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Magnetization Relaxation in Layered Superconductors.

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Abstract. – The energy of a pointlike vortex is calculated for a layered superconductor with very weak interlayer Josephson coupling. An energy barrier existing near the sample surface is found. Magnetization relaxation due to thermally activated penetration of pointlike vortices and quantum tunnelling of pointlike vortices are considered. An initial avalanche-type decay of magnetization is predicted.

Magnetization relaxation measurements are an effective method to investigate flux dynamics phenomena, current-voltage characteristics and critical current in superconductors [1-3]. In type-II superconductors magnetization relaxation is determined by vortices penetration, flow and pinning. The flux penetration process begins when the external magnetic field H becomes higher than a certain edge field H^*. In the case of a cylinder subjected to a parallel field the value of H^* is bigger than the lower critical field H_{c1} and smaller than the thermodynamical critical field H_c [4]. The difference between H^* and H_{c1} depends on interaction of vortices with pinning centres and sample surface. In continuous superconductors attraction of Abrikosov vortices to sample surface results in the Bean-Livingston barrier [5]. This barrier prevents penetration of Abrikosov vortices inside the sample. The value of H^* determined by the Bean-Livingston barrier depends on the surface roughness. It is maximal in the case of an ideal flat surface. In the absence of pinning centres in the bulk the edge field H^* is equal to the thermodynamical critical field H_c [4].

The discovery of Bi- and Tl-based high-T_c superconductors stimulated theoretical studies of layered superconductors with weak interlayer Josephson coupling. In particular, specific pointlike (or pancake) vortices were introduced and investigated [6-8]. Each of these pointlike vortices is residing only in one of the superconducting layers. The self-energy of an isolated pointlike vortex is proportional to $\ln(L/\xi)$, where L is the characteristic size of the sample in layers plane, and ξ is the coherence length. Thus, the self-energy of an isolated pointlike vortex diverges when $L/\xi \to \infty$ and it cannot exist in the bulk of a macroscopic sample.

The interaction of a pointlike vortex with sample surface consists of repulsion and attraction. The repulsion results from the interaction with Meissner screening currents. The attraction results from the increase of the superconducting current density of the pointlike vortex caused by sample surface. The correlation between these two interactions is determined by the external magnetic field H. At a certain value of H competition of attraction and repulsion can lead to a stable state localized near the surface. The existence of this stable state effects the flux penetration process and, in particular, magnetization relaxation.

In this letter we study the penetration of pointlike vortices into a layered superconductor. We show that the energy of a pointlike vortex G_v has a minimum G_m detached by an energy barrier G_g from the surface. We show that G_m is negative if the external magnetic field H is higher than a certain value H_1, which is of the order of the lower critical field H_{c1}. We present a scenario of magnetization relaxation due to thermally activated penetration pointlike vortices inside the sample. We consider a case characteristic of high-T_c superconductors, when $\xi \ll \lambda$ and $d \ll \lambda$, where λ is the London penetration depth, and d is the distance between the layers.

To calculate magnetization relaxation, we find first the energy of a pointlike vortex residing in one superconducting layer in the vicinity of the sample surface. We use the Lawrence-Doniach model [9], where, for simplicity, we neglect the interlayer Josephson coupling. This approach is valid, when the space scale of the considered phenomena is less than the Josephson length λ_J. As we take into account only electromagnetic interactions, the space scale is determined by λ. Thus, we assume that $\lambda \ll \lambda_J$.

Consider a semi-infinite layered superconductor subjected to a magnetic field $\boldsymbol{H}$ parallel to the surface and perpendicular to the layers. Suppose the z-axis is parallel to $\boldsymbol{H}$ and the x-axis is perpendicular to the surface. In the case $\xi \ll \lambda$, the energy of the pointlike vortex $G_v(x)$ is determined mostly by the magnetic field $\boldsymbol{B}(\boldsymbol{r})$, which for an isolated pointlike vortex can be written as

$$\boldsymbol{B}(\boldsymbol{r}) = \boldsymbol{H} \exp\left[-\frac{x}{\lambda}\right] + \boldsymbol{b}(\boldsymbol{r}). \tag{1}$$

