Defect pair in the elastic lattice of pancake vortices

M. Slutzky and R. G. Mints

School of Physics and Astronomy, Raymond and Beverly Sackler Faculty of Exact Sciences, Tel Aviv University, Tel Aviv 69978, Israel

E. H. Brandt

Institut für Physik, Max-Planck-Institut für Metallforschung, D-70506 Stuttgart, Germany

(Received 21 February 1997)

An additional pancake-antipancake vortex pair is considered in the vortex lattice of layered superconductors. Within linear elastic continuum theory, the relaxation of the background lattice screens the long-range logarithmic interaction of the defect pair, reducing the factor $\ln(r_0/\xi)$ to $\ln(a/\xi)$ where r_0 is the pair spacing, ξ the in-plane coherence length, and a the vortex spacing. The finite tilt modulus does not destroy this ideal two-dimensional screening, yielding a small correction $\sim(a^2/8\pi\lambda^2)\ln(r_0/a)$, which in principle is of long range, but has a very small prefactor when the vortex spacing a is smaller than the in-plane penetration depth λ. [S0163-1829(97)02825-7]

I. INTRODUCTION

The thermodynamic and electrodynamic properties of type-II superconductors in a magnetic field are determined by the behavior of the lattice of Abrikosov vortex lines or the three-dimensional lattice of "pancake vortices" in high-T_c superconductors with layered structure.[1–3] Features which are widely discussed in this context are[4,5] the pinning of vortices by material inhomogeneities, thermally activated depinning, collective creep, glassy behavior, and as an intrinsic property, even in the absence of pinning, the melting of the vortex lattice. All these effects depend on the elastic and plastic behavior of the vortex lattice and on the properties of its structural defects like vacancies, interstitials, dislocations, dislocation pairs and loops, stacking faults, etc. In particular, melting of a lattice formally may be described by the spontaneous generation and proliferation of such defects.

The crucial property of structural defects in a point lattice or line lattice is their free energy, composed of the self-energy of each defect and the interaction energy between defect pairs. The self-energy and interaction energy themselves consist of two terms, namely, the energy of the defect in the ideal (undeformed) host lattice and the elastic energy of the host lattice when this is allowed to relax around the defect. This lattice relaxation reduces the defect energy, and in some cases the two terms nearly compensate each other. The reduction is particular large in lattices with long-range interaction, e.g., in vortex lattices. For example, the vortex lines in superconductors interact over a distance λ, the magnetic penetration depth, which is typically much larger than the vortex spacing $a\approx(\Phi_0/B)^{1/2}$ (Φ_0 is the flux quantum and B the average magnetic induction). It was shown in Ref. 6 that the elastic relaxation of the vortex lattice drastically reduces the self-energy of a vacancy (defect line): The displacements of the vortices change the local induction $B(\mathbf{r})$ by an amount $\delta B(\mathbf{r})$ which looks similar to the induction $B_v(\mathbf{r})$ of the removed vortex line. Thus, the local depression of $B(\mathbf{r})$ caused by removing a vortex line is largely "repaired" by the relaxation of the surrounding vortex lattice such that $B(\mathbf{r})\approx B$ remains almost constant. Note that the

individual vortex fields strongly overlap, yielding an almost constant $B(\mathbf{r})$ when the vortex lattice is unperturbed and has not too large spacing $a\lesssim\pi\lambda$, corresponding to $B\gtrsim B_{c1}/\ln\kappa$ where B_{c1} is the lower critical field and $\kappa=\lambda/\xi$ the Ginzburg-Landau parameter.

