Rectangular normal domains in current-carrying superconductors

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We find a type of normal domain in composite superconductors—a traveling rectangular domain. Using the effective circuit model we study in detail the dynamics of this domain. We derive an analytical solution for the propagation velocity and the length of a traveling rectangular domain. Current–voltage characteristics are calculated for a superconductor with a rectangular domain. © *1998 American Institute of Physics.* [S0021-8979(98)03811-0]

I. INTRODUCTION

The origination and propagation of a normal zone in current-carrying superconducting wires has been continuously a subject of interest in the field of applied superconductivity (see, for example, Ref. 1, and references therein). If a normal seed nucleates in a current-carrying homogeneous superconductor it will either shrink when the current is less than a certain value I_p (minimum propagation current), or expand when the current is higher than I_p. Modern commercial superconducting wires consist of many fine filaments of a superconductor embedded in a matrix of a normal metal (stabilizer). If a normal seed nucleates in this composite superconductor the current in the vicinity of this normal seed redistributes into the normal metal stabilizer. This process is followed by a significant decrease of the Joule power and by the subsequent recovery of superconductivity.

Composite superconductors with a large amount of normal metal stabilizer have been tested for use in superconducting magnetic energy storage systems. Despite the above-described stabilizing mechanism, it was found experimentally that a normal zone of finite size (normal domain) can propagate along a composite superconductor for transport currents larger than a certain threshold value I_d.[2] The formation of these traveling normal domains was shown to be a result of a finite duration of the current redistribution process into the large stabilizer.[3–7] The Joule power generated in the superconducting filaments during this process is, consequently, high. This heat release results in a ''hot'' region at the front of the normal zone, and causes the expansion of the normal domain. After the current is expelled into the stabilizer, the superconductor cools down towards the stable state and superconductivity recovers.

The dynamics of a traveling normal domain was investigated both numerically and analytically in a number of theoretical studies. Dresner formulated a simplified model which could be treated analytically.[8,9] He performed explicit calculations of the propagation velocity approximating the decay of the Joule power during the process of current redistribution by an exponential term. His model predicts a current threshold I_d, for currents below I_d the composite is cryostable and for currents above I_d a traveling normal domain exists. At this threshold I_d, the propagation velocity of a normal domain jumps to a finite value rather than rising smoothly from zero. Lately, Kupferman *et al.* have found that the temperature profile in this traveling normal domain is spikelike.[10] The explicit equations for the propagation velocity of these traveling spike domains and for the threshold current I_d have been obtained.

In this paper we study the dynamics of a normal zone in the composite superconductors characterized by a very long duration of the current redistribution process into the stabilizer (this is typical, in particular, for Rutherford-type superconducting cables,[11,12] or for composite superconductors where a large amount of normal metal stabilizer and the superconductor itself are spatially segregated[7]). Our numerical simulations show that in this case a type of normal domain exists in a composite, namely, a traveling rectangular normal domain. The temperature profile of a rectangular domain is qualitatively different from the temperature profile of a spike domain. In particular, the length of a rectangular domain is about *two orders* of magnitude larger than the length of the spike domain. Modeling the process of current redistribution in the composite by an effective circuit,[10] we derive explicit expressions for the propagation velocity and length of the traveling rectangular domain. IV characteristics of the composite in the presence of a rectangular domain are calculated. We discuss the physical mechanism of normal domains propagation in composite superconductors. Finally, we use the effective circuit model to analyze the experimental data reported by Pfotenhauer *et al.*[2]

II. MAIN EQUATIONS

In this paper we consider a rectangular conductor consisting of a plane layer of a superconducting material, referred to as S, electrically and thermally bonded to a stabilizer, referred to as N. The thicknesses of the superconductor and the stabilizer are denoted by d_s and d_n, respectively. The conductor carries a transport current I and is kept in a thermal contact with a heat reservoir of temperature T_0 [see Fig. 1(a)].

