Normal zone in large composite superconductors*

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This paper considers the nucleation and propagation of a normal zone in large composite superconductors, considering the relatively long time of current redistribution in the stabilizer. A model is proposed for treating the composite as an effective electrical circuit, which yields two diffusion equations for the electric current and the temperature distributions along the conductor. Numerical simulations have been performed of normal-zone propagation for cryostable and non-cryostable conductors, and analytical expressions obtained for the velocity of propagation.

Keywords: normal zone propagation; composites; stability

Large composite superconductors have been recently proposed for use in energy storage devices. Such conductors are composed of superconducting strands embedded in a large normal metal matrix with high thermal and electrical conductivity to stabilize the conductor against superconducting-to-normal transition. The thermal stability of cylindrical conductors of 1 in (25 mm) diameter rated with currents up to 60 kA has been tested experimentally[1]. These experiments were done for a cryostable conductor: i.e., for all currents up to the critical current I_c, superconductivity recovers if the current is redistributed uniformly over the wire. However, it was found that these systems are unstable against the propagation of normal zones of finite size for currents above a threshold value I_d and below I_c. The formation of these travelling domains was shown to be a result of the high Joule power generated in the superconductor during the relatively long process of current redistribution between the superconductor and the stabilizer[1-11].

Theoretical studies have been performed to investigate the propagation of a normal zone of finite size in the cryostable regime. The first approach was presented by Huang and Eyssa[2,3] who performed numerical simulations of propagating normal domains. The results are in reasonable agreement with experimental observations[1]. An analytical approach, assuming that the time dependence of the Joule power is known, was first presented by Dresner[4,5]. He performed explicit calculations of the velocity for the case when the Joule heating decays exponentially.

In a recent study[10-11], the authors have proposed a model which allows one to investigate both numerically and analytically the nucleation and propagation of the normal zone in large composite superconductors. The composite is considered as an effective electrical circuit, consisting of two connected circuits, representing the superconductor and the stabilizer. This model yields two diffusion equations describing the dynamics of the temperature and the current density distributions along the conductor. Numerical simulations of these equations showed the existence of propagating normal domains in the cryostable regime, but the main advance was the analytical investigation. This approach supplied explicit formulas for the velocity of normal-zone propagation in superconductors with a relatively long time of current redistribution in the stabilizer.

This paper presents a detailed study on normal-zone propagation for cryostable and non-cryostable conductors, based on the above model. It describes both the numerical and the analytical methods, and presents results for the propagating normal domains, for the normal-to-superconductor switching wave, and for the margins of stability.

The basic equations

In this paper, we consider a rectangular conductor, consisting of a plane layer of superconducting material, electrically and thermally bonded to a stabilizing normal metal. The thickness of the superconductor and the stabilizer are denoted by d_s and d_n respectively. The conductor carries a transport current I, and is kept in thermal contact with a heat reservoir of temperature T_0.

In general[12,13], the dynamics of a normal zone in a composite superconductor are determined by both

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temperature and current density distributions. A complete treatment of the problem requires the solution of the heat diffusion equation which defines the dynamics of the temperature field, and the set of Maxwell equations which define the dynamics of the current distribution. These equations form a set of three-dimensional and time-dependent non-linear equations, which is too difficult for either analytical or numerical investigation. We have shown[10-11] that it is possible to reduce the complexity of the problem while preserving the main physical features.

1 The total longitudinal flow consists of parallel flows through the superconductor and the stabilizer.
2 A perpendicular (redistribution) current is allowed to flow from one component to the other at any point along the conductor.
3 Variations in the longitudinal current have a finite duration which is of the order of relaxation time of current redistribution in the stabilizer, $\tau_m \propto \mu_0 d_n^2/\rho_n$, where ρ_n is the resistivity of the stabilizer.