The reduction of the magnetic field variance and of the energy of a defect line by the lattice relaxation is the larger the more vortices are within the range of the interaction potential. Since the effective interaction range between the vortices $\lambda'=\lambda/(1-B/B_{c2})^{1/2}$ diverges as B approaches the upper critical field B_{c2}, the compensation of the energy term becomes almost complete at large inductions. Namely, while the energy of a vacancy *without* lattice relaxation formally diverges for $B\to B_{c2}$, the energy of the vacancy *with* relaxation decreases as $(B_{c2}-B)^2$, as does also the shear modulus c_{66} of the vortex lattice and the energies of all structural defects.[6]

A similar strong reduction of defect energies by the relaxation of the surrounding lattice is expected for the two-dimensional (2D) lattice of point vortices in layered high-T_c superconductors. In the limit of strong anisotropy, i.e., in the absence of Josephson coupling between the superconducting CuO layers, the interaction between two pancake vortices in the same layer is logarithmic and thus of very long range. The interaction of pancakes positioned in different layers is smaller by a factor of $s/\lambda\ll1$, where s is the layer spacing and λ the penetration depth of the supercurrent flowing in the layers.

In the present paper, as one such example, we consider the energy of a defect pair consisting of a pancake vortex and an antipancake vortex residing in the same plane. Since we are interested here in an approximate continuum-theoretical treatment, we disregard energy terms which depend on the position of the pancake and antipancake within the lattice cell. This is allowed when these two positions are equivalent, since then these terms cancel. In the general case the position-dependent energy terms have to be calculated numerically.

The main finding of our paper is that within continuum theory the linear elastic relaxation of the surrounding lattice

(mainly of the pancakes in the same layer) strongly reduces the energy of this defect pair, resulting in a short-range interaction, whereas the unscreened interaction before relaxation was logarithmic and thus of long range. A further reduction is expected, like in the case of vortex lines,[6] when the discreteness of the pancake lattice and the full nonlinear elastic response are accounted for. Such discrete-lattice computations are under way.

II. ENERGY OF A PANCAKE-ANTIPANCAKE PAIR

We consider a pancake-antipancake vortex pair added to a vortex lattice in an extremely anisotropic layered superconductor. As usual the superconductor is modeled by an infinite stack of superconducting layers of zero thickness parallel to the xy plane and located at $z_m = \pm ms$, where m is an integer and s is the interlayer distance. We disregard the Josephson coupling between the superconducting layers, treating the vortex lattice as a set of electromagnetically coupled stacks of pancakes. The additional pancake-antipancake vortex pair resides in the layer $m = 0$. The pancake is located at $\mathbf{r} = 0$ and the antipancake at $\mathbf{r} = \mathbf{r}_0 = (x_0, y_0, 0)$ with $|\mathbf{r}_0| \gg a$.

An additional pancake-antipancake pair generates a supercurrent $\mathbf{j}(\mathbf{r})$ which exerts forces $\mathbf{f}_\nu$ on the pancakes of the vortex lattice located at the points $\mathbf{r}_\nu = (x_\nu, y_\nu, z_\nu)$. This force causes vortex displacements $\mathbf{u}_\nu = (u_{\nu x}, u_{\nu y}, 0)$, which are determined by the elastic equilibrium.

Our aim is to calculate the energy of the additional vortex pair in an elastic vortex lattice assuming that the displacements are small, $|\mathbf{u}_\nu| \ll a$. The energy of the additional vortex pair in a vortex lattice then consists of three parts: the self-energy of the pair, i.e., its energy in the absence of the vortex lattice; the interaction energy between the vortex pair and the undeformed vortex lattice; and the energy of the elastic deformation. We focus our attention on the first and third terms. If the additional pancake and antipancake are located in equivalent points of the vortex lattice, the second contribution to the pair energy is zero, since in the absence of Josephson coupling the energy of the pancake arrangement is the linear superposition of all pairwise magnetic interactions. In the general case, however, the position-dependent interaction with the undeformed lattice (mainly with the nearest neighbors) has to be computed separately.

