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Nonlocal Josephson Electrodynamics

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We present a review of the main results of the recently developed nonlocal Josephson electrodynamics. A nonlinear integro-differential equation for the phase difference is derived and its applicability to different problems is discussed. Fluxons and electromagnetic waves propagating along a tunnel junction are examined in detail. Features specific for the limiting case of a Josephson junction in a very thin film are considered.

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1. INTRODUCTION

The electromagnetic properties of Josephson tunnel junctions are a subject of intensive studies over the past three decades.[1,2] In particular, substantial attention is attracted to the SIS-type Josephson contacts. In this case the electromagnetic properties are described by the sine-Gordon equation for the space and time dependent phase difference $\varphi(x,t)$ across the junction

$$\varphi_{\tau\tau} - \varphi_{\zeta\zeta} + \eta\varphi_\tau + \sin\varphi = \beta, \tag{1}$$

where the subscripts τ and ζ denote the derivatives over the dimensionless time $\tau = t\omega_J$ and coordinate $\zeta = x/\lambda_J$,

$$\omega_J = \sqrt{\frac{2ej_c}{\hbar C}} \tag{2}$$

is the Josephson plasma frequency, C is the specific capacitance of the junction, j_c is the critical current density across the Josephson junction,

$$\lambda_J = \sqrt{\frac{c\Phi_0}{16\pi^2\lambda j_c}} \tag{3}$$

is the Josephson penetration length, Φ_0 is the flux quantum, λ is the London penetration depth,

$$\eta = \frac{1}{\omega_J RC} \tag{4}$$

is the damping constant, R is the specific resistance of the junction, and $\beta = j/j_c$ is the dimensionless density of the bias current across the junction.

When applied to the electrodynamics of a Josephson tunnel contact the sine-Gordon equation (1) has its origin in a local relation between the phase difference $\varphi(x,t)$ and the magnetic field inside the junction $H(x,t)$, namely,

$$H(x,t) = \frac{\Phi_0}{4\pi\lambda}\frac{\partial\varphi}{\partial x}. \tag{5}$$

This local relation is valid if $\varphi(x,t)$ varies slowly over the lengths typical for spatial variations of $H(x,t)$. The characteristic space scale for the field $H(x,t)$ is determined by the London penetration depth λ for a tunnel contact with the thickness $d \gg \lambda$ and by the Pearl length[3]

$$\lambda_{\text{eff}} = \frac{\lambda^2}{d} \gg \lambda \tag{6}$$

for a thin film ($d \ll \lambda$). Introducing the characteristic space scale l for the spatial variations of the phase difference $\varphi(x,t)$ we can, therefore, summarize that the sine-Gordon equation (1) describes the electromagnetic properties of a long Josephson junction if $l \gg \lambda$ (for $d \gg \lambda$) and $l \gg \lambda_{\text{eff}}$ (for $d \ll \lambda$).

To illustrate the application of these relations let us consider the following two phenomena, namely, the Josephson fluxons and electromagnetic waves propagating along a Josephson tunnel contact with $d \gg \lambda$. We estimate $l \sim \lambda_J$ in the first case and $l \sim k^{-1}$, where k is the wave number, in the second case. Thus, the sine-Gordon (1) is applicable to treat a phase kink if $\lambda_J \gg \lambda$ and an electromagnetic wave if $k\lambda \ll 1$.

Usually the Josephson penetration depth λ_J is assumed to be much larger than the London penetration depth λ because of the small value of the critical current density across the tunnel junction. However, the copper-oxide high-temperature superconductors contain coherent planar defects such as twins, stacking faults, low-angle grain boundaries, etc.[4] These structural defects do not cause strong crystalline lattice distortions and can be treated as intrinsic Josephson junctions with high values of j_c and therefore with small values of the Josephson penetration depth λ_J. The extremely anisotropic Bi and Tl based copper oxide compounds consist of a periodic stack of weakly coupled two-dimensional CuO layers were the superconductivity resides. A variety of linear structural defects result from the crossing of the superconducting layers with planar structural defects. These linear defects can be treated as intrinsic Josephson junctions with $d \ll \lambda$ and high values of the critical current density j_c, *i.e.*, as Josephson junctions with small values of the Josephson penetration depth λ_J and large values of the effective penetration depth.

