Normal domains in Rutherford-type superconducting cables

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We study the formation and dynamics of normal domains in Rutherford-type superconducting cables. We use an effective circuit model to account for the electric current redistribution process between the multifilamentary strands in the presence of a normal zone. We obtain and integrate numerically the diffusion equations for the temperature and the current-density distributions in the cable. Our simulations show the formation of stable normal domains propagating along the cable. We derive an analytical expression for the threshold current I_d above which the propagating normal domains exist. © *1995 American Institute of Physics.*

I. INTRODUCTION

The study of normal zone in current-carrying superconductors has been continuously a subject of interest in the field of applied superconductivity (see, for example, Ref. 1 and references therein). It is well known that in a homogeneous superconductor an initial normal seed is always unstable. If a normal seed nucleates, it will shrink when the current is less than a certain value I_p, while for the higher currents, $I>I_p$, this normal seed will expand. If inhomogeneities in the physical properties of the superconductor are present, stable normal domains (normal zone of a finite size) can exist in their vicinity. These localized normal domains were also found in composite superconductors, in the presence of a high-resistance transition layer between the superconductor and the stabilizer.[2] In this case the domains are stable for a finite range of currents. Namely, a normal domain shrinks when the current is less than a certain value I_*, while for currents higher than $I^*(I^*>I_*)$, the initial domain undergoes a periodic process of splitting. This splitting results in a string of stationary normal domains formed along the composite.[3]

Recently it was found experimentally that normal domains can propagate along the multifilamentary composite superconductor with a large amount of stabilizer outside the multifilamentary area.[4] A number of theoretical studies were performed to investigate this effect.[5–7] It was shown that the existence of these propagating normal domains is a result of the high Joule power generated in the superconductor during a relatively long current redistribution process between the superconductor and the stabilizer.

Rutherford-type superconductor cables are considered for use in superconducting particle accelerator magnets.[8–10] These cables consist of multifilamentary composite strands, twisted together, and shaped into a flat keystoned form. Due to the twisting, each strand goes successively from the inner edge of the cable to the outer edge, and back to the inner edge, over a characteristic length l_p. Over the distance l_p the strand crosses over and has electrical contact with all the other cable strands (see Fig. 1). If a normal seed nucleates in some part of the cable, the current in this region starts to redistribute between the strands due to the existence of the interstrand electrical contact. The current redistribution process affects the normal zone dynamics in the cable, as in the case of a multifilamentary composite superconductor with a large stabilizer.

In a recent study[11] we considered the formation and propagation of a normal zone in Rutherford-type superconducting cables. We used an effective circuit model[7] to account for the current redistribution process between the multifilamentary strands in the presence of a normal zone. Our results show the existence of two different regimes of normal zone propagation. In the first regime, the nucleation of a normal seed in one of the multifilamentary strands composing the cable results in a quench propagation. In the second regime (cryostable regime) we observed the formation of two normal domains propagating in opposite directions in this particular strand, while the other strands remain in the superconducting state.

In this article we present a detailed study of the propagating normal domains in Rutherford-type superconducting cables based on the effective circuit model. We consider both numerically and analytically the threshold current I_d above which a stable normal domain can propagate along the cable. We obtain the current I_d as a function of the dimensionless parameters characterizing the cable and the cooling conditions. This article is organized as follows. In Sec. II we review the effective circuit model and the main equations describing the temperature and the current-density distributions in a cable in the presence of a normal zone. In Sec. III we present the results of the numerical simulations and describe the analytical method to calculate the threshold current I_d. A brief summary is given in Sec. IV.

