Josephson junctions with alternating critical current density

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The magnetic-field dependence of the critical current $I_c(H)$ is considered for a short Josephson junction with the critical current density j_c alternating along the tunnel contact. Two model cases, periodic and randomly alternating j_c, are treated in detail. Recent experimental data on $I_c(H)$ for grain-boundary Josephson junctions in $YBa_2Cu_3O_x$ are discussed. [S0163-1829(97)51614-6]

Considerable progress has recently been reported in understanding properties of grain-boundary Josephson junctions in $YBa_2Cu_3O_x$ (YBCO) films.[1,2] The boundaries were found to have facets with a variety of orientations, the fact which, in conjunction with the d-wave symmetry of the order parameter, led to the conclusion that the critical current density j_c may differ both in value and the sign at different facets.[2] This is offered as the reason for the grain-boundary critical current I_c being significantly suppressed relative to the bulk value.[2,3]

The dependence of I_c on the applied field H, one of the major junction properties relevant for applications, has also been studied. The observed patterns $I_c(H)$ are manifestly non-Fraunhofer and difficult for interpretation.[2–5] We show in this paper that qualitative features of these patterns can be attributed to the basic fact that the local critical current density j_c changes sign from one facet to another. Moreover, the alternating character of j_c results in a shift of the major maximum in $I_c(H)$ from $H=0$ of the standard Fraunhofer pattern to a position related to periodicity in the distribution of $j_c(x)$, where x is the axis along the tunnel contact. Random deviations from periodicity change dramatically patterns of $I_c(H)$.

In the following we calculate the critical current I_c for a Josephson junction with the length $L \ll \lambda_J$, a typical value of the local Josephson penetration depth. The current density across the junction is $j(x)=j_c(x)\sin\varphi(x)$, where $\varphi(x)$ is the phase difference. The magnetic field H is nearly constant inside a short Josephson junction; in this case[6]

$$\varphi=\varphi_0+kx, \quad k=2\pi\Phi/\Phi_0 L, \tag{1}$$

where $\varphi_0=\text{const}$, $\Phi=2\lambda LH$ is the total flux in the junction, λ is the London penetration depth, and Φ_0 is the flux quantum.

To evaluate the total current I through the junction,

$$I=\int_{-L/2}^{L/2} j_c(x)\sin(\varphi_0+kx)dx, \tag{2}$$

we write $j_c(x)$ as Fourier series

$$j_c(x)=\sum_n [a_n\cos(2\pi nx/L)+b_n\sin(2\pi nx/L)] \tag{3}$$

and integrate with the result:

$$I=\sin(\pi\phi)(a\sin\varphi_0+b\cos\varphi_0), \tag{4}$$

$$a=\sum_n (-1)^n \frac{a_n L}{\pi}\frac{\phi}{\phi^2-n^2}, \tag{5}$$

$$b=\sum_n (-1)^n \frac{b_n L}{\pi}\frac{n}{\phi^2-n^2}, \tag{6}$$

where $\phi=\Phi/\Phi_0$ is the dimensionless flux.

The critical current I_c at a given field is found by maximizing I relative to the still free parameter φ_0:

$$I_c=|\sin(\pi\phi)|\sqrt{a^2+b^2}. \tag{7}$$

Equation (7) follows also from a general relation $I_c(\phi)=|\tilde{j}_c(k)|$, where $\tilde{j}_c(k)$ is the Fourier transform of $j_c(x)$.[7]

Functions $a(\phi)$ and $b(\phi)$ are divergent at $\phi=m$ with an integer m; nevertheless I_c is finite at integer ϕ's due to $\sin(\pi\phi)=0$. Using Eqs. (5), (6), and (7) we obtain that at $\phi=m$, the critical current is determined only by corresponding Fourier transforms:

$$I_c=\begin{cases} a_0 L, & \text{for } \phi=0, \\ 0.5\sqrt{a_m^2+b_m^2}L, & \text{for } \phi=m. \end{cases} \tag{8}$$

In particular, we see that in zero magnetic field

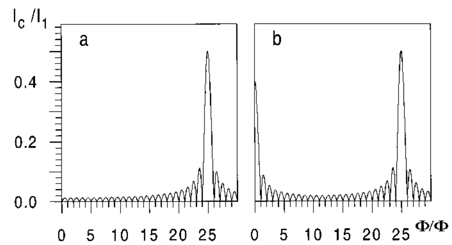

FIG. 1. The dependence of I_c/I_1 on Φ/Φ_0 for $N=25$ and (a) $j_0=0$, (b) $j_0=0.4j_1$.

