Quench propagation velocity for highly stabilized conductors

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Quench propagation velocity in conductors having a large amount of stabilizer outside the multifilamentary area is considered. The current redistribution process between the multifilamentary area and the stabilizer can strongly affect the quench propagation. A criterion is derived determining the conditions under which the current redistribution process becomes significant, and a model of effective stabilizer area is suggested to describe its influence on the quench propagation velocity. As an illustration, the model is applied to calculate the adiabatic quench propagation velocity for a conductor geometry with a multifilamentary area embedded inside the stablizer.

Keywords: stablized superconductors; quench propagation velocity; multifilamentary

The development of conductors with an aluminium superstabilizer for applications such as detector magnets for high energy physics[1] and energy storage devices[2-4] has led to some new problems. One of them is the effect of the current redistribution process between the superconductor and the stabilizer on quench propagation[5]. This effect can also be important for the conductors considered for the next generation of high energy particle accelerator magnets, because these conductors contain a large amount of stabilizer outside the multifilamentary area.

The quench propagation velocity is determined by the Joule heating in the vicinity of the transition front. During the transition from the superconducting to the resistive state, the current is redistributed from the superconductor to the stabilizer. This redistribution occurs in two phases. First, the current is expelled from the superconducting filaments to the copper in the multifilamentary area. Second, the current diffuses into the stabilizer outside the multifilamentary area. If the interfilament spacing is small, the first phase is very fast. Conversely, if most of the stabilizer is located outside of the multifilamentary area, the second phase can be relatively long. In the vicinity of the transition front, where the quench-driving heat release occurs, the current may thus remain confined in a small fraction of stabilizer around the multifilamentary area. This results in a relatively high local value of Joule heating, leading to high quench propagation velocity.

This paper: 1, considers the case where the quench propagation is affected by the current redistribution process; 2, introduces the characteristic velocity at which this process becomes significant; 3, provides a model of effective stabilizer area for fast quench propagation; and 4, presents a transcendental equation for the quench propagation velocity in highly stablized conductors.

Concept of highly stabilized conductor

Most of the papers on quench propagation velocity consider the current redistribution process as instantaneous (see the review in reference 6). To discuss the applicability of this assumption, let us estimate the characteristic times of the phenomena involved. The current redistribution time, t_d, may be estimated as

$$t_d = \frac{\mu_0 d^2}{\rho_n} \tag{1}$$

where

$$d = \frac{A_n}{P_n} \tag{2}$$

Here d, A_n and P_n are the effective thickness, the cross-sectional area and the contact perimeter of the stabilizer

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(see *Figure 1*), and ρ_n is the resistivity of the stabilizer.

The characteristic time associated with the quench propagation, t_p, is given by

$$t_p = \frac{L}{v} \qquad (3)$$

where v is the quench propagation velocity and L is the thickness of the zone where the quench-driving heat release occurs. In other words, L is the thickness of the region, in the vicinity of the transition front from the resistive to the superconducting state, where the Joule heating determining the propagation velocity takes place.

In this paper, we shall represent the power of the Joule heating in the superconductor as a step function of temperature. In other words, we shall assume that the Joule heating is equal to zero for temperatures below a certain temperature, T_t, and is non-zero above T_t. In the following, we shall represent T_t by