II. MAIN EQUATIONS

In this section we review the effective circuit model. We derive the equations describing the dynamics of the temperature and the current density distributions in a Rutherford-type superconducting cable in the presence of a normal zone. We consider the case when an initial normal seed nucleates in one of the multifilamentary strands composing the cable. To simulate the process of current redistribution between the strands, we model the cable by a rectangular conductor. The geometry of the model is shown in Fig. 2(a). We represent

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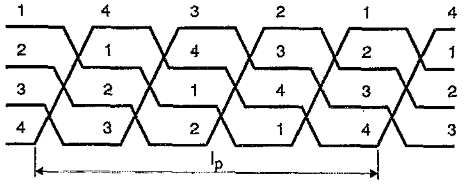

FIG. 1. Principal scheme of transposition of a Rutherford-type cable in case of four multifilamentary strands.

the strand consisting the initial normal seed by a flat superconductor (referred to as S1) of thickness d_1, while other strands are represented by a flat superconductor (referred to as S2) of thickness $d_2 (d_2 > d_1)$. To account for the relatively high interstrand resistance in the cable[12] we consider the presence of a transition layer of the thickness $d_i (d_i \ll d_1, d_2)$ between these flat superconductors. This rectangular conductor carries a transport current I, and is kept in thermal contact with a heat reservoir of temperature T_0.

The dynamics of a normal zone in the described above conductor is determined by both the temperature and the current density distributions. A complete treatment of the problem requires the solution of the heat diffusion equation, which defines the dynamics of the temperature field and the set of Maxwell equations, which define the dynamics of the current-density distribution. These equations form a set of three-dimensional time-dependent, nonlinear equations, which are difficult for either analytical or numerical investigation.

We simplify the problem by using the effective circuit model proposed by Kupferman and co-workers.[7] In the framework of this model the temperatures of both superconductors T_1 and T_2, as well as the corresponding current densities j_1 and j_2, may be regarded as almost uniform in the plane transverse to the sample axis (x axis). In this case we can consider their average values $T_1(x,t)$, $T_2(x,t)$, $j_1(x,t)$, and $j_2(x,t)$ which are functions only of the coordinate along the conductor, x, and time, t.

The process of current redistribution is modeled by the effective electrical circuit sketched in Fig. 2(b). Each component of the rectangular conductor is described by a discrete chain of resistors. The upper chain of resistors represents the superconductor S1, each resistor of resistance $R_1 = \rho_1(T_1, j_1)\Delta x / d_1$, where Δx is an arbitrary discretization length. Similarly the lower chain of resistors represents the superconductor S2, each resistor of resistance $R_2 = \rho_2(T_2, j_2)\Delta x / d_2$. Here ρ_1, ρ_2 are the resistivities of the superconductors S1, S2, which depend on both the local temperature and the current density in the corresponding superconductor. The values of ρ_1 and ρ_2 are equal to zero in the superconducting phase, and are finite above the normal transition. Both chains are linked through a third kind of resistor $R_i = \rho_i d_i / \Delta x$ representing the transition layer (ρ_i is the resistivity of the transition layer). Finally, the inclusion of a characteristic time scale in the electric current diffusion process is accomplished by taking into account the inductances of both superconductors, $\mathscr{L}_1 = \gamma_l \mu_0 d_1 \Delta x$ and $\mathscr{L}_2 = \gamma_l \mu_0 d_2 \Delta x$, where γ_l is a numerical factor of the order of one. Applying Kirchhow's laws on this circuit we obtain the following equation for the current-density distributions $j_1(x,t)$ and $j_2(x,t)$ in the superconductors S1 and S2:

$$\mathscr{L}_2 d_2 \frac{\partial j_2}{\partial t} - \mathscr{L}_1 d_1 \frac{\partial j_1}{\partial t} = \rho_i d_i d_2 \frac{\partial^2 j_2}{\partial x^2} + \rho_1(T_1, j_1) j_1 - \rho_2(T_2, j_2) j_2 . \tag{1}$$