The intrinsic tunnel contacts specific for the high-temperature superconductors initiated a considerable interest to investigate the electromagnetic properties of the SIS-type Josephson junctions where the space scale of variations of the phase difference is less than the space scale of variations of the magnetic field.[5-26] In this case the relation between $H(x,t)$ and $\varphi(x,t)$ is an integral relation, *i.e.*, it is nonlocal. As a result the phase difference $\varphi(x,t)$ is described by an integro-differential equation,[6,7,18] which is the basis for the nonlocal Josephson electrodynamics.

In particular, when applied to Josephson fluxons in a long tunnel contact with $d \gg \lambda$ this nonlocality means that $\lambda_J < \lambda$. In terms of the critical current density across the junction it takes the form $j_d/\kappa < j_c < j_d$, where $j_d = c\Phi_0/12\sqrt{3}\pi^2\lambda^2\xi$ is the depairing current density and $\kappa = \lambda/\xi$ is the Ginzburg-Landau parameter.[6,12] Note that for the extreme type-II superconductors with $\kappa \gg 1$ the relation $\lambda_J < \lambda$ holds in a wide region of j_c.

In this paper we review the nonlocal Josephson electrodynamics of a one-dimensional SIS-type tunnel junction. We derive the integro-differential equation describing the phase difference for the cases $d \gg \lambda$ and $d \ll \lambda$. We apply this equation to treat two phenomena, namely, the Josephson fluxons and the electromagnetic waves propagating along a tunnel contact.[1] It is shown that nonlocality significantly changes the spatial distributions of the magnetic field and current for a Josephson vortex as well as the dispersion relations for an electromagnetic wave.

The paper is organized in the following way. In Sec. 2, we consider the nonlocal Josephson electrodynamics of a one-dimensional SIS-type tunnel junction. The integro-differential equation describing the phase difference $\varphi(x,t)$ is derived for the two limiting cases $d \gg \lambda$ and $d \ll \lambda$. In Sec. 3, we apply this equation to consider Josephson fluxons. In Sec. 4, the dispersion relation is derived for an electromagnetic wave propagating along a tunnel contact. In Sec. 5, we summarize the overall conclusions.

2. MAIN EQUATIONS

Let us treat a planar SIS-type Josephson junction parallel to the xz-plane and assume that $\lambda \gg \xi$. In this case the magnetic field $B_z(x,y,t)$ and current density $\mathbf{j}(x,y)$ inside the superconductor are described by the

[1]The nonlocality effect on pinning of Abrikosov and Josephson vortices by a high-j_c planar defect and a network of such defects is discussed in detail in Refs.[6,12,17,26]

London equations taking the form

$$\lambda^2 \Delta \mathbf{B} - \mathbf{B} = 0, \tag{7}$$

$$\mathbf{j} = -\frac{c}{4\pi\lambda^2}\left(\mathbf{A} - \frac{\Phi_0}{2\pi}\nabla\theta\right), \tag{8}$$

where $\mathbf{A}$ is the vector potential and θ is the phase of the order parameter. The magnetic field $H(x,t)$ inside the Josephson junction is determined by the boundary conditions $B_z(x, \pm 0, t) = H(x,t)$ and the phase difference $\varphi(x,t)$ is defined as $\varphi(x,t) = \theta(x, +0, t) - \theta(x, -0, t)$.

Let us first derive the equation for $\varphi(x,t)$ in the case of a thick tunnel contact, *i.e.*, for $d \gg \lambda$. We begin this derivation with the relations

$$j_x(x,+0) - j_x(x,-0) = \frac{c\Phi_0}{8\pi^2\lambda^2}\frac{\partial\varphi}{\partial x}, \tag{9}$$

$$j_x(x,+0) - j_x(x,-0) = \frac{c}{4\pi}\left[\frac{\partial B}{\partial y}\bigg|_{y=+0} - \frac{\partial B}{\partial y}\bigg|_{y=-0}\right], \tag{10}$$

which follow from Eq. (8) and the Maxwell equation $\mathrm{rot}\mathbf{B} = (4\pi/c)\,\mathbf{j}$. Taking into account that $B(x,y) = B(x,-y)$ and combining Eqs. (9) and (10) we find the boundary conditions for the magnetic field $B(x,y)$ in the form

$$\frac{\partial B}{\partial y}\bigg|_{y=+0} = -\frac{\partial B}{\partial y}\bigg|_{y=-0} = \frac{\Phi_0}{4\pi\lambda^2}\frac{\partial\varphi}{\partial x}. \tag{11}$$