The first term represents here the magnetic-field penetration into the sample in the absence of vortices. The magnetic field $\boldsymbol{b}(\boldsymbol{r})$ results from a pointlike vortex. It can be calculated by means of the method of images, *i.e.* to a pointlike vortex located at (x, y) we add an image pointlike antivortex located at $(-x, y)$ and take for $\boldsymbol{b}(\boldsymbol{r})$ the sum of the fields of this vortex and antivortex. Using the magnetic-field distribution $\boldsymbol{B}(\boldsymbol{r})$ we calculate $G_v(x)$:

$$G_v = \left(\frac{\Phi_0}{4\pi\lambda}\right)^2 \mathrm{d} \ln\left(\frac{2x}{\xi}\right) + \frac{\Phi_0}{4\pi} Hd\left[\exp\left[-\frac{x}{\lambda}\right] - 1\right], \qquad x \geq \xi. \tag{2}$$

The first term in eq. (2) represents the attraction between the pointlike vortex and its image. It is minimal near the surface and increases monotonically with increase of x. The second term in eq. (2) represents the repulsion of the pointlike vortex from the surface due to the external magnetic field and associated screening current. It is maximal at $x = 0$ and decreases monotonically with the increase of x. The function $G_v(x)$ is shown in fig. 1 for different magnetic fields H. The dependence of G_v on x increases monotonically if $H < H_0$, where

$$H_0 = e\frac{\Phi_0}{4\pi\lambda^2}. \tag{3}$$

For $H > H_0$ the curve $G_v(x)$ has a maximum G_g at $x = x_g$ and a minimum G_m at $x = x_m$. The

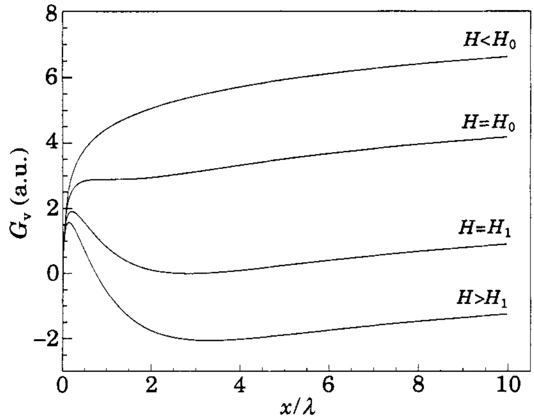

Fig. 1. – Dependence of the pointlike vortex energy G_v on x for different values of the external magnetic field H.

explicit formulae for G_g, x_g, G_m and x_m can be derived by means of eq. (2) in the case when $H \gg H_0$ and $\ln(\lambda/\xi) \gg 1$:

$$G_g \approx \left(\frac{\Phi_0}{4\pi\lambda}\right)^2 d \ln\left[\frac{\Phi_0}{2\pi}\frac{1}{\lambda\xi H}\right], \tag{4}$$

$$G_m \approx \left(\frac{\Phi_0}{4\pi\lambda}\right)^2 d \ln\left[2e\frac{\lambda}{\xi}\ln\left(\frac{4\pi\lambda^2}{\Phi_0}H\right)\right] - \frac{\Phi_0}{4\pi} dH, \tag{5}$$

$$x_g \approx \frac{\Phi_0}{4\pi\lambda}\frac{1}{H} < \lambda, \qquad x_m \approx \ln\left(\frac{4\pi\lambda^2}{\Phi_0}H\right) > \lambda. \tag{6}$$

Note that while deriving formulae (4)-(6) we neglect the contribution arising from Josephson coupling of superconducting layers. The accuracy of this approach is of the order of $x_g/\lambda_J < x_m/\lambda_J \ll 1$ as we assume that $\lambda \ll \lambda_J$.

It follows from eq. (5) that the minimum energy G_m becomes negative when $H > H_1 > H_{c1}$, where

$$H_1 \approx \frac{\Phi_0}{4\pi\lambda^2} \ln\left[2e\frac{\lambda}{\xi}\ln\ln\left(\frac{\lambda}{\xi}\right)\right]. \tag{7}$$

We now consider magnetization relaxation for the magnetic field from the interval $H_1 < H < H^*$. Then the energy of an Abrikosov vortex is negative in the bulk. However, as the Bean-Livingston barrier is proportional to the vortex length, the Abrikosov vortex will not enter into a macroscopic sample. On the other hand, for $H > H_1$ the minimum energy of a pointlike vortex G_m is negative and the energy barrier G_g is finite. At non-zero temperature it can lead to a thermally activated penetration of pointlike vortices into a sample and thus to a specific mechanism of magnetization relaxation in layered superconductors (1). This mechanism is especially effective if $H_1 < H < H^*$.