The process of current redistribution in the conductor is modeled by the effective electrical circuit sketched in Fig. 1(b). Each component of the conductor is described by a discrete chain of resistors. The lower chain represents the stabilizer, each resistor being attributed a resistance $R_n = \rho_n \Delta x / d_n$, where Δx is an arbitrary discretization length

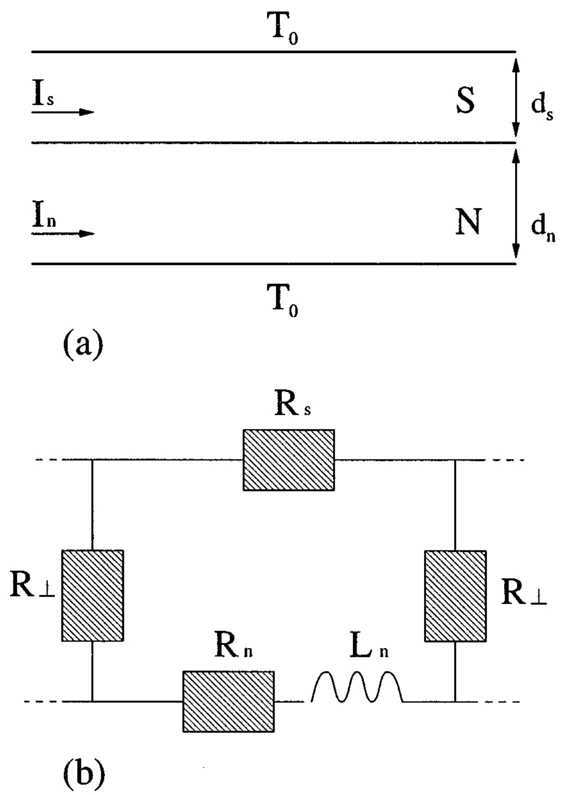

FIG. 1. (a) rectangular conductor model; (b) effective electrical circuit describing the current distribution in the conductor.

(x is the axis along the conductor). The upper chain of resistors represents the superconductor, with $R_s=\rho_s\Delta x/d_s$, where $\rho_s=\rho_s(T,j_s)$ is the resistivity of the superconductor depending on both the local temperature and current density in the superconductor. It vanishes in the superconducting state, and it is finite above the normal transition. The two chains are linked through a third kind of resistor, $R_\perp=\gamma_R\rho_n d_n/\Delta x$, representing a resistance to the perpendicular current. Here, γ_R is a numerical factor of the order of 1, which depends on the geometry of the conductor. The inclusion of a characteristic time scale in the electric current diffusion process is accomplished by taking into account the inductance of the stabilizer, $L=\gamma_L\mu_0 d_n\Delta x$, where $\gamma_L\sim 1$ is another numerical factor. Let j_s be the current density in the superconductor and $j=I/d_s$ be the current density in the superconductor if all the current flows through it. Then, the current density in the stabilizer j_n is given by $j_n=(j-j_s)d_s/d_n$, and the current density in the perpendicular direction $j_\perp$ is given by $j_\perp=-d_s\partial j_s/\partial x$. Applying now Kirchhoff's laws on this circuit we obtain the following equation for the current density in the superconductor:

$$\gamma_L\mu_0 d_n d_s\,\frac{\partial j_s}{\partial t}=\gamma_R\rho_n d_n d_s\,\frac{\partial^2 j_s}{\partial x^2}+\rho_n\,\frac{d_s}{d_n}\,(j-j_s)-\rho_s(T,j_s)j_s. \tag{1}$$

Next, we consider the temperature distribution in the conductor. We assume that the thermal relaxation time over the cross section is much shorter than the thermal relaxation time between the conductor and the coolant. In this case, the temperature distribution over the cross section is uniform and is a function only of the coordinate along the conductor and time.[10] In this case, the temperature $T(x,t)$ satisfies the one-dimensional heat equation

$$C\,\frac{\partial T}{\partial t}=\kappa\,\frac{\partial^2 T}{\partial x^2}-W(T)+Q(T,j_s), \tag{2}$$

where C and κ are the heat capacity and the heat conductivity of the conductor, respectively, both averaged values, taken here as constants. The term $W(T)$ is the rate of heat transfer to the coolant per unit volume, which we write as $W(T)=h(T-T_0)/d$, where h is the heat-transfer coefficient (we consider for simplicity the case where h is constant) and $d\equiv d_s+d_n$. The function $Q(T,j_s)$ is the rate of Joule heating per unit volume, $Q(T,j_s)=d_s\rho_s(T,j_s)j_s^2/d$. We neglect the contributions to the function $Q(T,j_s)$ from the current in the stabilizer and from the perpendicular current. This simplification reduces the complexity of the resulting equations while preserving the main physical features of the model.