The solution of Eq. (7) matching the equations (11) results in the following expressions for the magnetic field $H(x,t)$ and current density across the Josephson junction $j_y(x,t) = j_y(x, \pm 0, t)$

$$H(x,t) = \frac{\Phi_o}{4\pi^2\lambda^2}\int_{-\infty}^{\infty} K_0\left(\frac{|x-u|}{\lambda}\right)\frac{\partial\varphi}{\partial u}\,du, \tag{12}$$

$$j_y(x,t) = \frac{c\Phi_0}{16\pi^3\lambda^2}\int_{-\infty}^{\infty} K_0\left(\frac{|x-u|}{\lambda}\right)\frac{\partial^2\varphi}{\partial u^2}\,du, \tag{13}$$

where $K_0(x)$ is the zero order modified Bessel function. At the same time the current $j_y(x,t)$ is a sum of the tunnel current $j_c \sin\varphi$, the displacement current $\hbar C\ddot{\varphi}/2e = j_c\varphi_{\tau\tau}$, and the resistive current $\hbar\dot{\varphi}/2eR = j_c\eta\varphi_\tau$, *i.e.*,

$$j_y(x,t) = j_c \sin\varphi + j_c\varphi_{\tau\tau} + j_c\eta\varphi_\tau. \tag{14}$$

We equate now the two expressions (13) and (14) for the current density $j_y(x,t)$. This results in the equation[6,7]

$$\varphi_{\tau\tau} + \eta\varphi_\tau = \frac{l_J}{\pi}\int_{-\infty}^{\infty} K_0\left(\frac{|x-u|}{\lambda}\right)\frac{\partial^2\varphi}{\partial u^2}\,du - \sin\varphi + \beta, \tag{15}$$

where β accounts for the bias current $j = j_c\beta$ and

$$l_J = \frac{\lambda_J^2}{\lambda} = \frac{c\Phi_0}{16\pi^2\lambda^2 j_c}. \tag{16}$$

The integro-differential equation (15) determines the nonlocal Josephson electrodynamics of a tunnel contact with the thickness $d \gg \lambda$. It is valid for any relation between l and λ. Note that if the phase difference $\varphi(x,t)$ varies slowly on the length scale of the order of λ, *i.e.*, if $l \gg \lambda$, then the function $K_0(x)$ can be replaced by $\pi\delta(x)$. As a result the integro-differential equation (15) transforms to the sine-Gordon equation (1) and the integral relation (12) transforms to the local relation (5).

Let us now consider the equation for $\varphi(x,t)$ for a tunnel contact in a thin film with $d \ll \lambda$. In this case the stray field outside the superconductor is important and significantly affects the current density across the junction $j_y(x,t)$. As a result the space scale for the field $H(x,t)$ is determined by the Pearl length λ_{eff} and the equation determining the nonlocal Josephson electrodynamics for a thin film takes the form[18]

$$\varphi_{\tau\tau} + \eta\varphi_\tau = \frac{l_J}{\pi}\int_{-\infty}^{\infty} Q_0\left(\frac{|x-u|}{2\lambda_{\text{eff}}}\right)\frac{\partial^2\varphi}{\partial u^2}\,du - \sin\varphi + \beta, \tag{17}$$

where

$$Q_0(x) = \int_0^\infty \frac{J_0(v)}{v+x}\,dv, \tag{18}$$

and $J_0(x)$ is the zero-order Bessel function.

In the limiting case when the characteristic space scale of the phase difference $\varphi(x,t)$ variation is extremely small, *i.e.*, for $l \ll \lambda$ ($d \gg \lambda$) and $l \ll \lambda_{\text{eff}}$ ($d \ll \lambda$), the kernels of Eqs. (15) and (17) can be replaced by their expansions at small argument,

$$K_0(x) \approx Q_0(x) \approx \ln(2/x) - \mathbf{C}, \tag{19}$$

where $\mathbf{C} \approx 0.577$ is the Euler constant. As a result both equations (15) and (17) take the same form[8,18]

$$\varphi_{\tau\tau} + \eta\varphi_\tau = \frac{l_J}{\pi}\int_{-\infty}^{\infty} \frac{\partial\varphi}{\partial u}\frac{du}{u-x} - \sin\varphi + \beta. \tag{20}$$