The process of current redistribution is modelled by the effective electrical circuit sketched in *Figure 1*. Here, each component is described by a discrete chain of resistors. $R_n = \rho_n \Delta x/d_n$, represents the stabilizer (Δx is an arbitrary discretization length; x is the axis along the conductor). $R_s = \rho_s \Delta x/d_s$ represents the superconductor: $\rho_s(j_s, T)$ is the resistivity of the superconductor, j_s is the current density in the superconductor, and T is the temperature. The two chains are linked through a third kind of resistor, $R = \gamma_R \rho_n d_n/\Delta x$ (γ_R is a numerical factor of the order of one, which depends on the geometry of the conductor). Finally, we attribute to the normal resistors an inductance $\mathcal{L} = \gamma_{\mathcal{L}} \mu_0 d_n \Delta x$, where $\gamma_{\mathcal{L}} \sim 1$ is another numerical factor. Applying Kirchhoff's laws on this circuit, we obtain the equation for the current density in the superconductor

$$\left(\frac{\gamma_{\mathcal{L}}\mu_0 d_n^2}{\rho_n}\right)\frac{\partial j_s}{\partial t} = \gamma_R d_n^2 \frac{\partial^2 j_s}{\partial x^2} - j_s\left(1 + \frac{\rho_s d_n}{\rho_n d_s}\right) + j \qquad (1)$$

where $j = I/d_s$.

We suppose that $\kappa_s/d_s + \kappa_n/d_n \gg h$, where κ_s and κ_n are the heat conductivities of the superconductor and the stabilizer respectively, taken here as constant, and $h(T)$ is the heat transfer coefficient. It means that the thermal relaxation time over the cross-section is much shorter than the thermal relaxation time between the conductor and the coolant. In this case the temperature distribution over the cross-section is almost uniform, and we can consider its averaged value $T(x)$, which is a function only of the coordinate along the conductor. The temperature, $T(x)$, satisfies the heat equation

$$c\,\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(\kappa\,\frac{\partial T}{\partial x}\right) - W(T) + Q(T) \qquad (2)$$

where c is the heat capacity, and κ is the heat conductivity, both average values, obtained by weighting the values in each component with their relative cross-sectional area. The term $W(T)$ is the rate of heat transfer to the coolant per unit volume, which can be written as $W(T) = h(T)(T - T_0)/d$, where $d = d_s + d_n$. Finally, $Q(T, j_s)$ is the rate of Joule heating per unit volume, which has three contributions: from the current in the superconductor when it is in the normal state, from the current in the stabilizer, and from the perpendicular current. As a result (see also *Figure 1*), $Q(T)$ is given by

$$Q(T) = \frac{1}{d}\left[d_s\rho_s j_s^2 + \frac{d_s^2\rho_n}{d_n}(j - j_s)^2 + \gamma_R d_s^2 \rho_n\left(\frac{\partial j_s}{\partial x}\right)^2\right] \qquad (3)$$

It is convenient to introduce the following dimensionless fields: θ, the temperature, and i_s, the current density in the superconductor

$$\theta \equiv \frac{T - T_0}{T_c - T_0}, \qquad i_s \equiv \frac{j_s}{j_c} \qquad (4)$$

where j_c is the critical current density in the superconductor at temperature T_0. We define L_{th}, the characteristic thermal length and τ_{th}, the characteristic thermal relaxation time, by

$$L_{th}^2 \equiv \frac{(d_n + d_s)\kappa}{h}, \qquad \tau_{th} \equiv \frac{(d_n + d_s)c}{h} \qquad (5)$$

with $h \equiv h/T_0$). We define L_m, the characteristic length of variations in the current distribution, and τ_m, the corresponding relaxation time, by

$$L_m^2 \equiv \gamma_R d_n^2, \qquad \tau_m \equiv \frac{\gamma_{\mathcal{L}}\mu_0 d_n^2}{\rho_n} \qquad (6)$$