First, we calculate the displacements $\mathbf{u}_\nu$ using the equation of elastic equilibrium. The force per unit volume exerted by the additional vortex pair on the other pancakes is

$$\mathbf{f} = n\mathbf{f}_\nu = \frac{sn\Phi_0}{c}\mathbf{j} \times \hat{\mathbf{z}}, \qquad (1)$$

$$\mathbf{j} = -\frac{c}{4\pi}\nabla^2\mathbf{A}^{\text{pair}}, \qquad (2)$$

where $n = B/(\Phi_0 s)$ is the pancake density, $\hat{\mathbf{z}}$ is the unit vector along the z axis, and $\mathbf{A}^{\text{pair}}(\mathbf{r})$ is the vector potential of the magnetic field generated by the vortex pair. Using the Fourier transform

$$\mathbf{A}(\mathbf{r}) = \int \frac{d^3\mathbf{p}}{8\pi^3}\mathbf{A}_\mathbf{p}e^{i\mathbf{p}\cdot\mathbf{r}}, \quad \mathbf{A}_\mathbf{p} = \int d^3\mathbf{r}\mathbf{A}(\mathbf{r})e^{-i\mathbf{p}\cdot\mathbf{r}}, \qquad (3)$$

with $\mathbf{p} = (p_x, p_y, p_z)$, we obtain

$$\mathbf{A}_\mathbf{p}^{\text{pair}} = \mathbf{A}_\mathbf{p}^0[1 - \exp(-i\mathbf{q}\cdot\mathbf{r}_0)]. \qquad (4)$$

Here $\mathbf{q} = (p_x, p_y, 0)$ (since $\mathbf{r}_0$ is in the plane $z = 0$) and $\mathbf{A}_\mathbf{p}^0$ is the Fourier transform of the vector potential of a single pancake[7] (the solution of the London equation with a singularity at $\mathbf{r} = 0$),

$$\mathbf{A}_\mathbf{p}^0 = \frac{i\Phi_0 s}{q^2(1 + \lambda^2\widetilde{p}^2)}\frac{\widetilde{p}^2}{p^2}\mathbf{q} \times \hat{\mathbf{z}}, \qquad (5)$$

$$\widetilde{p}^2 = q^2 + \frac{2[1 - \cos(ks)]}{s^2} = q^2 + \widetilde{k}^2, \quad k = p_z. \qquad (6)$$

The equation of the elastic equilibrium reads

$$n\mathbf{f}_\mathbf{p} - \mathbf{\Phi}_\mathbf{p}\cdot\mathbf{u}_\mathbf{p} = 0, \qquad (7)$$

where $\mathbf{\Phi}_\mathbf{p}$ is the dynamic matrix of the vortex lattice; see Appendix A. The Fourier transform of the discrete functions $\mathbf{u}_\nu$ and $\mathbf{f}_\nu$ is defined as

$$\mathbf{u}_\nu = \int_{\text{BZ}} \frac{d^3\mathbf{p}}{8\pi^3}\mathbf{u}_\mathbf{p}e^{i\mathbf{p}\cdot\mathbf{R}_\nu}, \quad \mathbf{u}_\mathbf{p} = \frac{1}{n}\sum_\nu \mathbf{u}_\nu e^{-i\mathbf{p}\cdot\mathbf{R}_\nu}, \qquad (8)$$

and the $\mathbf{p}$ integration is over the Brillouin zone of the 3D pancake lattice.

From Eqs. (1) to (5) we obtain

$$\mathbf{f}_\mathbf{p} = -\frac{\Phi_0^2 s^2}{4\pi}\frac{i\mathbf{q}}{q^2}\frac{\widetilde{p}^2}{1 + \lambda^2\widetilde{p}^2}[1 - \exp(-i\mathbf{q}\cdot\mathbf{r}_0)]. \qquad (9)$$

Inverting the elastic matrix in Eq. (7) (see Appendix A) we find the displacements

$$\mathbf{u}_\mathbf{p} = \mathbf{u}_\mathbf{p}^0[1 - \exp(-i\mathbf{q}\cdot\mathbf{r}_0)],$$

$$\mathbf{u}_\mathbf{p}^0 = -\frac{i\mathbf{q}}{q^2 n}\frac{\lambda^2\widetilde{p}^2}{\lambda^2\widetilde{p}^2 + \gamma(\widetilde{k})(1 + \lambda^2\widetilde{p}^2)}. \qquad (10)$$

Here the dimensionless function

$$\gamma(\widetilde{k}) = \frac{4\pi\lambda^2 c_{44}(\widetilde{k})\widetilde{k}^2}{B^2} = \frac{a^2}{8\pi\lambda^2}\ln\left(1 + \frac{\widetilde{k}^2}{k_{\text{BZ}}^2}\right) \ll 1 \qquad (11)$$

originates from the tilt modulus c_{44} of the vortex lattice. Using the Fourier transform (8) and formula (10) for the displacements $\mathbf{u}_\mathbf{p}$ we estimate $|\mathbf{u}_\nu| \sim a^2/r_\nu$ and therefore $|\mathbf{u}_\nu| \ll a$ at large distances, $r_\nu \gg a$.