We define the following dimensionless variables: θ, the temperature of the conductor; and i_s, the current density in the superconductor,

$$\theta\equiv\frac{T-T_0}{T_c-T_0},\quad i_s\equiv\frac{j_s}{j_c}, \tag{3}$$

where T_c is the critical temperature of the superconductor and j_c is the critical current density at the temperature T_0. We define l_{th}, the characteristic thermal relaxation length, and τ_{th}, the characteristic thermal relaxation time,

$$l_{\text{th}}^2\equiv\frac{(d_s+d_n)\kappa}{h},\quad \tau_{\text{th}}\equiv\frac{(d_s+d_n)C}{h}, \tag{4}$$

the characteristic length of the current redistribution l_m, and the corresponding relaxation time τ_m,

$$l_m^2\equiv\gamma_R d_n^2,\quad \tau_m\equiv\frac{\gamma_L\mu_0 d_n^2}{\rho_n}. \tag{5}$$

We assume here the "step model" for the resistivity of the superconductor,[1] $\rho_s(T,j_s)=\rho_s H[j_s-j_c(T)]$, where H is the Heaviside step function $[H(x)=0$ if $x<0$ and $H(x)=1$ if $x>0]$, and $j_c(T)$ is the critical current density in the superconductor given by[9]

$$j_c(T)=j_c\left(1-\frac{T-T_0}{T_c-T_0}\right)=j_c(1-\theta). \tag{6}$$

Then, we define two dimensionless parameters

$$\xi\equiv\frac{\rho_s d_n}{\rho_n d_s},\quad \alpha\equiv\frac{d_s^2\rho_n j_c^2}{d_n h(T_c-T_0)}, \tag{7}$$

where ξ is the ratio of the resistances of the superconductor in the normal state and the stabilizer, and α is the ratio of the characteristic rates of Joule power and heat flux to the coolant (Stekly parameter). Finally, we use dimensionless scales for time and length, and express time in units of τ_{th} and length in units of l_{th}. As a result, Eqs. (1) and (2) take the form

$$\frac{\partial\theta}{\partial t}=\frac{\partial^2\theta}{\partial x^2}-\theta+\alpha\xi H[i_s+\theta-1]i_s^2, \tag{8}$$

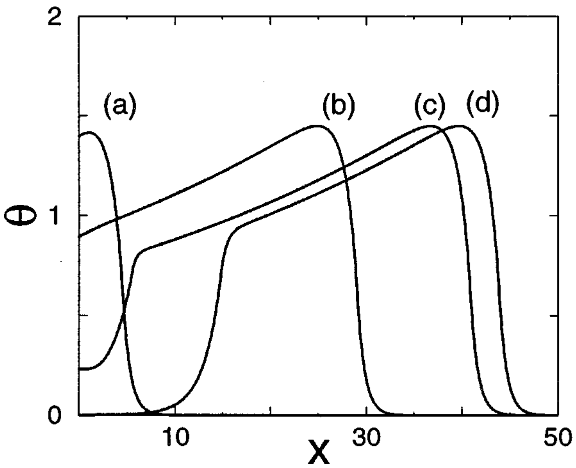

FIG. 2. The temperature distribution in the composite superconductor during the formation of a traveling rectangular normal domain. The parameters are $\tau=1000$, $\xi=5$, $\alpha=2$, $\lambda=0.1$, and $i=0.4$, and (a) $t=10$, (b) $t=60$, (c) $t=83$, and (d) $t=90$.