Next, we consider the heat diffusion equations for the temperature distributions $T_1(x,t)$ and $T_2(x,t)$ in the superconductors S1 and S2,

$$C_1 \frac{\partial T_1}{\partial t} = \frac{\partial}{\partial x}\left(k_1 \frac{\partial T_1}{\partial x} \right) - \frac{h_0}{d_1}(T_1 - T_0) \frac{k_i}{d_i d_1}(T_1 - T_2) + Q_1 , \tag{2}$$

$$C_2 \frac{\partial T_2}{\partial t} = \frac{\partial}{\partial x}\left(k_2 \frac{\partial T_2}{\partial x} \right) - \frac{h_0}{d_2}(T_2 - T_0) - \frac{k_i}{d_i d_2}(T_2 - T_1) + Q_2 , \tag{3}$$

where k_i is the heat conductivity of the transition layer and h_0 is the heat transfer coefficient to the coolant with the temperature T_0. We assume that the heat capacities C_1 and C_2 and the heat conductivities k_1 and k_2 of the superconductors S1 and S2 are equal, i.e., $C_1 = C_2 \equiv C_s$ and $k_1 = k_2 \equiv k_s$. As usually,[1] we suppose that the values of C_s, k_s, k_i, and h_0 are constant. This assumption simplifies the numerical simulations and does not change qualitatively the obtained results. The functions Q_1 and Q_2 are the effective rates of Joule heating per unit volume in the superconductors S1 and S2. They both have contributions coming from the superconductor when it is in the normal state and from the transition layer. As a result, Q_1 and Q_2 are given by the following formulas:

$$Q_1 = \rho_1 r_1(T_1, j_1) j_1^2 + \frac{1}{2}\rho_i \frac{d_i d_2^2}{d_1}\left(\frac{\partial j_2}{\partial x} \right)^2 , \tag{4}$$

$$Q_2 = \rho_2(T_2, j_2) j_2^2 + \frac{1}{2}\rho_i d_i d_2 \left(\frac{\partial j_1}{\partial x} \right)^2 . \tag{5}$$

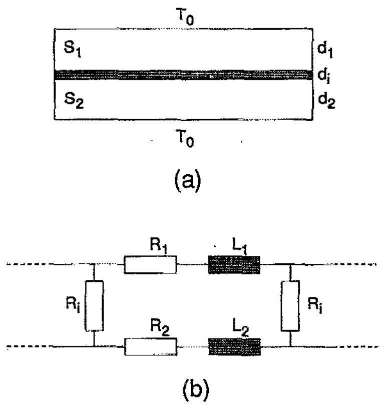

FIG. 2. (a) Schematic structure of the rectangular conductor representing the cable; (b) effective electrical circuit describing the current distribution in the rectangular conductor.

We consider the "step model" for the resistivities of the superconductors,[1] i.e., we assume that $\rho_1(T_1,j_1)$ and $\rho_2(T_2,j_2)$ are given by

$$\rho_{1,2}(T_{1,2},j_{1,2})=\rho_s\eta[j_{1,2}-j_c(T_{1,2})], \tag{6}$$

where η is Heaviside step function $[\eta(x)=0$ if $x<0$ and $\eta(x)=1$ if $x>0]$, and $j_c(T)$ is the critical current density in the superconductor given by

$$j_c(T)=j_c\left(1-\frac{(T-T_0)}{(T_c-T_0)}\right), \tag{7}$$

where T_c is the critical temperature of the superconductor.

We introduce, for convenience, the following dimensionless variables, the temperatures θ_1, θ_2 and the current densities i_1,i_2 in the superconductors S1, S2

$$\theta_1\equiv\frac{T_1-T_0}{T_c-T_0},\quad \theta_2\equiv\frac{T_2-T_0}{T_c-T_0},\quad i_1\equiv\frac{j_1}{j_2},\quad i_2\equiv\frac{d_2}{d_1}\frac{j_2}{j_c}. \tag{8}$$

We define $l_{\rm th}$, the characteristic thermal length, and $t_{\rm th}$, the characteristic thermal relaxation time,

$$l_{\rm th}^2\equiv\frac{d_1k_s}{h},\quad \tau_{\rm th}\equiv\frac{d_1C_s}{h}, \tag{9}$$

where $h\equiv h_0+k_i/d_i$. We define l_m, the characteristic length, and t_m, the corresponding characteristic time, for the current redistribution process,

$$l_m^2\equiv d_id_1\frac{\rho_i}{\rho_s},\quad t_m\equiv(\mathscr{L}_1+\mathscr{L}_2)\frac{d_1}{\rho_s}. \tag{10}$$

We introduce also the dimensionless parameters

$$\alpha\equiv\frac{d_1\rho_sj_c^2}{2h(T_c-T_0)},\quad W\equiv\frac{k_i}{d_ih}, \tag{11}$$

where α is the ratio of characteristic rates of Joule heating and the heat flux to the coolant, and W characterizes the thermal coupling between the superconductors.