Knowing the displacements we can evaluate the reduction of the spatial variation of the magnetic field around a defect pair, i.e., the screening of this pair by the elastic vortex lattice. This effect is analogous to the screening of an extra charge in an ionic crystal. The total vector potential created by the additional pair and by the pancakes of the vortex lattice is

$$\mathbf{A}^{\text{tot}}(\mathbf{r}) = \mathbf{A}^{\text{pair}}(\mathbf{r}) + \sum_\nu \mathbf{A}^0(\mathbf{r} - \mathbf{r}_\nu), \qquad (12)$$

where $\mathbf{A}^0(\mathbf{r})$ is the vector potential of a single pancake. Writing $\mathbf{r}_\nu = \mathbf{R}_\nu + \mathbf{u}_\nu$, where $\mathbf{R}_\nu$ are the unperturbed positions of the lattice pancakes, we obtain the Fourier transform of the

vector potential $\mathbf{A_p}$ induced by the pancake-antipancake defect pair in an elastic vortex lattice up to the first order in the displacements,

$$\mathbf{A_p} = \mathbf{A_p^{pair}} - i\mathbf{q} \cdot \mathbf{u_p} n \mathbf{A_p^0}. \qquad (13)$$

Combining Eqs. (10) and (13) we find

$$\mathbf{A_p} = \mathbf{A_p^{pair}} \frac{\gamma(\tilde{k})(1+\lambda^2\tilde{p}^2)}{\lambda^2\tilde{p}^2 + \gamma(\tilde{k})(1+\lambda^2\tilde{p}^2)}; \qquad (14)$$

i.e., the value of $\mathbf{A_p}$ is proportional to the dimensionless function $\gamma(\tilde{k}) \ll 1$ originating from the tilt modulus c_{44} of the vortex lattice. With the accuracy of $\gamma(\tilde{k}) \ll 1$ one can neglect the tilt modulus. In this approximation one has $\mathbf{A_p} = 0$, which means that the displaced pancakes of the lattice entirely screen out the defect pair at distances $R_\nu \gg a$. But accounting for c_{44} leads to a power law decay of the $\mathbf{u_\nu}$ and of $\mathbf{A(r)}$ at $r_\nu \to \infty$ and to the dependence of the energy of the defect pair on its size r_0. Note that $c_{44} = 0$ corresponds to a liquid of pancake vortices or to completely decoupled 2D lattices of pancake vortices. The energy of a pancake in a pancake liquid is considered in Appendix B following the lines of the Debye-Hückel theory of strong electrolytes.

We now calculate the energy of the added pancake-antipancake pair, $F_{pair} = F_{pair}^0 + W_{rel}$, consisting of two parts: the self-energy of the pair in the absence of the vortex lattice,

$$F_{pair}^0 = \frac{\Phi_0^2 s^2}{4\pi} \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{\tilde{p}^2}{q^2(1+\lambda^2\tilde{p}^2)} [1 - \cos(\mathbf{q} \cdot \mathbf{r_0})]$$

$$= \frac{\Phi_0^2 s}{8\pi^2\lambda^2} \ln\left(\frac{r_0}{\xi}\right), \qquad (15)$$

and the elastic relaxation energy

$$W_{rel} = -\frac{1}{2} \int_{BZ} \frac{d^3\mathbf{p}}{(2\pi)^3} \mathbf{f_p} \cdot \mathbf{u_{-p}}, \qquad (16)$$