$$T_t = \frac{i}{2} T_0 + \left(1 - \frac{i}{2}\right) T_c \qquad (4)$$

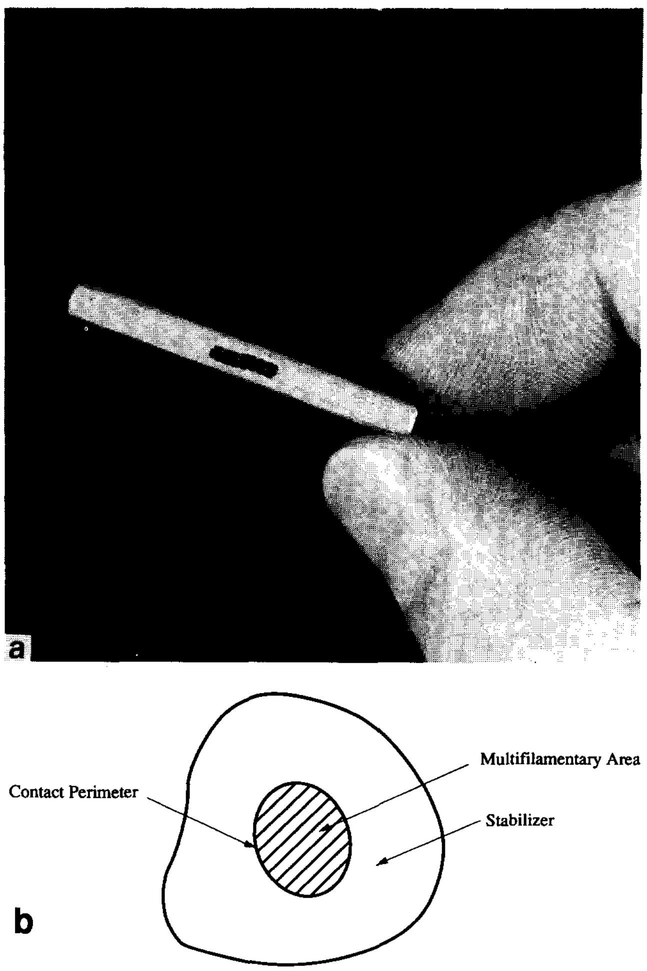

Figure 1 Example of highly stabilized conductors: (a) cross-sectional view of the aluminium stabilized conductor used for the ALEPH solenoid[1]; (b) cross-sectional schematic of a highly stabilized conductor

Here T_0 is the coolant temperature, T_c is the critical temperature at the given field and zero current and i is the dimensionless current defined as

$$i = \frac{I}{I_c} \qquad (5)$$

where I is the transport current and I_c is the critical current at the given field and T_0. It was shown[7] that Equation (4) leads to satisfactory results when computing the quench propagation velocity.

In most cases of practical interest, the cooling conditions are weak. Then L is determined by the thermal diffusion along the conductor and can be estimated as[6]

$$L = \frac{k}{Cv} \qquad (6)$$

where k and C are the thermal conductivity and the heat capacity per unit volume averaged over the conductor cross-section, and taken at the given field and T_t.

Thus, the current redistribution process can be considered as instantaneous only if the dimensionless parameter $\tau = t_d/t_p$ is less than one. Using Equations (1)–(3) and (6), it is convenient to rewrite τ as

$$\tau = \frac{\mu_0 C d^2}{k\rho_n} v^2 = \left(\frac{v}{v_c}\right) \qquad (7)$$

where we have introduced the characteristic velocity v_c defined as

$$v_c = \frac{1}{d}\left(\frac{k\rho_n}{\mu_0 C}\right)^{1/2} = \frac{P_n}{A_n}\left(\frac{k\rho_n}{\mu_0 C}\right)^{1/2} \qquad (8)$$

As an illustration, let us estimate v_c for the case of an aluminium superstabilized conductor[8]. Using the data in *Table 1* and Equation (8), one obtains $v_c \approx 0.7$ m s^{-1}.

Table 1 Data used in calculations

Multifilamentary area	
Cu:Nb–Ti	1.35:1
Cu RRR	200
A_2 (mm^2)	8
Superstabilizer	
Al RRR	2200
A_n (mm^2)	118
P_n (mm)	12
P (mm)	3.6
Critical current at the given field, B	
$I_c = \frac{B_0}{B + B_0} I_0$	
B_0 (T)	1.04
I_0 (A)	25820
All estimations are done at	
T_0 (K)	4.2
I (A)	5000
B (T)	1.5

These values appear to be of the same order of magnitude or even less than the quench propagation velocities measured experimentally.

It follows from the preceding discussion that actual conductors can exhibit quench propagation velocities larger than v_c. In these cases, the dimensionless parameter τ is larger than one and the current redistribution has to be taken into account while calculating v. The conductors where values of v occur that are of the order of, or higher than, v_c will be defined as highly stabilized.