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(1) One has also to consider penetration into the sample of a nucleus of Abrikosov vortex (a vortex loop) [10]. The energy barrier in this case is of the order of $\lambda G_g/d \gg G_g$.

We consider here, as an illustration, magnetization relaxation for the following problem. A semi-infinite layered superconductor is cooled down to a certain temperature T below the critical temperature in zero magnetic field. Then, the magnetic field $\boldsymbol{H}$ parallel to the sample surface is instantaneously turned on. We suppose that superconducting layers are perpendicular to $\boldsymbol{H}$ and $H_1 < H < H^*$. There is only one mechanism of magnetization relaxation in this case, *i.e.* the thermally activated penetration of the pointlike vortices into the sample. We treat here the following scenario for this mechanism of relaxation.

The initial magnetization $\boldsymbol{M}$ is equal to $\boldsymbol{M} = -\boldsymbol{H}/4\pi$. The thermally activated penetration of pointlike vortices into the sample leads to decay of magnetization. The rate of this process is determined mostly by the energy barrier for pointlike vortices, and it depends exponentially on this barrier. After penetration into a sample the pointlike vortices reside randomly in superconducting layers in the vicinity of the plane $x = x_{\mathrm{m}}$. The interaction of the incoming vortex with these vortices changes the energy barrier. Thus, to find the magnetization relaxation rate, we have to determine this energy barrier shift δG_{g}.

The value of δG_{g} is determined by repulsion of pointlike vortices residing in the same layer and attraction of pointlike vortices residing in different layers. To find the energy barrier shift, we use formulae for the interaction energy of pointlike vortices and antivortices [7]. We also suppose that pointlike vortices are distributed randomly in each of the superconducting layers ($z = nd$, where n is the number of the layers) along the line $x = x_{\mathrm{m}}$, $z = nd$. The result of the calculation shows that δG_{g} is proportional to the average linear concentration N of pointlike vortices:

$$\delta G_{\mathrm{g}} = -\frac{Nd}{2\gamma_{\mathrm{g}} M_0}\left(\frac{\Phi_0}{4\pi\lambda}\right)^3, \tag{8}$$

where $M_0 = H/4\pi$, and γ_{g} is a number of the order of one. The main contribution to the decrease of the energy barrier results from attractive interaction with pointlike vortices residing in the nearest $\lambda/d \gg 1$ layers. The entire number of these pointlike vortices is of the order of $N x_{\mathrm{m}} \lambda/d \gg 1$. The formula given by eq. (8) is valid if the average distance $l = N^{-1}$ between pointlike vortices in each of the superconducting layers is less than the penetration depth, *i.e.* $N\lambda < 1$.

Similar calculations show that the increase of magnetization δM due to pointlike vortices residing in superconducting layers is also proportional to N:

$$\delta M = N\frac{\Phi_0}{8\pi\lambda}\ln\left(\frac{4\pi\lambda^2}{\Phi_0}H\right). \tag{9}$$

Using eqs. (8) and (9) we present δG_{g} as

$$\delta G_{\mathrm{g}} = -\frac{\delta M}{\gamma_{\mathrm{g}} M_0} d\left(\frac{\Phi_0}{4\pi\lambda}\right)^2 \ln^{-1}\left(\frac{4\pi\lambda^2}{\Phi_0}H\right). \tag{10}$$

Thus, the energy barrier decreases with the increase of magnetization. This dependence results in initial avalanche-type thermally activated magnetization relaxation.