Finally, we use dimensionless time t expressed in units of $\tau_{\rm th}$, and dimensionless coordinate x expressed in units of $l_{\rm th}$. Equations (1)–(3) take then the form[11]

$$\frac{\partial\theta_1}{\partial t}=\frac{\partial^2\theta_1}{\partial x^2}-\theta_1+W\theta_2+2\alpha i_1^2\eta(i_1-1+\theta_1)+\alpha\lambda^2\left(\frac{\partial i_2}{\partial x}\right)^2, \tag{12}$$

$$r\frac{\partial\theta_2}{\partial t}=\frac{\partial^2\theta_2}{\partial x^2}-\theta_2+W\theta_1+\frac{2\alpha}{r}i_2^2\eta\left(\frac{i_2}{r}-1+\theta_2\right)+\alpha\lambda^2\left(\frac{\partial i_2}{\partial x}\right)^2, \tag{13}$$

$$\tau\frac{\partial i_2}{\partial t}=\lambda^2\frac{\partial^2 i_2}{\partial x^2}-\frac{i_2}{r}\eta\left(\frac{i_2}{r}-1+\theta_2\right)+i_1\eta(i_1-1+\theta_1). \tag{14}$$

The dimensionless parameters r,τ, and λ are defined by

$$r\equiv\frac{d_2}{d_1},\quad \tau\equiv\frac{t_m}{\tau_{\rm th}},\quad \lambda\equiv\frac{l_m}{l_{\rm th}}. \tag{15}$$

The perpendicular current density in the transition layer is small, i.e., $j_\perp\ll j_n,j_s$ if the electrical resistance of the conductor is dominated by the transition layer. In this case the relation between the dimensionless current densities i_1,i_2 can be written as

$$i_1+i_2=(1+r)i, \tag{16}$$

where $i\equiv j/j_c$ is the dimensionless total current density in the conductor.

III. RESULTS AND DISCUSSION

In order to study the behavior of a normal zone in Rutherford-type superconducting cable, we perform numerical simulations of model Eqs. (11)–(14). We observe how the temperature and the current-density distributions evolve in time, when the conductor is initially in the superconducting state, except for a normal seed of length $2l_{\rm th}$ located in the superconductor S1. As we already mentioned two different regimes of normal zone propagation exist. In the first regime the initial normal seed results in a quench propagation. In the second regime the initial normal seed results in the formation of two stable normal domains propagating in the opposite directions along the superconductor S1. In the second regime the superconductor S2 remains in the superconducting state during the current and heat diffusion process, and functions as a stabilizer with zero electrical resistance.

To study the dynamics of the propagating normal domains we consider the values of the dimensionless parameters α, τ, λ, W, and r for which the conductor is in the second regime. We observe that for a given set of the dimensionless parameters there is a dimensionless threshold current i_d, above which normal domains are formed. A sequence of the temperature distributions θ_1 and θ_2 in the superconductors S1 and S2 for the values of the dimensionless parameters in the second regime and for $i>i_d$ is shown in Fig. 3 (note that due to the symmetry of temperature distributions we show only the left-hand-side half of the sample). We observe that the initial normal seed starts to expand during the diffusion of current into the superconductor S2. After it reaches a certain length, the center of the normal zone starts to cool down, while the outer sides continue to expand (the heat generation there is maximal). As a result, superconductivity recovers at the center of the normal zone, and we find two separated normal domains traveling away in the opposite directions. The system tends to a steady state with two normal domains propagating along the superconductor S1 with a constant velocity, while superconductivity recovers behind. Note that the superconductor S2 remains in the superconducting state ($\theta_2<1$) during the formation of the propagating normal domains in the superconductor S1. For the values of current below the threshold current $i_d,i<i_d$ the initial normal seed shrinks and disappears, i.e., the superconductivity recovers in a whole conductor.