which is the work of the force field $\mathbf{f(R_\nu)}$ towards the elastic equilibrium. The integration in Eq. (15) is over the intervals $q < \pi/\xi$, $|k| < \pi/s$, whereas in Eq. (16) it is over the first 3D Brillouin zone, i.e., $q < k_{BZ}$, $|k| < \pi/s$, and $k_{BZ} = (4\pi B/\Phi_0)^{1/2}$. Using Eqs. (9) and (10), we find

$$F_{pair} = \frac{\Phi_0^2 s^2}{4\pi} \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{\tilde{p}^2[1-\cos(\mathbf{q} \cdot \mathbf{r_0})]}{q^2(1+\lambda^2\tilde{p}^2)}$$

$$- \frac{\Phi_0^2 s^2}{4\pi} \int_{BZ} \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{\gamma(\tilde{k})[1-\cos(\mathbf{q} \cdot \mathbf{r_0})]}{q^2\lambda^2}. \qquad (17)$$

In the first integral, the integration is over the interval $k_{BZ} < q < \pi/\xi$; we also neglected a small term proportional to $\gamma(\tilde{k})$ in the denominator of the second integrand. The first integral in Eq. (17) is

$$F_1 \approx \frac{\Phi_0^2 s}{8\pi^2\lambda^2} \ln\frac{a}{\xi}. \qquad (18)$$

This main term may be interpreted as the energy of a pancake pair at the short distance a, the lattice spacing. It is thus of the same order as the position-dependent interaction of the additional pancake and antipancake with the pancake lattice in the same plane. This term is, however, much smaller than the unscreened energy of the pair, $F_{pair}^0 \propto \ln(r_0/\xi)$, Eq. (15). The second integral in Eq. (17) is

$$F_2 \approx \frac{\Phi_0^2 s a^2}{64\pi^3\lambda^4} \ln\left(\frac{r_0}{a}\right) \ln\left(\frac{a^2}{2\pi s^2}\right). \qquad (19)$$

If $a^2 \ll 4\pi\lambda^2$, one has $F_1 \gg F_2$, so that the main contribution to the energy comes from F_1, i.e., from the unscreened pancake and antipancake currents in the plane $z = 0$. In the general case we keep both terms and write

$$F_{pair} \approx \frac{\Phi_0^2 s}{8\pi^2\lambda^2} \left[\ln\frac{a}{\xi} - \frac{a^2}{8\pi\lambda^2} \ln\left(\frac{r_0}{a}\right) \ln\left(\frac{a^2}{2\pi s^2}\right) \right]. \qquad (20)$$

III. CONCLUSION

We have shown that in the presence of an additional pancake-antipancake vortex pair in the same layer, the relaxation of the surrounding pancake lattice reduces the logarithmic factor $\ln(r_0/\xi)$ in the energy of the defect pair, Eq. (15), to the smaller factor $\ln(a/\xi)$, Eq. (18), where $r_0 \gg a$ is the pair extension and a the equilibrium spacing of the vortex lattice. The energy of the pair plus the relaxation energy of the background lattice becomes thus independent of the pair spacing r_0. The sum (18) of these two energy terms is comparable to the third energy term, namely, the strongly position-dependent interaction of the pancake and of the antipancake with their nearest neighbors, which contains also factors of order $\ln(a/\xi_{ab})$. This remaining energy term has to be calculated numerically, even as the corrections due to the discreteness of the lattice and to the full nonlinear elastic relaxation. Such more detailed calculations will not alter our result of screened interaction of the two defects qualitatively. Therefore, for $r_0 \gg a$ it will be sufficient to compute the energy of each defect separately and then calculate their much smaller interaction energy as a perturbation.

These results were obtained in the limit of high anisotropy $\Gamma \gg 1$ corresponding to vanishing Josephson coupling. They are, however, more general, applying to any two-dimensional (2D) lattice with long-range interaction. In the above problem the in-plane interaction of the pancakes is by far dominating, rendering the problem almost 2D. The small correction due to the finite tilt modulus c_{44} of the vortex lattice, the second term in Eq. (20), in principle is of long range, but has a very small prefactor if $a \ll \lambda$.