$$\tau \frac{\partial i_s}{\partial t}=\lambda^2 \frac{\partial^2 i_s}{\partial x^2}-(1+\xi H[i_s+\theta-1])i_s+i, \tag{9}$$

where the dimensionless parameters i, τ, and λ are

$$i\equiv j/j_c, \quad \tau\equiv\tau_m/\tau_{\mathrm{th}}, \quad \lambda\equiv l_m/l_{\mathrm{th}}. \tag{10}$$

To complete the presentation of the model, we identify the characteristic time and length scales of the resulting equations. Equation (8) has one set of characteristic scales, namely, the thermal relaxation time and the corresponding relaxation length, both defined here to be equal to unity. Equation (9) has two sets of characteristic scales, depending whether the system is in its superconducting state ($H=0$) or in its normal state ($H=1$). In the superconducting state the current diffuses from the stabilizer to the superconductor with the relaxation time τ and the characteristic length λ. In the normal state the current redistributes from the superconductor to the stabilizer with the relaxation time $\tau/(\xi+1)$, and the characteristic length $\lambda/\sqrt{\xi+1}$.

III. RESULTS

A. Numerical simulations

In order to study the propagation of normal domains in the limit of a very long process of current redistribution in the composite superconductor we perform numerical simulations of model Eqs. (8) and (9) considering the values of the dimensionless parameters τ and ξ in the range $\tau\gg\xi$. The rest of the dimensionless parameters are evaluated using experimental data,[1,9] which gives $\alpha\sim1-10$, $\lambda\sim0.1-1$. We observe how the temperature and the current density distributions evolve in time, when the conductor is initially in the superconducting rest state ($\theta=0$, $i_s=i$), except for a normal seed of length $2l_{\mathrm{th}}$ in which the temperature is raised above the critical value $\theta=1$.

For a given set of the dimensionless parameters there is a threshold current i_d, above which traveling normal domains are formed. A sequence of temperature distributions in the conductor is shown in Fig. 2 for $i>i_d$ (note that due to the symmetry of the temperature distribution we show only the right-hand side of the sample). We observe that the initial normal seed starts to expand and change its shape [Fig. 2(a)]. The temperature field at the boundary of the expanding normal zone reaches a steady shape after a short time interval of the order of thermal relaxation time τ_{th} (one in dimensionless units). During this time interval the current inside the normal zone remains constant. After the normal zone reaches a certain length the current starts to diffuse into the stabilizer with characteristic time $\tau/(\xi+1)\gg1$ (in dimensionless units). As a result, the center of a normal zone cools down [Fig. 2(b)]. The expansion of the normal zone accompanied by slow diffusion of the current into the stabilizer continues until the current at the center of a normal zone reaches a certain value. At this point, the temperature at the center drops down below its critical value, over a short time interval of the order of τ_{th}, and superconductivity recovers [Fig. 2(c)]. As a result, we find two separated normal domains with a nearly rectangular shape of the temperature profile traveling away in the opposite directions. The system tends to a steady state with two rectangular normal domains propagating along the conductor with a constant velocity, while superconductivity recovers behind [Fig. 2(d)].

B. Analytical solution of the steady state

Let us consider now the propagation of a rectangular normal domain analytically. For $i>i_d$ the temperature and the current density distributions of the steady state are given by the stationary solutions of Eqs. (8) and (9), with $\theta=\theta(x-vt)$ and $i_s=i_s(x-vt)$, which correspond to a reference frame moving along the conductor with velocity v,

$$\frac{d^2\theta}{dz^2}+v\frac{d\theta}{dz}-\theta+\alpha\xi H[\theta+i_s-1]i_s^2=0, \tag{11}$$

$$\lambda^2\frac{d^2 i_s}{dz^2}+v\tau\frac{di_s}{dz}-(1+\xi H[\theta+i_s-1])i_s+i=0, \tag{12}$$

where v still has to be determined. We define $z=x-vt=0$ to be the point where the normal transition occurs and $z=-D$ to be the point where superconductivity recovers behind the normal domain (see Fig. 3).