Let us now study the dependence of the normal zone propagation velocity v in the conductor on the entire current i. To calculate the function $v(i)$ analytically we consider the process of current redistribution between the superconductor (S1) and the stabilizer (S2) in the presence of a normal zone.

We begin with a case of an unstablized superconductor, where the current inside the normal zone is constant and equal to i. In this case the propagation velocity v of the normal zone can be calculated exactly.[1] The dependence $v(i)$ is given by the formula

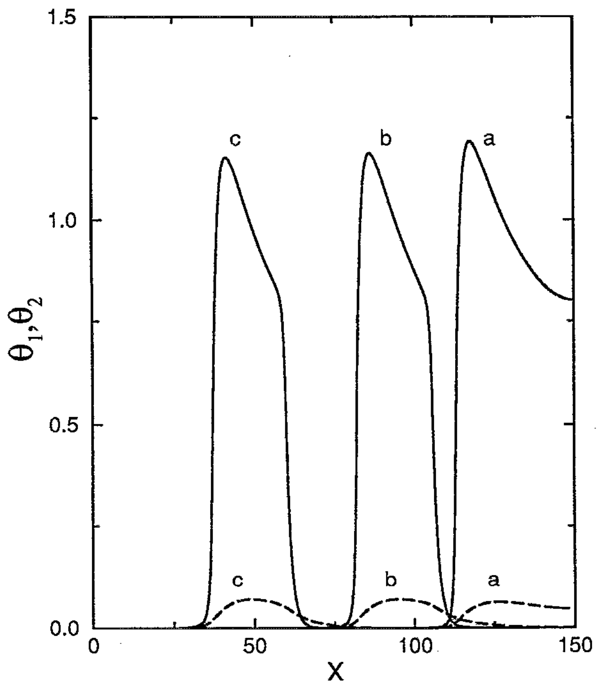

FIG. 3. Temperature distributions θ_1 (solid line) and θ_2 (dashed line) in the superconductors S1 and S2 for the current $i=0.55(i>i_d)$. The parameters are $\alpha=2$, $W=0.05$, $\lambda=20$, $r=8$, and $\tau=100$; (a) $t=40$, (b) $t=80$, (c) $t=140$.

$$v(i)=v_{\text{th}}\frac{\alpha_{\text{un}}i^2+2i-2}{\sqrt{(1-i)(\alpha_{\text{un}}i^2+i-1)}}, \tag{17}$$

where $v_{\text{th}}\equiv l_{\text{th}}/\tau_{\text{th}}$ is a thermal velocity, and α_{un} is a Stekly parameter for the unstabilized superconductor. This parameter is determined by the ratio of the characteristic rate of Joule heating inside the normal zone and characteristic heat flux to the coolant and is equal to $\alpha_{\text{un}}\equiv d_s\rho_s j_c^2/2h(T_c-T_0)$ (d_s is a thickness of the unstablized superconductor). The velocity $v(i)$ is positive, i.e., the normal zone expands along the superconductor, when the current i is higher than i_p, $i>i_p$. The current i_p is known as the minimum normal zone propagation current[1] and is given by the formula

$$i_p=\frac{1}{\alpha_{\text{un}}}(\sqrt{1+2\alpha_{\text{un}}}-1). \tag{18}$$

The velocity $v(i)$ of the normal zone expansion can be considered as a linear function of the current for the current interval $0<i-i_p\ll i_p$, where

$$v(i)=v_0(i-i_p) \tag{19}$$

and v_0 is obtained by means of Eq. (17) and is given by the expression

$$v_0\equiv 2v_{\text{th}}\frac{\alpha_{un}i_p+1}{\sqrt{(1-i_p)(\alpha_{\text{un}}i_p^2-1+i_p)}}. \tag{20}$$