To complete we present here the expression for the energy $\mathcal{E}$ of an ideal tunnel contact ($\eta = 0$) per unit area

$$\mathcal{E} = \frac{\hbar j_c\lambda_J}{4e}\int_{-\infty}^{\infty}\left[\varphi_\tau^2 + 2(1-\cos\varphi) + \frac{\lambda_J}{\pi\lambda}\int_{-\infty}^{\infty} K(\zeta-\zeta')\varphi_\zeta\varphi_{\zeta'}\,d\zeta\right]d\zeta', \tag{21}$$

where the subscript ζ denotes the derivative over the coordinate $\zeta = x/\lambda_J$, $K(\zeta) = K_0(\lambda_J|\zeta|/\lambda)$ for $d \gg \lambda$, and $K(\zeta) = Q_0(\lambda_J|\zeta|/2\lambda_{\text{eff}})$ for $d \ll \lambda$.

3. JOSEPHSON VORTICES

Let us first assume that there is no bias current ($\beta = 0$) and consider a stationary Josephson fluxon, *i.e.*, a phase kink described by a phase difference distribution $\varphi(x)$ matching the boundary conditions $\varphi(-\infty) = 0$ and $\varphi(\infty) = 2\pi$. Explicit solutions describing a single stationary Josephson vortex are known in two limiting cases, namely, in the local ($l \gg \lambda$) and the nonlocal ($l \ll \lambda$ for $d \gg \lambda$ and $l \ll \lambda_{\rm eff}$ for $d \ll \lambda$) limits.

In the local case the stationary dependence $\varphi(x)$ for a phase kink is given by the well-known solution of the sine-Gordon equation[1]

$$\varphi(x) = 4 \arctan \left[\exp\left(\frac{x}{\lambda_J}\right)\right]. \tag{22}$$

Note that the asymptotic dependence of the phase difference (22) on x at $|x| \to \pm\infty$ is exponential and therefore the current density and magnetic field in the region $x > \lambda_j$ also decay exponentially.[1]

In the nonlocal case the stationary dependence $\varphi(x)$ for a phase kink is given by a stationary solution of Eq. (20), which has the form[6,8]

$$\varphi(x) = 2 \arctan\left(\frac{x}{l_J}\right) + \pi. \tag{23}$$

Note that contrary to the local case the dependence (23) decays as a power law ($\propto 1/|x|$) at $|x| \to \pm\infty$. Using Eq. (23) we find the following explicit expressions for $j_x(x, \pm 0)$, $j_y(x)$, and $H(x)$ at $x \ll l$

$$j_x(x, \pm 0) = \pm j_c \frac{2l_J^2}{x^2 + l_J^2} = \pm \frac{c\Phi_0}{8\pi\lambda^2} \frac{l_J}{x^2 + l_J^2}, \tag{24}$$

$$j_y(x) = j_c \frac{2xl_J}{x^2 + l_J^2} = \frac{c\Phi_0}{8\pi\lambda^2} \frac{x}{x^2 + l_J^2}, \tag{25}$$

$$H(x) = \frac{\Phi_0}{4\pi\lambda^2} \left[\ln \frac{4\lambda^2}{x^2 + l_J^2} - 2\mathrm{C}\right], \tag{26}$$

where $\mathrm{C} \approx 0.577$ is the Euler constant. It follows from Eqs. (24) - (26) that in the nonlocal case the current density and magnetic field of a Josephson vortex decay as power laws in the region $l_J \ll x \ll l$. Note that the same asymptotic dependencies are characterizing Abrikosov vortices[27] with the only difference that $l_J \to \xi$ and $l \to \lambda$, where ξ is the coherence length.

In the presence of a bias current ($\beta \neq 0$) a Josephson vortex is moving along the tunnel contact. This motion results in an electric field $E \propto \varphi_\tau$ localized inside the Josephson junction and therefore in an increase of the energy described by the first term in Eq. (21). Let us consider a tunnel contact with a high specific resistance R resulting in a low damping ($\eta \ll 1$).