APPENDIX A: ELASTIC MODULI OF A VORTEX LATTICE

Within continuum theory the dynamic matrix $\mathbf{\Phi_p}$ of the vortex lattice with uniaxial symmetry is given by[8,9]

$$\Phi_{\mathbf{p}}^{xx} = c_{11}q_x^2 + c_{66}q_y^2 + c_{44}\widetilde{k}^2,$$

$$\Phi_{\mathbf{p}}^{yy} = c_{66}q_x^2 + c_{11}q_y^2 + c_{44}\widetilde{k}^2, \tag{A1}$$

$$\Phi_{\mathbf{p}}^{xy} = \Phi_{\mathbf{p}}^{yx} = (c_{11}-c_{66})q_x q_y.$$

The elastic moduli derived from the anisotropic London theory for $B \gg \Phi_0/(4\pi\lambda^2)$ are[10,11]

$$c_{11} = \frac{B^2[1+\lambda_c^2\widetilde{p}^2]}{4\pi[1+\lambda_{ab}^2\widetilde{p}^2](1+\lambda_c^2q^2+\lambda_{ab}^2\widetilde{k}^2)}, \tag{A2}$$

$$c_{66} = \frac{B\Phi_0}{(8\pi\lambda_{ab})^2}, \tag{A3}$$

$$c_{44} = \frac{B^2}{4\pi(1+\lambda_c^2q^2+\lambda_{ab}^2\widetilde{k}^2)} + \frac{B\Phi_0}{32\pi^2\lambda_c^2}\ln\left(\frac{\xi^{-2}}{k_{\mathrm{BZ}}^2+\Gamma^{-2}s^{-2}}\right)$$

$$+ \frac{B\Phi_0}{32\pi^2\lambda_{ab}^4\widetilde{k}^2}\ln\left(1+\frac{\widetilde{k}^2}{k_{\mathrm{BZ}}^2}\right), \tag{A4}$$

where λ_{ab} and λ_c are the penetration depths for currents flowing in the crystalline ab plane and along the c axis, respectively, $\Gamma = \lambda_c/\lambda_{ab}$, ξ_{ab} is the in-plane coherence length, and $k_{\mathrm{BZ}}^2 = 4\pi B/\Phi_0$. In the limit of purely electromagnetic coupling between the superconducting layers ($\Gamma\to\infty$) we have, denoting λ_{ab} by λ,

$$c_{11} = \frac{B^2\widetilde{p}^2}{4\pi q^2[1+\lambda^2\widetilde{p}^2]}, \tag{A5}$$

$$c_{66} = \frac{B\Phi_0}{(8\pi\lambda)^2}, \tag{A6}$$

$$c_{44} = \frac{B\Phi_0}{32\pi^2\lambda^4\widetilde{k}^2}\ln\left(1+\frac{\widetilde{k}^2}{k_{\mathrm{BZ}}^2}\right). \tag{A7}$$

The inverse of the dynamical matrix (A1) is

$$\mathbf{\Phi}_{\mathbf{p}}^{-1} = \frac{1}{\det[\mathbf{\Phi}_{\mathbf{p}}]}\begin{bmatrix}\Phi_{\mathbf{p}}^{yy} & -\Phi_{\mathbf{p}}^{yx}\\ -\Phi_{\mathbf{p}}^{xy} & \Phi_{\mathbf{p}}^{xx}\end{bmatrix}, \tag{A8}$$

with $\det[\mathbf{\Phi}_{\mathbf{p}}] = (c_{11}q^2+c_{44}\widetilde{k}^2)(c_{66}q^2+c_{44}\widetilde{k}^2)$.

APPENDIX B: A PANCAKE VORTEX IN A VORTEX LIQUID

In this appendix we calculate the energy of a pancake in a liquid of pancakes and the screening length for the pancake's magnetic field and interaction with other pancakes. We follow the ideas of the Debye-Hückel theory of strong electrolytes.