To find the equation determining magnetization relaxation, we consider diffusion of pointlike vortices in the surface layer $x < x_{\mathrm{m}}$. To estimate the appropriate diffusion coefficient, we treat the motion of pointlike vortices as a viscous flux flow [11]. We also consider the dependence of the energy of the pointlike vortices G_{v} on x as an external potential. Finally, this approach results in the following equation:

$$\frac{dM}{dt} = \frac{\alpha M_0}{\tau} \exp\left[- \frac{\delta G_g}{k_B T} \right], \tag{11}$$

where

$$\alpha = \gamma_g \frac{k_B T}{d} \left(\frac{4\pi\lambda}{\Phi_0} \right)^2 \ln\left(\frac{4\pi\lambda^2}{\Phi_0} H \right), \tag{12}$$

$$\tau = \gamma_\tau \frac{\lambda^2}{\rho_n c^2} \exp\left[\frac{G_g}{k_B T} \right], \tag{13}$$

γ_τ is a number of the order of one, ρ_n is the resistivity in the normal state, k_B is the Boltzmann constant, and $\lambda^2/\rho_n c^2$ is the characteristic time constant appearing in theory of non-equilibrium superconductivity [11].

The solution of eq. (11) has the form

$$M = M_0 \left[-1 + \alpha \ln\left(\frac{\tau}{\tau - t} \right) \right]. \tag{14}$$

Thus, the thermally activated pointlike-vortices penetration into the sample leads to a specific initial avalanche-type dependence of magnetization on time. The characteristic time of magnetization decay τ strongly (exponentially) depends on energy barrier and temperature. The dimensionless amplitude α of magnetization relaxation is proportional to temperature and slowly (logarithmically) depends on the applied magnetic field.

The dependence given by eq. (14) is valid until

$$\delta M = \alpha M_0 \ln\left(\frac{\tau}{\tau - t} \right) < M_0 , \tag{15}$$

and the density of vortices N is less than a certain critical value $N_c \sim \lambda^{-1}$. When N becomes of the order of N_c Abrikosov vortices self-assemble from pointlike vortices and then penetrate inside the bulk. It follows from eqs. (9) and (15) that a noticeable increase of N starts when $t \to \tau$.

To estimate the values of the characteristic time constant τ and the dimensionless amplitude of magnetization relaxation α we use $\gamma_g = \gamma_\tau = 1$ and the data obtained for a monocrystal $Bi_2Sr_2CaCu_2O_8$ [12]: $\lambda \approx 3 \cdot 10^{-5}$ cm, $\xi = \xi_{ab} \approx 1.5 \cdot 10^{-7}$ cm, $d \approx 1.5 \cdot 10^{-7}$ cm, $\rho_n \approx$ $\approx 10^{-5}\,\Omega$ cm. Using eqs. (5), (15) and (16) we find that if $T \ll T_c$ and $H \geqslant H_1 = 0.013T$, then

$$\tau \approx 10^{-13} \exp\left[\frac{300}{T} \ln\left(\frac{0.7}{H} \right) \right] \text{s}, \tag{16}$$

$$\alpha \approx \frac{T}{300} \ln(500H), \tag{17}$$

where the temperature T is given in kelvin and the magnetic field H is given in tesla. Substituting in eqs. (16) and (17) $T = 25$ K and $H = 0.04$ T, we find $\tau \approx 82$ s and $\alpha \approx 0.25$. These numbers seem to be reasonable for experimental observation of the main peculiarities of magnetization relaxation due to thermally activated penetration of pointlike vortices.

At low temperatures the thermally activated mechanism of magnetization relaxation becomes ineffective. Instead we consider quantum tunnelling of pointlike vortices in the potential $G_v(x)$. We treat this process in the limit of overdamped pointlike-vortices dynamics following the methods developed in [13, 14]. This approach results in initial avalanche-type magnetization relaxation given by eq. (14) but with α and τ different from eqs. (12) and (13). Numerical estimations show that in case of quantum tunnelling $\alpha \approx 0.1$ and $\tau \approx 8 \cdot 10^3$ s, *i.e.* the dimensionless amplitude of magnetization relaxation α becomes smaller and the characteristic time of magnetization decay becomes sufficiently bigger.

To summarize, we have shown that in an external magnetic field higher than H_0 (eq. (3)) the energy of a pointlike vortex G_v has a minimum G_m detached from the sample surface by an energy barrier G_g. The value of G_m becomes negative in a magnetic field higher than H_1 (eq. (7)). We have calculated the time-dependence of magnetization relaxation due to thermally activated penetration of pointlike vortices inside the sample. We have shown that this process results in initial avalanche-type decay of magnetization. We considered quantum tunnelling of pointlike vortices as a mechanicm of low-temperature magnetization relaxation. We have shown that this process results in the same initial avalanche-type decay as the thermally activated penetration.

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