In this case for small velocities v of a Josephson fluxon ($v \ll l_J\omega_J$) the phase difference distribution is given by Eq. (23) with the substitution $x \to x - vt$. The energy of the electric field inside the Josephson junction is then

$$\mathcal{E}_{el} = \frac{\hbar j_c}{4e}\int_{-\infty}^{\infty}\varphi_\tau^2\,dx = \frac{v^2C\hbar^2}{8e^2}\int_{-\infty}^{\infty}\Big(\frac{\partial\varphi}{\partial x}\Big)^2dx = \frac{\pi C\hbar^2v^2}{4e^2l_J}, \tag{27}$$

i.e., $\mathcal{E}_{el} \propto v^2$ and in the nonlocal case as well as in the local case a Josephson vortex is characterized by the mass per unit length m, where

$$m = \frac{\pi\hbar^2C}{2e^2l_J}. \tag{28}$$

Let us now find the mobility per unit length μ of a Josephson fluxon. To do it we calculate the total dissipation rate $\dot{Q}$

$$\dot{Q} = \frac{1}{R}\Big(\frac{\hbar\omega_J}{2e}\Big)^2\int_{-\infty}^{\infty}\varphi_\tau^2dx = \frac{v^2}{R}\Big(\frac{\hbar}{2e}\Big)^2\int_{-\infty}^{\infty}\Big(\frac{\partial\varphi}{\partial x}\Big)^2dx = \frac{\Phi_0^2v^2}{2\pi c^2l_JR} \tag{29}$$

and equate it to v^2/μ. As a result we obtain the following expression for μ

$$\mu = \frac{2\pi c^2l_JR}{\Phi_0^2} = \frac{RC}{m}. \tag{30}$$

In the local case the expressions for m and μ have the same form as the expressions given by Eqs. (28) and (30) with l_J replaced by $4\lambda_J/\pi \sim \lambda_J$.[1] Thus in the nonlocal case a Josephson vortex has a bigger mass and a lower mobility as compared to the local case.

We consider now a tunnel contact in the overdamped regime ($\eta \gg 1$). In this case the first term in the left part of Eq. (20) can be neglected. The reduced integro-differential equation

$$\eta\varphi_\tau = \frac{l_J}{\pi}\int_{-\infty}^{\infty}\frac{du}{u-x}\,\frac{\partial\varphi}{\partial u}\,du - \sin\varphi + \beta. \tag{31}$$

has an exact solution for a phase kink matching the boundary conditions $\varphi(-\infty,t) = \arcsin\beta$ and $\varphi(\infty,t) = \arcsin\beta + 2\pi$. This solution describes a moving ($\beta \neq 0$) Josephson vortex and has the form[12]

$$\varphi(x,t) = \arcsin\beta + \pi + 2\arctan\Big(\frac{x-vt}{L}\Big), \tag{32}$$

where

$$L = \frac{l_J}{\sqrt{1-\beta^2}}, \tag{33}$$

$$v = -\frac{\beta v_0}{\sqrt{1-\beta^2}}, \tag{34}$$

and $v_0 = l_J\omega_J/\eta = Rc^2/8\pi\lambda^2$. The length scale L and the velocity v increase with β and diverge when $j \to j_c$ ($\beta \to 1$). Therefore for the nonlocal case and $\eta \gg 1$ the Josephson vortex expands with the increase of v contrary to its contraction for the local case and $\eta \ll 1$. Note that at $\beta \to 1$, *i.e.*, at $j \to j_c$ the solution (32) should be modified as it is valid only for $L \ll l$.[12]

It is interesting to note that Eq. (20) with $\eta = 0$ has a solution $\varphi(x - vt)$ describing a 4π-phase-kink ($\varphi(+\infty) - \varphi(-\infty) = 4\pi$) moving with a constant velocity $v = \omega_J l_J$ and carrying two flux quanta.[10] This solution has the form

$$\varphi(x - vt) = 4\arctan\Big(\frac{x - vt}{l_J}\Big). \tag{35}$$

In the static case Eq. (20) describing the nonlocal Josephson electrodynamics is similar to the equation describing dislocations in the Peierls potential. Using the results of the dislocation theory[28] one can obtain new classes of periodic solutions for vortex structures.[15]