Let us now consider the current density distribution inside the normal domain in the superconductor S1 while the superconductor S2 functions as an electric current stabilizer (cryostable regime). In this case the current in the normal domain is not a constant anymore and redistributes into the stabilizer by diffusion. As the redistribution of current requires a finite time interval, the stabilizing mechanism suffers an effective delay time of the order of τ. During this time interval the superconductor S1 behaves as a temporarily unstablized superconductor. The Stekly parameter α_{un} associated with this temporarily unstabilized superconductor is equal to $\alpha_{\text{un}}=2\alpha$, where α is defined by Eq. (11). The length l_d of this temporarily unstabilized superconductor, i.e., the length of the region at the front of the normal zone where the current redistributes into the stabilizer, can be obtained from Eq. (14). The value of l_d is given by the formula

$$l_d=\frac{2\lambda^2}{-v\tau+\sqrt{(v\tau)^2+4\lambda^2}}, \tag{21}$$

where v is the propagation velocity of a normal domain. The parameters τ and λ are the characteristic time and length of the current redistribution process defined by Eq. (10).

The normal zone propagation velocity is determined mainly by the temperature distribution in the vicinity of the front of the normal domain. We now define l_v, the length of this region, by the following reasoning.

If the characteristic time τ of the current redistribution process is sufficiently large, $\tau\gg 1$, then $l_d\gg l_v$, i.e., the current in the region of the length l_v at the front of the domain is confined in the superconductor S1 and is equal to the entire current i. In this case the velocity $v(i)$ of the domain propagation coincides with the normal zone propagation velocity for the unstabilized superconductor and is given by Eq. (17). The threshold current i_d coincides with the minimum propagation current i_p and is given by Eq. (18).

To estimate the propagation velocity of the normal domain for a wider range of τ we introduce an effective current i_{eff} in the superconductor S1. The effective current i_{eff} is a function of the ratio l_v/l_d and is equal to the entire current i in the case of the unstabilized superconductor ($l_v\ll l_d$). In the first (linear) approximation, we obtain the formula

$$i_{\text{eff}}=i\left(1-\frac{l_v}{l_d}\right). \tag{22}$$

Substituting the effective current i_{eff} instead of the entire current i in Eq. (17), and using Eq. (21) for l_d we obtain the following implicit formula for the propagating velocity v of a normal domain:

$$v=v_0\left(i-i_p-i\frac{l_v}{2\lambda^2}[-v\tau+\sqrt{(v\tau)^2+4\lambda^2}]\right). \tag{23}$$

This equation has a solution for the velocity v if the current i is in the range $i_d<i<1$, where the threshold current i_d is obtained from Eq. (23) and is given by the following implicit formula:

$$v_0\tau(i_d-i_p)^2=4l_v i_d, \tag{24}$$

where the values of i_p and v_0 can be calculated by means of Eqs. (18) and (19). Given the value of the length l_v one can use Eq. (24) to calculate the dependence of the threshold current i_d on τ. In general the length l_v depends on τ as well as on other dimensionless parameters (α, W, λ, and r). We

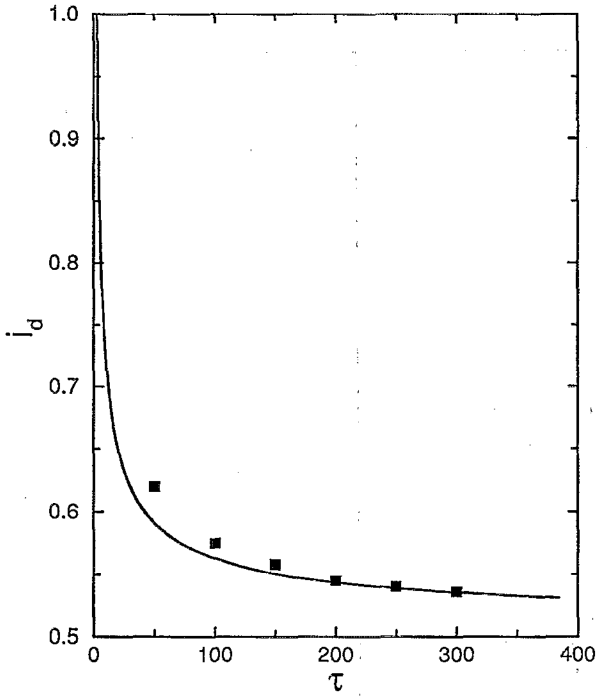

FIG. 4. The threshold current i_d as a function of τ. The dots represent the results of numerical simulations. The parameters are $\alpha=2$, $W=0.05$, $\lambda=20$, and $r=8$. The solid line represents the threshold current i_d calculated by means of the analytical expression Eq. (24).