To simplify calculations, we consider in this paper a diffusionless limit for the current density distribution. We suppose that the electric current diffusion into the stabilizer in front of a propagating normal domain does not affect significantly the propagation velocity and the domain's shape. Our numerical simulations show that this assumption holds with a high degree of accuracy for the relevant range of parameters, $\lambda^2\ll v\tau$. In the first approximation, we set $\lambda^2=0$, dropping the diffusion term in Eq. (12). The resulting equation

$$v\tau\frac{di_s}{dz}-(1+\xi H[\theta+i_s-1])i_s+i=0, \tag{13}$$

is of first order and nonlinear. It can be solved, however, in the three regions, $z>0$ ($H=0$), $-D\le z\le0$ ($H=1$), and $z<-D$ ($H=0$). In each of these regions Eq. (13) is a linear

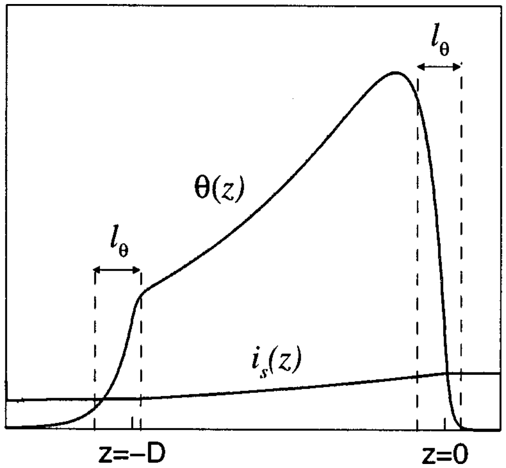

FIG. 3. The temperature and the current density distributions in a traveling rectangular normal domain.

equation with constant coefficients. The solution of Eq. (13) with boundary conditions at infinity $i_s(\pm\infty)=i$ is given by

$$i_s(x)=\begin{cases} i, & z>0,\\ \dfrac{i}{\xi+1}\left[1+\xi\,\exp\left(\dfrac{\xi+1}{v\tau}\,z\right)\right], & -D\leqslant z\leqslant 0,\\ i-[i-i_b]\,\exp\left(\dfrac{z+D}{vt}\right), & z<-D, \end{cases} \tag{14}$$

where we define $i_b\equiv i_s(-D)$, the value of the current density at the transition point $z=-D$. We use this solution to calculate the length of a traveling rectangular normal domain. Matching conditions at the transition point $z=-D$ an explicit equation for the rectangular normal domain length is obtained,

$$D(i)=\frac{v\tau}{\xi+1}\,\ln\frac{\xi_i}{(\xi+1)i_b-i}, \tag{15}$$

where the velocity v, and the current i_b have to be determined by considering the temperature distribution in the domain. For this purpose, the explicit expressions for $i_s(z)$ can be substituted into Eq. (11), yielding linear equations for $\theta(z)$ in each one of three regions. The boundary and matching conditions form a closed set of equations for the integration constants v and i_b. These implicit equations, however, are very cumbersome and can be only solved numerically.

To obtain simple explicit expressions for the propagation velocity v and the current i_b, we use the results of our numerical simulations. These simulations show that a fast temperature relaxation process results in formation of two boundary layers in the vicinity of transition points $z=0$ and $z=-D$, where the temperature distribution has large gradients (see Fig. 3). The characteristic length of these boundary layers l_θ is approximately equal to the product of the propagation velocity and the characteristic time of the temperature relaxation, $l_\theta\approx v\tau_{\mathrm{th}}$ (v in dimensionless units). On the other hand, as it follows from Eq. (14), the electric current diffuses into the stabilizer with the characteristic length $l_i\approx v\tau/(\xi+1)$. In the limit $\tau\gg\xi>1$, which is considered in this paper, we obtain $l_i\gg l_\theta$. This means that the current varies only slightly in the boundary layers, and in the first approximation it can be set constant and equal to its values at the transition points. In this approximation the propagation of the front and the back boundary layers can be treated independently and described by Eq. (11) with a constant $i_s=i$ for the front boundary, and $i_s=i_b$ for the back boundary. As a result, the temperature distribution at the front boundary is a solution of the equation

$$\frac{d^2\theta}{dz^2}+v\,\frac{d\theta}{dz}-\theta+\alpha\xi H[\theta+i-1]i^2=0, \tag{16}$$