4. ELECTROMAGNETIC WAVES

Let us now consider an electromagnetic wave propagating along a Josephson junction. To do it we treat the phase distribution in the form of a plain wave with a small amplitude, namely, $\varphi(x,t) = \varphi_a\exp(-i\omega t + ikx)$ with $|\varphi_a| \ll 1$. In the local case the dispersion relation $\omega(k)$ is determined by the sine-Gordon equation (1) and has the form

$$\omega = \omega_J\sqrt{1 + k^2\lambda_J^2}, \tag{36}$$

where k is the wave number. The well-known Swihart electromagnetic wave[29] corresponds to the limit $k\lambda_J \gg 1$ of Eq. (36). It has a linear dispersion relation $\omega \approx \omega_J\lambda_J k$ and propagates with the velocity $c_s = \omega_J\lambda_J = c/\sqrt{8\pi\lambda C}$, which is independent on the critical current density j_c.

In the framework of the nonlocal Josephson electrodynamics one has Eqs. (15) and (17) to determine the dispersion relation for an electromagnetic wave. In the case of a tunnel contact with $d \gg \lambda$ and zero dissipation we have[7,18]

$$\omega = \omega_J\sqrt{1 + \frac{k^2\lambda_J^2}{\sqrt{1 + k^2\lambda^2}}}. \tag{37}$$

If $k\lambda \gg 1$, then the dispersion relation (37) takes the form $\omega \approx \omega_J\sqrt{1 + kl_J}$ and in the region $kl_J \gg 1$ we have $\omega \approx c_s\sqrt{k/\lambda}$, *i.e.*, in the nonlocal limit the phase velocity of an electromagnetic wave propagating along a tunnel contact $v_{\rm ph} = \omega/k \approx c_s/\sqrt{k\lambda}$ tends to zero proportionally to $k^{-1/2}$. In particular, this decrease of $v_{\rm ph}$ with the increase of k results in Josephson vortex Cherenkov radiation.[20]

In the case of a thin film with $d \ll \lambda$ the dispersion relation $\omega(k)$ follows from Eq. (17) and is given by[18]

$$\omega = \omega_J \sqrt{1 + 2k^2\lambda_{\rm eff} l_J Q(2k\lambda_{\rm eff})}, \tag{38}$$

where

$$Q(x) = \int_{-\infty}^{\infty} Q_0(u) \exp(ixu)\, du. \tag{39}$$

The function $Q(x)$ has the following explicit form

$$Q(x) = \begin{cases} \dfrac{1}{\pi\sqrt{1-x^2}} \ln \dfrac{1+\sqrt{1-x^2}}{1-\sqrt{1-x^2}}, & \text{if } x < 1; \\ \dfrac{1}{\sqrt{x^2-1}} \left[1 - \dfrac{2}{\pi} \arctan \dfrac{1}{\sqrt{x^2-1}}\right], & \text{if } x > 1. \end{cases} \tag{40}$$

Using Eqs. (38) and (40) we find, in particular, the dispersion relation $\omega(k)$ in the limiting cases $k\lambda_{\rm eff} \ll 1$ and $k\lambda_{\rm eff} \gg 1$

$$\omega = \begin{cases} \omega_J \sqrt{1 - \dfrac{4k^2\lambda_{\rm eff} l_J}{\pi} \ln(k\lambda_{\rm eff})}, & \text{if } \ k\lambda_{\rm eff} \ll 1; \\ \omega_J\sqrt{1 + kl_J}, & \text{if } \ k\lambda_{\rm eff} \gg 1. \end{cases} \tag{41}$$

It follows from Eq. (41) that in the case of a thin film with $d \ll \lambda$ the phase velocity of an electromagnetic wave propagating along a Josephson junction tends to zero proportionally to $k^{-1/2}$ in the region $kl_J \gg 1$.

5. SUMMARY

To summarize we present the integro-differential equations (15) and (17) determining the electromagnetic properties of an SIS-type tunnel contact with the thickness $d \gg \lambda$ and $d \ll \lambda$. An arbitrary relation between the typical scales of spatial variations of the magnetic field and the phase difference is allowed for. We treat in detail the extremely nonlocal Josephson electrodynamics described by Eq. (20) and focus on the Josephson fluxons and the electromagnetic waves propagating along a tunnel contact. The known exact solutions for single static (23) and moving (32) Josephson vortices are presented. The mass and mobility per unit length are calculated for these phase kinks. An exact solution (35) describing a traveling 4π-kink is presented. We derive dispersion relations for electromagnetic waves propagating along a Josephson junction.

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