Criterion of highly stabilized conductor

Let us first consider the case of instantaneous current redistribution. The quench propagation velocity is determined by the Joule heating in the vicinity of the transition front. The thickness of this region is L and the power of the quench-driving heat release, q, can be written as

$$q = \rho \frac{I^2}{A} L \qquad (9)$$

where ρ is the longitudinal electrical resistivity of the conductor, defined as

$$\rho = \frac{A\rho_n\rho_s}{A_n\rho_s + A_s\rho_n} \qquad (10)$$

In Equation (10), A is the conductor cross-sectional area, and A_s and ρ_s are the cross-sectional area and the resistivity of the multifilamentary area. While deriving Equation (9), we considered that the currents I_n and I_s flowing in the stabilizer and the multifilamentary area were uniform and the ratio $R = I_n/I_s$ was equal to

$$r = \frac{A_n\rho_s}{A_s\rho_n} \qquad (11)$$

In all cases of practical interest, the resistance per unit length of the stabilizer, ρ_n/A_n, is much smaller than that of the multifilamentary area, ρ_s/A_s, and thus, $r \gg 1$.

On the other hand, q is equal to the heat flux which heats up the superconducting zone to the transition temperature. It follows that

$$q = vA\Delta H \qquad (12)$$

where

$$\Delta H = \int_{T_0}^{T_t} C \, dT \qquad (13)$$

and is the difference in enthalpy per unit volume of conductor between T_0 and T_t. If we assume adiabatic cooling conditions, the value of L is given by Equation (6). Equating the two expressions of q and substituting the expression of L, lead to

$$v = \frac{I}{A}\left(\frac{k\rho}{C\Delta H}\right)^{1/2} \qquad (14)$$

(The same formula can be derived by considering a non-zero heat transfer coefficient to the coolant and letting the Stekly parameter tend towards infinity[6].)

In this model, the maximum of the quench propagation velocity, v_m, is obtained for $I = I_c$ and thus, $T_t = (T_0 + T_c)/2$. Let us estimate v_m for the superstabilized conductor considered above. Using the data from *Table 1* and Equation (14), one obtains $v_m \approx 13 \text{ m s}^{-1}$. As we can see, for actual conductors, the value of v_m can be much higher than the value of v_c. In these cases, the current redistribution process has to be taken into account while calculating the quench propagation velocity.

For adiabatic cooling conditions, a criterion defining highly stabilized conductors may be derived by comparing v_m and v_c. Let us define the dimensionless parameter β

$$\beta = \left(\frac{v_m}{v_c}\right)^2 \qquad (15)$$

Then, a highly stabilized conductor is a conductor with β larger than one. Combining Equations (8) and (14) leads to the following criterion

$$\beta = \frac{\rho}{\rho_n}\left(\frac{A_n}{A}\right)^2 \frac{\mu_0 I_c^2}{\Delta H_c P_n^2} \geq 1 \qquad (16)$$

where ΔH_c is calculated by mean of Equation (13) at $T_t = (T_0 + T_c)/2$, i.e. for $I = I_c$. For the superstabilized conductor considered above $\beta \approx 300$, which is much larger than one.

Model of effective stabilizer area

Let us now consider the case where the current redistribution has to be taken into account while calculating the quench propagation velocity, i.e. $v > v_c$. Then, in the vicinity of the transition front, the current remains confined to a certain fraction of stabilizer around the multifilamentary area, leading to non-uniform quench-driving heat release. The cross-sectional area occupied by the current is determined by the ratio, τ, of the characteristic times associated with the current redistribution and the quench propagation. The larger τ, i.e the larger the ratio of v to v_c, the smaller the fraction of stabilizer where the current has diffused.

On the other hand, as we mentioned before, in most cases of practical interest the cooling is weak, i.e. the Biot parameter, Bi, is much less than one

$$Bi = \frac{A}{P}\frac{h}{k_t} \ll 1 \qquad (17)$$

where P is the cooling perimeter, k_t is the transverse thermal conductivity of the stabilizer and h is the heat transfer coefficient to the coolant. In particular, for the superstabilized conductor considered above, and for $h = 10^3$ W m^{-3}, we obtain $Bi \approx 0.006$, which appears to be much smaller than one.