with the boundary conditions,

$$\theta(\infty)=0,\quad \theta(-\infty)=\alpha\xi i^2, \tag{17}$$

while the temperature distribution at the back boundary is a solution of the equation,

$$\frac{d^2\theta}{dz^2}+v\,\frac{d\theta}{dz}-\theta+\alpha\xi H[\theta+i-1]i_b^2=0, \tag{18}$$

with the boundary conditions,

$$\theta(\infty)=\alpha\xi i_b^2,\quad \theta(-\infty)=0. \tag{19}$$

Equation (16) with the boundary conditions (17) is an eigenvalue problem for the propagation velocity v parametrized by the current i. It represents propagation of the superconducting-to-normal switching wave in a homogeneous superconductor with a constant current equal to i. The solution of this problem is well known, and the propagation velocity $v(i)$ as a function of total current is given by[1]

$$v(i)=\frac{\alpha\xi i^2+2i-2}{\sqrt{(1-i)(\alpha\xi i^2+i-1)}}. \tag{20}$$

The propagation of a normal domain corresponds to the positive velocity $v(i)$. For the negative values of velocity the initial normal seed shrinks and disappears. Thus, equating propagation velocity $v(i)$ to zero we calculate the value of the threshold current i_d. Particularly simple expressions for $v(i)$ and i_d are obtained if we consider the following approximations, $\sqrt{\alpha\xi}\gg 1$, and $\alpha\xi i^2\gg 1$, which hold for most cases of practical interest. Then, we obtain

$$v(i)\approx i\sqrt{\frac{\alpha\xi}{1-i}},\quad i_d\approx\sqrt{\frac{2}{\alpha\xi}}. \tag{21}$$

Figure 4 shows the comparison between the propagation velocity of a rectangular normal domain obtained by the numerical simulations, and calculated by means of formulas (20) and (21). As it is seen from Fig. 4, the velocity v is a monotonically increasing function of the current i, rising smoothly from zero at the threshold rather than starting from a finite value as it happens for the traveling spike domains.[7,10] Comparing the numerical results with the results of Eq. (20), we find a high degree of accuracy in the entire range of currents $i>i_d$, with the maximum deviation less than 3%. The velocity calculated by Eq. (21) is very close to the numerical values for larger currents fitting the condition $\alpha\xi i^2\gg 1$.

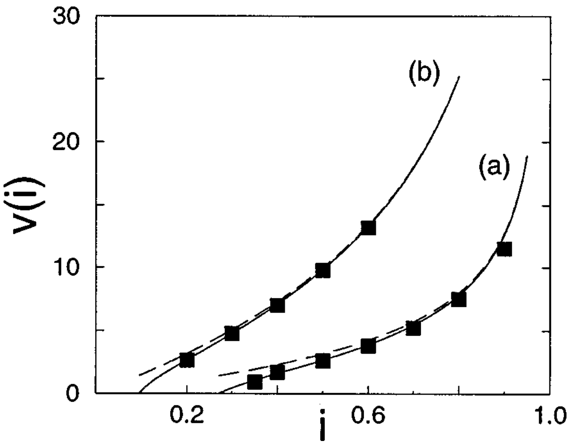

FIG. 4. Velocity of a rectangular normal domain vs current. The dots represent the results obtained in the numerical simulations, the solid lines represent formula (20), and the dashed lines represent formula (21). The parameters are $\alpha=2$, $\lambda=0.1$, (a) $\tau=300$, $\xi=10$, and (b) $\tau=3000$, $\xi=100$.