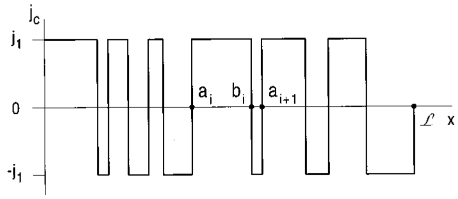

FIG. 2. A model for a random critical current density distribution.

$$I_c(0)=\int_0^L j_c(x)dx. \qquad (9)$$

This equation is a generalization of the formula $I_c(0)=j_cL$ for uniform junctions with $j_c=\text{const.}$ to inhomogeneous junctions with $j_c(x)$. Equation (2) shows that if $j_c(x)$ is positive within a junction, the critical current reaches its absolute maximum at $H=0$. For an arbitrary $j_c(x)$, the last statement is not necessarily true. Consider, for example, a junction made of equal numbers of identical facets with negative and positive j_c's so that the integral in Eq. (9) vanishes; $I_c(0)=0$ for this case, and the pattern $I_c(H)$ has a zero at $H=0$ instead of the central Fraunhofer maximum.

In general, if the average value of $j_c(x)$ is small, i.e.,

$$I_c(0)=\int_0^L j_c(x)dx \ll \int_0^L |j_c(x)|dx, \qquad (10)$$

$I_c(0)$ can be much less than the maximum value of the critical current achieved at a certain magnetic-field $H_{\max}\neq 0$. Qualitatively, this happens because the sign change (and the current suppression) due to the field-dependent phase factor $\sin\varphi(x)$ can be compensated by the sign change of $j_c(x)$ provided these two are accurately correlated. Therefore, a pattern $I_c(H)$ with $I_c(0)\ll I_c(H_{\max})$ is a clear signature of the critical current density taking both positive and negative values.

To demonstrate the main features of the pattern $I_c(\phi)$ caused by an alternating critical current density, we consider two model dependencies for $j_c(x)$.

First we treat a simple periodic dependence

$$j_c(x)=j_0+j_1\sin(2\pi Nx/L), \qquad (11)$$

with an integer N. In zero magnetic field the critical current $I_c(0)=j_0L$, as is seen from Eq. (9). There are only two nonzero Fourier coefficients in the expansion (3): $a_0=j_0$ and $b_N=j_1$. Therefore, Eq. (7) yields

$$I_c=\frac{|\sin(\pi\phi)|}{\pi}\left(\frac{I_0^2}{\phi^2}+\frac{I_1^2N^2}{(\phi^2-N^2)^2}\right)^{1/2}, \qquad (12)$$

where $I_0=j_0L$ and $I_1=j_1L$. We show in Fig. 1 the field dependence of I_c for $N=25$ and $j_0=0$ (a), $j_0=0.4j_1$ (b). It is seen that $I_c(\phi)$ oscillates with a slightly varying amplitude when the field increases. A strong peak occurs at $\phi=25$; this value corresponds to one flux quantum per the period L/N.

The shift of the peak from the central position $\phi=0$ to $\phi=N$ for the case $j_0=0$ [zero average of $j_c(x)$] can be understood as follows: The maximum contribution from the oscillating term in $j_c(x)$ to the total current I corresponds to such a flux for which the term $j_1\sin(2\pi Nx/L)$ and the phase factor $\sin\varphi(x)$ change signs simultaneously. Comparing Eqs. (1) and (11) we find that this happens if $\varphi_0=0$ and $\phi=N$. Thus, precise correlation between the phase factor and the $j_c(x)$ oscillations causes I_c to reach its maximum value of $0.5j_1L$ at $\phi=N$.

Note that for the nonzero average critical current density ($j_0\neq 0$), the pattern $I_c(H)$ still has the standard central Fraunhofer peak at $\phi=0$ with the height proportional to j_0. The central peak constitutes the main difference between patterns for $j_0=0$ and $j_0\neq 0$.

We now turn to the effect of randomness in the spatial distribution of $j_c(x)$ on the field dependence of critical current I_c. We use a model dependence $j_c(x)$ shown schematically in Fig. 2, namely, the critical current density alternates sequentially taking two values j_1 and $-j_1$:

$$j_c=\begin{cases} j_1, & \text{if } a_i<x<b_i, \\ -j_1, & \text{if } b_i<x<a_{i+1}, \end{cases} \qquad (13)$$

where $i=1,2,\ldots,N$. Thus $j_c=j_1$ within N intervals with the lengths $l_i^+=b_i-a_i$, and $j_c=-j_1$ within N intervals $l_i^-=a_{i+1}-b_i$. The sequences l_i^+ and l_i^- are random with average values

$$l^\pm=\frac{1}{N}\sum_{i=1}^N l_i^\pm. \qquad (14)$$

We characterize the distribution of j_c by its average $j_0=j_1(l^+-l^-)/L$, and by the dispersion

$$\sigma=\left[\frac{1}{N}\sum_{i=1}^N (l_i^\pm)^2-(l^\pm)^2\right]^{1/2}. \qquad (15)$$

We treat here the case when both sequences l_i^+ and l_i^- have the same value of σ.