At the same time, the dimensionless ratio, δ, of the transverse thermal diffusivity, $D_t = k_t/C$, to the magnetic flux diffusivity, $D_m = \rho_n/\mu_0$, is much larger than one, i.e.

$$\delta = \frac{D_t}{D_m} = \mu_0 \frac{k_t}{\rho_n C} \gg 1 \tag{18}$$

For example, for the superstabilized conductor considered above, $\delta \approx 10^5$, which is much larger than one. It results from Equations (17) and (18) that the temperature is uniform over the conductor cross-sectional area, even if the heat release is non-uniform.

As the temperature is uniform over the conductor cross-sectional area, we shall treat the temperature distribution as one-dimensional, depending only on the coordinate along the conductor. The main difference between highly stabilized and conventional conductors is thus the non-uniformity in the quench-driving heat release. To find the exact expression of the Joule heating, we should solve the system of Maxwell's and heat diffusion equations. For most cases of practical interest, this cannot be done analytically and is a complicated problem for numerical analysis.

In this paper, we shall calculate the Joule heating by considering that the current is uniformly redistributed between the multifilamentary area and a certain area of the stabilizer, which we shall introduce as an effective area, A_{eff}. As the fraction of the stabilizer where the current has diffused depends on the quench propagation velocity, the effective area of the stabilizer is determined by the ratio v/v_c, i.e.

$$A_{eff} = A_n f\left(\frac{v}{v_c}\right) \tag{19}$$

To find an expression for f, let us first discuss its asymptotic behaviour for small and large values of v/v_c.

When the ratio v/v_c is small, the current redistribution process is almost instantaneous and the current occupies the whole stabilizer cross-sectional area. This means that A_{eff} tends towards A_n and

$$f\left(\frac{v}{v_c}\right) = 1 \quad \text{for } \frac{v}{v_c} \to 0 \tag{20}$$

On the other hand, when the ratio v/v_c is large, the current only diffuses into a thin layer of stabilizer, l, and the current redistribution process can be treated as in the case of a semi-infinite slab of stabilizer. Then, l is determined by the magnetic flux diffusion length for a characteristic time of the order of t_p

$$l = (D_m t_p)^{1/2} = \frac{A_n}{P_n}\frac{v_c}{v} \tag{21}$$

Thus, the effective area, i.e. the cross-sectional area of stabilizer occupied by the current, is

$$A_{eff} = lP_n = A_n \frac{v_c}{v} \tag{22}$$

and it follows that

$$f\left(\frac{v}{v_c}\right) = \frac{v_c}{v} \quad \text{for } \frac{v}{v_c} \to \infty \tag{23}$$

Having determined the asymptotic dependences for small and large values of v/v_c, we shall now define f for the full range of velocities. To match smoothly Equations (20) and (23) we suggest the following function

$$f\left(\frac{v}{v_c}\right) = \tanh\left(\frac{v_c}{v}\right) \tag{24}$$

Thus, we shall calculate the quench-driving heat release by considering that the stabilizer cross-sectional area is equal to A_{eff} as given by Equations (19) and (24).

Adiabatic quench propagation velocity for highly stabilized conductors

In this section, we shall apply the above model of effective stabilizer area to the computation of the adiabatic quench propagation velocity. To do this, we have to calculate the quench-driving heat release. In the case of adiabtic cooling conditions, it is given by Equation (9), where the expression of ρ is given by Equation (10). Then, substituting A_n by A_{eff} in Equation (10), and combining Equations (9) and (10), it follow that

$$q = \frac{\rho_n \rho_s I^2}{A_{eff}\rho_s + A_s\rho_n}\frac{k}{Cv} \tag{25}$$

where we have replaced L by Equation (6). An equation determining v can be derived by equating Equations (25) and (12), and replacing A_{eff} by Equation (19). It then follows that

$$v = \frac{I}{A}\left(\frac{k\rho}{C\Delta H}\right)^{1/2}\left(\frac{1+r}{1+rf(v/v_c)}\right)^{1/2} \tag{26}$$

Equation (26) is similar to the equation derived in reference 9, the solutions of which were shown to be in good agreement with experimental data[8].

Let us now qualitatively discuss the dependence of the quench propagation velocity on the transport current. When the ratio v/v_c is small, the current occupies the whole cross-sectional area of the stabilizer, i.e. $f \approx 1$. In this case, and as expected, the dependence of v on I coincides with that given by Equation (14). Let us note that Equation (14) gives the lower limit of the quench propagation velocity.