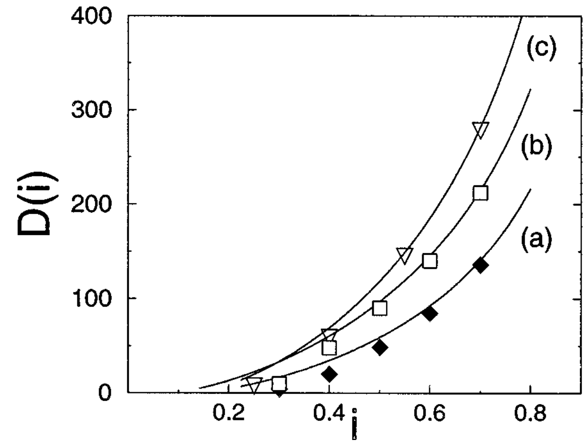

FIG. 5. The length of a traveling rectangular normal domain vs current. The dots represent the results obtained in the numerical simulations, the solid lines represent formula (23). The parameters are $\alpha=2$, $\lambda=0.1$, (a) $\tau=200$, $\xi=20$, (b) $\tau=400$, $\xi=20$, and (c) $\tau=400$, $\xi=50$.

To calculate the characteristic current i_b, we use Eq. (18) with boundary conditions (19). It corresponds to propagation of the normal-to-superconducting switching wave in a homogeneous superconductor with a constant current equal to i_b. In this case, the propagation velocity v is parametrized by the current i_b, $v=v(i_b)$, and is negative (the superconducting phase expels the normal phase). The propagation of a normal domain with steady shape requires that the front and the back boundaries of the domain propagate with the same velocity, namely,

$$v(i_b)=-v(i). \tag{22}$$

Substituting expressions (20) for $v(i)$ and $v(i_b)$, and using the above-considered approximations, $\sqrt{\alpha\xi}\gg 1$ and $\alpha\xi i^2 \gg 1$, we find for the current i_b, $i_b\approx 1/\sqrt{\alpha\xi}$.

Knowing the dependence of the velocity and the characteristic current i_b on the dimensionless parameters and the total current, we calculate the length of a traveling rectangular normal domain as a function of the total current. Substituting the expressions for the velocity, Eq. (21), and the current, $i_b\approx 1/\sqrt{\alpha\xi}$, into Eq. (15) we obtain for the length of a rectangular domain,

$$D(i)\approx \tau i \sqrt{\frac{\alpha}{\xi(1-i)}} \ln \frac{\xi_i}{\sqrt{\frac{\xi}{\alpha}}-i}. \tag{23}$$

Figure 5 shows the dependence $D(i)$ calculated by means of Eq. (23) and by numerical simulations. We find that the analytical result is in good agreement with the numerical results for the entire range of currents $i_d<i<1$. It becomes extremely accurate for larger currents and for the values of the dimensionless parameters α and ξ, fitting the conditions $\sqrt{\alpha\xi}\gg 1$, and $\alpha\xi i^2\gg 1$.

The presence of a traveling normal domain results in a potential drop U given by

$$U=j_c\rho_s l_{\rm th}\int dz\, i_s(z). \tag{24}$$

Using the solution for the current density, Eq. (14), we obtain the dependence of the voltage U on the propagation velocity and the length of a traveling rectangular domain,

$$U=\frac{i}{\xi}\left[D+\tau v\left(1-\exp\left(-\frac{\xi}{\tau v}D\right)\right)\right]. \tag{25}$$

The voltage $U(i)$ as a function of the total current is calculated by means of Eqs. (15), (20), and (25); the result of these calculations is shown in Fig. 6. The curves $U(i)$ start at $i=i_d$ with a zero voltage and are strongly nonlinear for the entire range of currents $i_d<i<1$.

C. Discussion and conclusions

We discuss now the physical mechanism of normal domains formation and propagation in composite superconductors. Our numerical simulations show that for the same type of composite superconductor different values of the parameters characterizing the composite and cooling conditions result in formation of either spike or rectangular normal domains. Assume an initial normal seed originates in some part