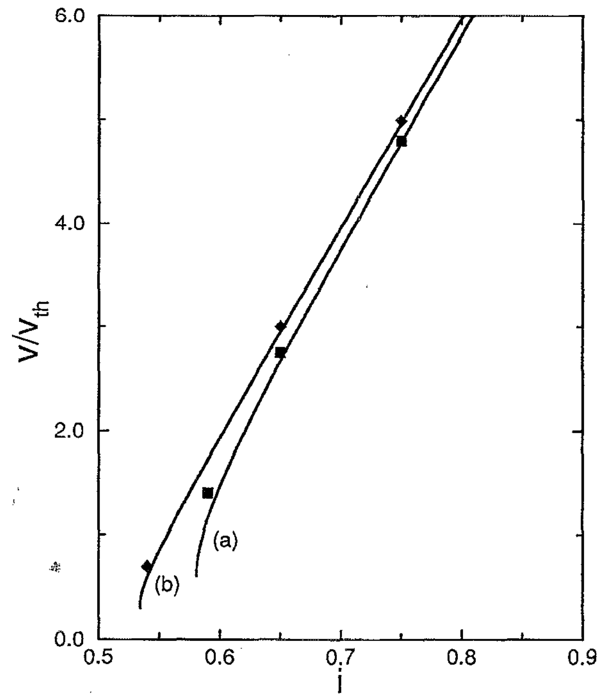

FIG. 5. The propagation velocity in units of v_{th} vs current. The dots represent the results obtained in numerical simulations, the solid lines are the solutions of Eq. (23). The parameters are $\alpha=2$, $W=0.05$, $\lambda=20$, and $r=8$; (a) $\tau=50$, (b) $\tau=300$.

found however, that the dependence of l_v on τ becomes weak for large τ ($\tau>100$) and the length l_v can be considered as a constant for this range of τ. Therefore, we calculate the value of l_v from numerical simulations for a given set of parameters α, W, λ, r, and $\tau>100$. We use this value then to calculate the function $i_d(\tau)$ from Eq. (24).

We show in Fig. 4 the threshold current i_d as a function of τ for the values of the dimensionless parameters $\alpha=2$, $W=0.05$, $\lambda=20$, and $r=8$. We calculate the corresponding value of the length l_v as $l_v=3.5$ (in the units of the thermal length l_{th}). Points represent the results of the numerical simulations. The solid line represents the function $i_d(\tau)$, calculated by means of Eq. (24). For large values of τ ($\tau>100$), the implicit formula [Eq. (24)] gives the values of the threshold current i_d with a high degree of accuracy. In this range the maximum deviation from the numerical results is less than 2%. For small values of τ, deviation between the numerical and analytical results becomes sufficient mainly due to the fact that the linear approximation which was used to obtain Eq. (21) is insufficient for small values of τ.

In Fig. 5 we show a comparison of the velocity $v(i)$ obtained by the analytical solution [Eq. (23)] with the velocity obtained by the numerical simulations. For large values of i, the roots of the implicit Eq. (23) give the velocity with a high degree of accuracy, deviating from the numerical results in less than 3%.

IV. SUMMARY

To summarize, we use the effective circuit model to study the formation and dynamics of normal domains in a Rutherford-type superconducting cable. Our simulations show the formation of stable normal domains propagating along the cable with a constant velocity. We find both numerically and analytically the threshold current density i_d above which the normal domains propagate in the cable. We calculate the propagation velocity of the normal domains.

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