When the ratio v/v_c is large, $f \approx v_c/v$. It follows from Equation (26) that the dependence of v on I is given by the solution of the following second order equation

$$v^2 + rvv_c - \frac{I^2}{A^2}\frac{k\rho r}{C\Delta H} = 0 \quad \text{for } v_c < v \tag{27}$$

where we assumed $r \gg 1$.

To illustrate these results, *Figure 2* shows plots of the quench propagation velocity as a function of the dimensionless current, i, for the superstabilized conductor considered above. The solid line represents the velocity calculated by the combination of Equations (24) and (26), which takes into account the current redistribution process. The dashed line represents the velocity calculated by means of Equation (14), which assumes an instantaneous current redistribution. As can be seen in *Figure 2*, the difference in the results can be up to eight times. Note that the solution of the approximate equation (27) practically coincides with the solution of the complete Equation (26) for the whole range of quench propagation velocities larger than v_c. *Figure 3* shows a plot of the dimensionless ratios A_{eff}/A_n and I_n/I as a function of the dimensionless current, i. It can be seen that, even for relatively low values of effective stabilizer area, i.e. $A_{eff}/A_n < 0.01$, more than 40% of the transport current is still flowing in the stabilizer.

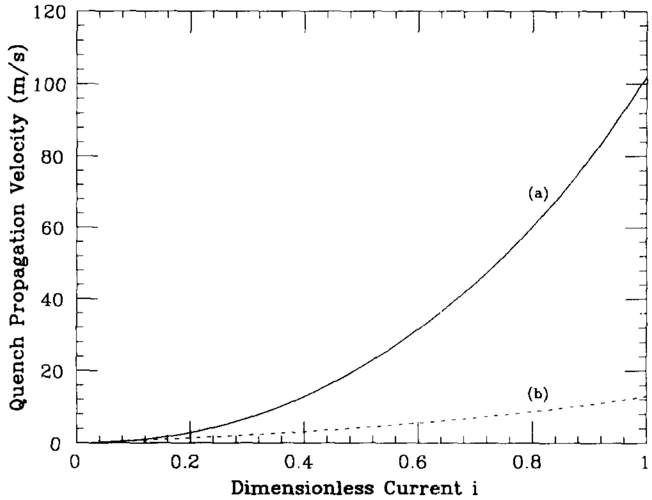

Figure 2 Quench propagation velocity as a function of dimensionless current, i: (a) taking into account current redistribution; (b) assuming instantaneous current redistribution

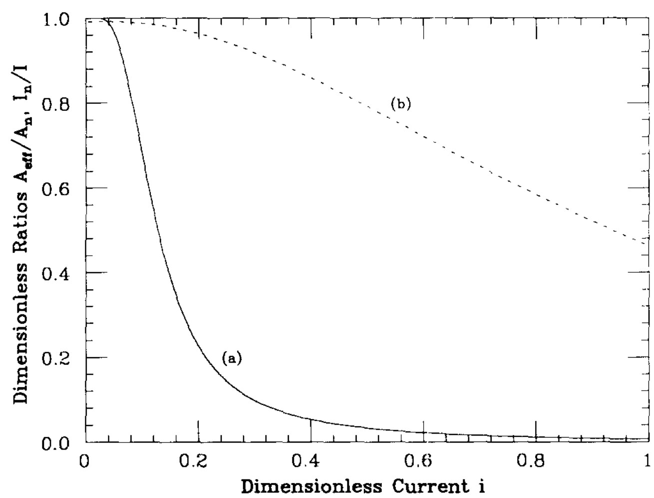

Figure 3 Dimensionless ratios characterizing the effectiveness of the stabilizer as a function of dimensionless current, i: (a) effective stabilizer area normalized to total stabilizer area, A_{eff}/A_n; (b) current in stabilizer normalized to total transport current, I_n/I

Conclusions

For conductors having a large amount of stabilizer outside the multifilamentary area, the current redistribution process has to be taken into account when calculating quench propagation velocity. To do so, we developed a model of effective stabilizer area. We applied this model to the case of weak cooling conditions and to a conductor geometry where the multifilamentary area is embedded inside the stabilizer. We derived a transcendental equation, Equation (26), which determines the quench propagation velocity.

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