Consider a probing pancake vortex added to the point $\mathbf{r}=0$ in a liquid of pancakes with average density $n = B/\Phi_0 s$. As a result the energy of a pancake located at arbitrary position $\mathbf{r}=\mathbf{R}$ will change by

$$\Delta E(\mathbf{R}) = \int d^3\mathbf{r}\frac{\mathbf{A}(\mathbf{r})\cdot\mathbf{j}^0(\mathbf{r}+\mathbf{R})}{c} = \int\frac{d^3\mathbf{p}}{(2\pi)^3}\frac{\mathbf{A}_{\mathbf{p}}\cdot\mathbf{j}_{-\mathbf{p}}^0}{c}e^{i\mathbf{p}\cdot\mathbf{R}}, \tag{B1}$$

where $\mathbf{A}(\mathbf{r})$ is the total vector potential, $\mathbf{j}^0(\mathbf{r})$ is the current induced by the pancake located at $\mathbf{r}=0$, and the Fourier transform $\mathbf{j}_{-\mathbf{p}}^0$ is

$$\mathbf{j}_{-\mathbf{p}}^0 = \frac{c}{4\pi}p^2\mathbf{A}_{\mathbf{p}}^0, \tag{B2}$$

with the vector potential $\mathbf{A}_{\mathbf{p}}^0$ taken in the limit $ks \ll 1$ [see Eq. (5)],

$$\mathbf{A}_{\mathbf{p}}^0 = i\frac{\Phi_0 s}{(1+\lambda^2 p^2)}\frac{\mathbf{q}\times\hat{\mathbf{z}}}{q^2}. \tag{B3}$$

The interaction between the inserted pancake and the vortices of the liquid affects the local density of the pancake vortices $n(\mathbf{R})$,

$$n(\mathbf{R}) = n\exp\left[-\frac{\Delta E(\mathbf{R})}{k_BT}\right]. \tag{B4}$$

Assuming that $\Delta E(\mathbf{R})\ll k_BT$ we find

$$\delta n(\mathbf{R}) = n(\mathbf{R})-n = -n\frac{\Delta E(\mathbf{R})}{k_BT}. \tag{B5}$$

Combining Eqs. (B1)–(B5) we obtain for the Fourier transform of $\delta n(\mathbf{R})$

$$\delta n_{\mathbf{p}} = -\frac{np^2}{4\pi k_BT}\mathbf{A}_{\mathbf{p}}\cdot\mathbf{A}_{\mathbf{p}}^0. \tag{B6}$$

The total vector potential $\mathbf{A}_{\mathbf{p}}$ up to the first order in $\delta n(\mathbf{R})$ [compare with Eq. (13)] is

$$\mathbf{A}_{\mathbf{p}} - \mathbf{A}_{\mathbf{p}}^0 = \delta n_{\mathbf{p}}\mathbf{A}_{\mathbf{p}}^0. \tag{B7}$$

Using Eqs. (B6) and (B7) we obtain for $\mathbf{A}_{\mathbf{p}}$ the equation

$$\mathbf{A}_{\mathbf{p}} = \frac{\mathbf{A}_{\mathbf{p}}^0}{\mu_{\mathrm{liq}}(\mathbf{p})}, \tag{B8}$$

where

$$\mu_{\mathrm{liq}}(\mathbf{p}) = 1 + \frac{\alpha^2 p^2}{q^2(1+\lambda^2 p^2)^2} \tag{B9}$$

is an effective permeability and we have introduced the dimensionless parameter

$$\alpha^2 = \frac{B\Phi_0 s}{4\pi k_BT} \ll 1. \tag{B10}$$

In real space Eqs. (B8) and (B9) for $\rho\gg\lambda/\alpha$ yield

$$A_\varphi(\rho,z) \simeq \frac{\Phi_0 d}{2\pi}\frac{\rho}{(\alpha^2\rho^2+z^2)^{3/2}}. \tag{B11}$$

Note that the vector potential for a pancake in a liquid of pancakes decays along the z axis as a power law (for $z\gg\rho\alpha$), whereas the vector potential for a single pancake decays exponentially $\mathbf{A}^0(\mathbf{r})\propto\exp(-z/\lambda)$.