After straightforward algebra we obtain for the tunneling current

$$I=\frac{I_1}{2\pi\phi}(A\cos\varphi_0-B\sin\varphi_0), \qquad (16)$$

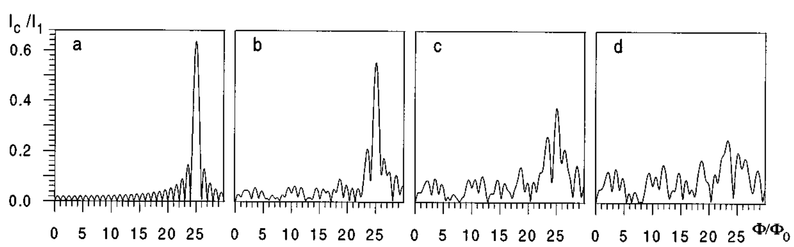

FIG. 3. The dependence of I_c/I_1 on Φ/Φ_0 for $j_0=0$ and (a) $\sigma=0$, (b) $\sigma=0.142$, (c) $\sigma=0.275$, (d) $\sigma=0.397$.

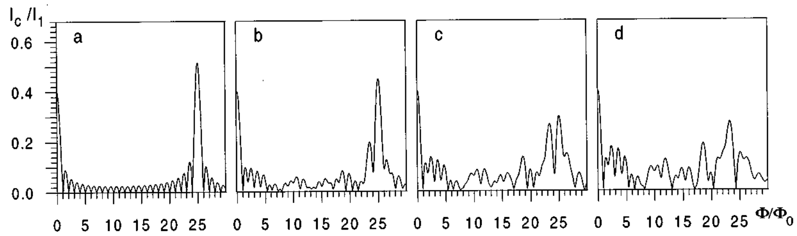

FIG. 4. The dependence of I_c/I_1 on Φ/Φ_0 for $j_0=0.4j_1$ and (a) $\sigma=0$, (b) $\sigma=0.142$, (c) $\sigma=0.275$, (d) $\sigma=0.397$.

where $I_1=j_1L$,

$$A=\sum_{i=1}^{N}(\cos ka_i+\cos ka_{i+1}-2\cos kb_i), \qquad (17)$$

$$B=\sum_{i=1}^{N}(\sin ka_i+\sin ka_{i+1}-2\sin kb_i). \qquad (18)$$

The maximum current at a given magnetic field is

$$I_c=\frac{I_1}{2\pi\phi}\sqrt{A^2+B^2}. \qquad (19)$$

Figures 3 and 4 show the field dependence of I_c for $N=25$ and different values of j_0 and σ.

The critical current density $j_c(x)$ is a periodic function when $\sigma=0$. The fingerprint of this periodicity is the peak seen at $\phi=25$ in Figs. 3(a) and 4(a). The peak corresponds to one flux quantum per one period $L/25$ of $j_c(x)$. Randomness of the spatial distribution of j_c smears the peak at $\phi=25$. Remarkably, the central peak at $\phi=0$, $I_c(0)=j_1(l^+-l^-)=j_0L$, is affected not by randomness, but only by total lengths where j_c is positive and negative [i.e., by the nonzero average of $j_c(x)$].

In conclusion, we have studied the effect of alternating critical current density $j_c(x)$ on the field dependence of the junction critical current $I_c(H)$. We have found that if the average j_c is small, the major peak in the pattern $I_c(H)$ is shifted aside from the central position of the standard Fraunhofer pattern. Two particular situations are considered: a smooth sinusoidal and a stepwise periodic $j_c(x)$ alternating between positive and negative values of equal size. Both model dependencies result in qualitatively similar patterns $I_c(H)$ with shifted major peaks. To simulate properties of real grain-boundary junctions, we introduced random distribution of steps and showed that the randomness smears the major peak and strengthens the minor ones, however, it leaves the position of the shifted peak in place for a weak randomness. We consider the shift of the major peak as the signature of the alternating nature of the critical current density. This feature is seen indeed in experimental $I_c(H)$ of the 45° grain boundaries in YBCO films.[8] It remains to be seen how much extra detail can be extracted from the observed patterns of $I_c(H)$.

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