Then, we define the dimensionless parameters

$$\alpha \equiv \frac{d_s^2\rho_n j_c^2}{d_n h(T_c - T_0)}, \qquad \xi(\theta, i_s) \equiv \frac{\rho_s d_n}{\rho_n d_s} \qquad (7)$$

where ξ is the ratio of the resistances of the superconductor and of the stabilizer per unit length, and α is the ratio of characteristic rates of Joule heating and heat flux to the coolant (the Stekly parameter[14]). Finally, we use dimensionless scales of time and length. We express time in units of τ_{th}, and length in units of L_{th}. Equations

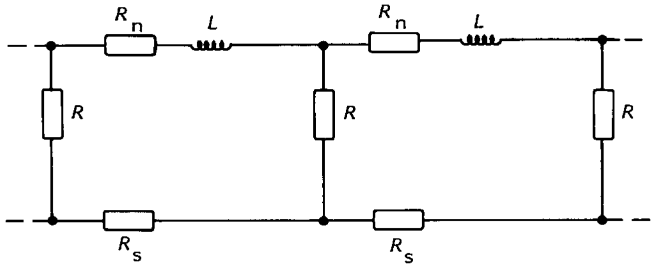

Figure 1 Effective electrical circuit describing the current distribution in the conductor

(1) and (2) then take the form

$$\frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\theta x^2} - \theta + \alpha(i - i_s)^2 + (\theta, i_s)\alpha i_s^2 + \alpha\lambda^2\left(\frac{\partial i_s}{\partial x}\right)^2 \tag{8}$$

$$\tau \frac{\partial i_s}{\partial t} = \lambda^2 \frac{\partial^2 i_s}{\partial x^2} - (1 + \xi(\theta, i_s))i_s + i \tag{9}$$

where the dimensionless parameters i, τ and λ are given by

$$i \equiv j/j_c, \qquad \tau \equiv \tau_m/\tau_{th}, \qquad \lambda \equiv L_m/L_{th} \tag{10}$$

In this paper we consider the so-called step model, where the resistivity of the superconductor is ρ_s in the normal state ($i_s > 1 - \theta$), and vanishes in the superconducting state ($i_s < 1 - \theta$)[13]. The minimum current of normal zone existence, i_m, is given in this specific model by $i_m = [\sqrt{\{1 + 4\alpha\xi(\xi + 1)\}} - 1]/2\alpha\xi$. The composite is therefore cryostable ($i_m = 1$), if $\alpha < 1$. It should be emphasized that all properties of the system depend only on the four dimensionless parameters α, ξ, τ and λ, as the final Equations (8) and (9) include only those parameters.

Results

In order to study the dynamics of normal zone, we first performed numerical simulations of Equations (8) and (9). The initial conditions were taken where the temperature is raised to the critical value $\theta = 1$ in a region of length $2L_{th}$. For currents in the range of values $i_m < i < 1$ the normal state is always the one which propagates into the superconducting state. *Figure 2* shows a time sequence of profiles of the temperature field for a cryostable conductor ($\alpha < 1$). This profile reaches a steady shape propagating with constant velocity after a time interval in the order of τ_m. A region of high temperature is formed at the front of the propagating wave. This temperature peak decreases rapidly in the direction of propagation, but decays moderately in the opposite direction, towards the stable superconducting state. We see how in the absence of a stable normal state, the superconducting state is recovered behind the propagating front. *Figure 3* shows a time sequence of profiles of the temperature field for a non-cryostable conductor ($\alpha > 1$). For $i < i_m$, there is a range of values $i_d < i < i_m$, where the system is unstable against the propagation of a normal zone of finite size. Again, we note the non-symmetric shapes of the fields on both sides of the normal zone. The velocity of propagation has a non-vanishing lower bound at $i = i_d$ and for a non-cryostable conductor the point $i = i_m$ is a regular point on the curve $v = v(i)$.