We calculate now the energy of a pancake in a vortex liquid. In the London approximation the free energy is

$$F = \frac{1}{2c} \int d^3\mathbf{r} \left(\mathbf{A} \cdot \mathbf{j} + \frac{4\pi\lambda^2}{c} \mathbf{j}^2 \right), \tag{B12}$$

$$\mathbf{j} = -\frac{c}{4\pi} \nabla^2 \mathbf{A}. \tag{B13}$$

Equation (B12) can be rewritten as

$$F = \frac{1}{2c(2\pi)^3} \int d^3\mathbf{p} \left(\mathbf{A_p} \cdot \mathbf{j_{-p}} + \frac{4\pi\lambda^2}{c} |\mathbf{j_p}|^2 \right),$$

$$= \frac{1}{2c(2\pi)^3} \int d^3\mathbf{p} \mathbf{A_p} \cdot \mathbf{j_{-p}} (1 + \lambda^2 p^2). \tag{B14}$$

Using $\mathbf{A_p}$ from Eq. (B8) and $\mathbf{j_p}$ from Eq. (B13) we obtain

$$F = \frac{\Phi_0^2 d^2}{8\pi} \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{p^2 q^2 (1+\lambda^2 p^2)^3}{[q^2(1+\lambda^2 p^2)^2 + \alpha^2 p^2]^2}. \tag{B15}$$

The integration here is over the region

$$0 \le q \le 1/\xi, \quad -\pi/s \le k \le \pi/s. \tag{B16}$$

For small α, the main contribution to the integral (B15) comes from $q \ll \xi^{-1}$ and from large $k \approx \pi/s$. We can therefore approximate it by integrating over the sphere $p \lesssim \pi/s$. As our final result we obtain the energy of a pancake in a liquid of pancakes in the form

$$F \approx \frac{\Phi_0^2 s}{(4\pi\lambda)^2} \ln\left[\frac{2}{\alpha} \left(\frac{\pi\lambda}{s} \right)^2 \right]. \tag{B17}$$

Comparing this energy with the self-energy of a single pancake, $F_s \approx (\Phi_0^2 s / 8\pi^2 \lambda^2) \ln(L/\xi)$ (where L is the size of the sample), we see that the screening length r_{scr} for a pancake in a liquid of pancakes is

$$r_{scr}^2 = \frac{2\pi^2 \lambda^2 \xi^2}{\alpha s^2} = \pi \frac{k_B T}{(\Phi_0^2 s / 8\pi^2 \lambda^2)} \frac{\xi^2}{s^2} a^2 \gg a^2. \tag{B18}$$

This screening length $r_{scr}^2 \propto T$ is analogous to the Debye-Hückel screening length in strong electrolytes.

[1] K. B. Efetov, Zh. Eksp. Teor. Fiz. **76**, 1781 (1979) [Sov. Phys. JETP **49**, 905 (1979)].

[2] S. N. Artemenko and A. N. Kruglov, Phys. Lett. A **143**, 485 (1990).

[3] J. R. Clem, Phys. Rev. B **43**, 7837 (1991).

[4] G. Blatter, M. V. Feigelman, V. B. Geshkenbein, A. I. Larkin, and V. M. Vinokur, Rev. Mod. Phys. **66**, 1125 (1994).

[5] E. H. Brandt, Rep. Prog. Phys. **58**, 1465 (1995).

[6] E. H. Brandt, Phys. Status Solidi **35**, 1027 (1969); **36**, 371 (1969); **36**, 381 (1969); **36**, 393 (1969).

[7] A. Buzdin and D. Feinberg, J. Phys. (Paris) **51**, 1971 (1990).

[8] E. H. Brandt, J. Low Temp. Phys. **26**, 709 (1977); **26**, 735 (1977).

[9] A. Sudbø and E. H. Brandt, Phys. Rev. Lett. **66**, 1781 (1991).

[10] L. N. Bulaevskii, M. P. Maley, and I. F. Schegolev, Physica B **197**, 506 (1994).

[11] L. I. Glazman and A. E. Koshelev, Phys. Rev. B **43**, 2835 (1991).