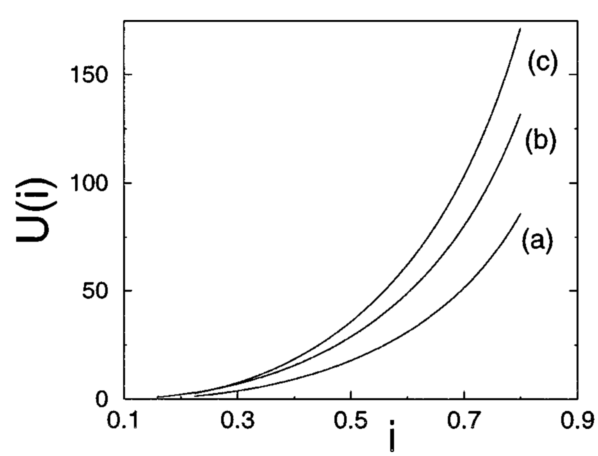

FIG. 6. Current–voltage characteristics of a composite superconductor in the presence of a traveling rectangular normal domain (V is in units of $\rho_s j_c l_{\rm th}$). The parameters are $\lambda=0.1$, (a) $\alpha=2$, $\xi=20$, $\tau=200$, (b) $\alpha=4$, $\xi=20$, $\tau=200$, and (c) $\alpha=2$, $\xi=20$, $\tau=400$.

of the superconductor. The current in this region starts to redistribute between the superconductor and the stabilizer by diffusion, a process which has a characteristic duration $\tau_m/(\xi+1)$. It was shown that in the case when the characteristic time of the current redistribution process is of the order of the thermal relaxation time, $\tau_m/(\xi+1)\sim\tau_{\text{th}}$, the initial normal seed results in the formation of traveling spike normal domains.[10] In this paper we have shown that in the case of a very long current redistribution process with $\tau_m/(\xi+1)\gg\tau_{\text{th}}$, a different type of traveling normal domain is formed, namely, a traveling rectangular normal domain. These two types of traveling normal domains are characterized by different propagation mechanisms, which we consider here.

In the regime of the spike domains, the relatively short process of current redistribution results in the formation of a thin region at the front of a normal zone where the current diffuses into the stabilizer. The Joule power generated in this thin region is, consequently, high. As a result, the region of high temperature (''hot spot'') is formed at the front of a normal zone. The length of this ''hot spot'' is determined by the product of the propagation velocity and the characteristic time of the current redistribution process and is equal to $v\tau_m/\xi$. Behind this ''hot spot'' there exists the region of length $v\tau_{\text{th}}$ where the current flows through the stabilizer and the temperature is decreasing toward the transition point. The fast expansion of a ''hot spot'' along the superconductor, accompanied with the local recovery of superconductivity behind it, is the origin of the spike normal domain propagation.

In the regime of the rectangular normal domain, the characteristic time of the current redistribution process is much larger than the thermal relaxation time $\tau_m/(\xi+1)\gg\tau_{\text{th}}$. We have shown that this long current redistribution process results in the formation of two regions at the normal zone boundaries, where the current remains confined in the superconductor while the temperature changes on the characteristic scale of the order of $v\tau_{\text{th}}$ (see Fig. 3). In this case the propagation of the normal zone boundaries can be treated as the propagation of two superconducting-to-normal switching waves in a superconductor with a constant current. The value of the current on the front switching wave is equal to the total current i. The value of the current on the back switching wave is determined by means of condition (22), which requires propagation of two switching waves with the same velocity. Between two switching waves there is a long region with a characteristic length of the order of $v\tau_m/(\xi+1)$ where the current diffuses into the stabilizer changing from i to i_b. The propagation of two switching waves coupled by the current is the origin of the traveling rectangular normal domain formation.

Traveling normal domains in composite superconductors with a large stabilizer were studied experimentally by Pfotenhauer *et al.*[2] They found that the propagation velocity of a normal domain jumps at a threshold current from zero to a finite value. As we have shown, this behavior at a threshold is typical for traveling spike normal domains. Using the experimental data of Ref. 2 we estimate the dimensionless parameters τ and ξ of the effective circuit model as $\tau\sim 100$ and $\xi\sim 100$. Numerical simulations of the model equations for these values of the parameters show the formation of traveling spike normal domains in agreement with the experimental result.

In conclusion, we have shown the existence of a type of traveling normal domain in composite superconductors, namely, rectangular normal domains. An effective circuit model is used to study the formation and propagation of these domains. An analytical solution is found for the propagation velocity and length of the rectangular normal domains in the case of the steady-state propagation. We discussed the physical mechanism of normal domain formation and propagation in composite superconductors.

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