsymbol{H}_\mathrm{e}. \quad (8)$$

All macroscopic fields $\boldsymbol{H}$ and $\boldsymbol{B}$ as well as the magnetization $\boldsymbol{M}$ inside the ellipsoid are uniform and related to the applied field $\boldsymbol{H}_\mathrm{e}$ by

$$(1-n)H_x + nB_x = H_{ex}, \quad (9)$$

$$2nH_z + (1-2n)B_z = H_{ez}, \quad (10)$$

where $n = n_x$ is the corresponding eigenvalue of the demagnetization tensor. Within the interval $|\gamma| = |\pi/2 - \alpha| \ll 1$, a simple algebra results in

$$\tilde{F} = \frac{H_\mathrm{e}\cos\gamma}{8\pi n}(H_0 - H_\mathrm{e}\cos\gamma). \quad (11)$$

This yields the torque for $\gamma \ll 1$ [3]

$$\tau_y = -\frac{\mathrm{d}\tilde{F}}{\mathrm{d}\gamma} = -\frac{H_\mathrm{e}}{8\pi n}(2H_\mathrm{e} - H_0)\gamma. \quad (12)$$

Thus, as expected, the torque is continuous at $\gamma = 0$ ($\alpha = \pi/2$) and fast increases in magnitude when $|\gamma|$ increases. The crossover between Eqs. (6) and (12) takes place at a certain angle γ_m which can be roughly estimated by equating torques of Eqs. (6) and (12). This estimate leads to the angle $\gamma_m \sim 2n\sqrt{k}$ and the maximum torque then is [3]

$$\tau_m \sim \frac{H_0 H_\mathrm{e}}{4\pi}\sqrt{k}. \quad (13)$$

To estimate τ_m assume that $\lambda \sim 10^{-5}$ cm and $\Lambda_J(0) \sim 10^{-4}$ cm, then $\tau_m \sim 10^2$ erg/cm^3 in fields on the order 100 G. Even for a tiny crystal with dimensions $(0.1\times0.1\times0.01)$ mm^3 $= 10^{-7}$ cm^3, one estimates the torque acting on the whole sample as 10^{-5} erg. The sensitivity of piezoresistive torque magnetometers is in the range of 10^{-7} erg, so the Josephson torque can, in principle, be measured.

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