The steady propagating solutions of Equations (8) and (9) can be found analytically in the limit of $L_m/\sqrt{\xi} \ll L_{th}$. We write the equations in a frame of reference moving along the conductor with an arbitrary velocity v to be determined. The matching conditions at the transition point yield an implicit equation for v, which has the

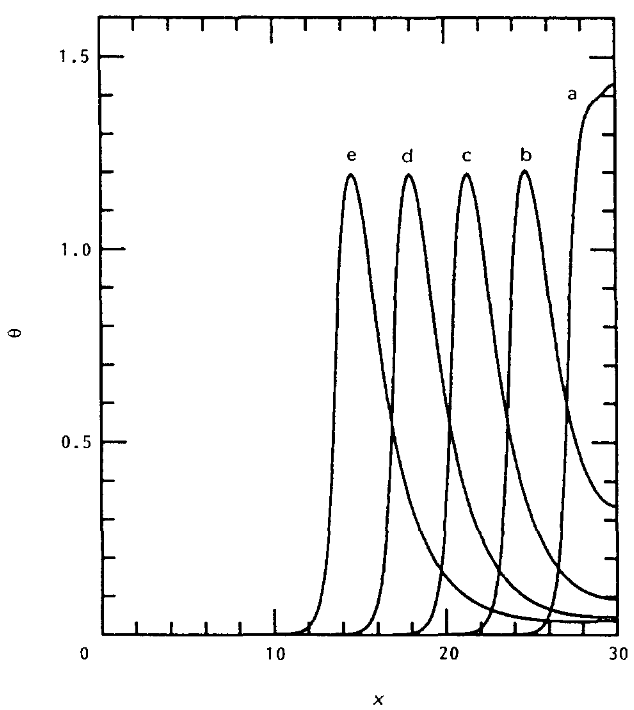

Figure 2 The formation of a travelling normal domain for $i > i_d$. The parameters are: $\tau = 100$, $\xi = 100$, $\alpha = 0.5$, $\lambda = 0.3$, $i = 0.34$, and (a) $t = 1$, (b) $t = 2$, (c) $t = 2$, (c) $t = 3$, (d) $t = 4$, (e) $t = 5$

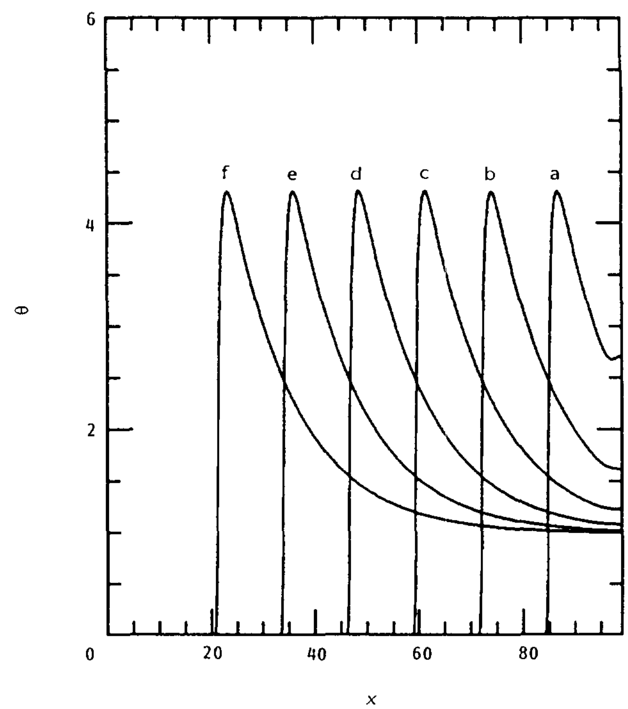

Figure 3 The formation of a superconductor-to-normal switching wave. The parameters are: $\tau = 100$, $\xi = 100$, $\alpha = 4.0$, $\lambda = 0.3$, $i = 0.50$, and (a) $t = 1$, (b) $t = 2$, (c) $t = 3$, (d) $t = 4$, (e) $t = 5$, (f) $t = 6$

following form.

$$(1 - i)(\omega_+ + \omega_-)$$

$$= -\frac{(\omega_+ - 2k_+)k^2}{(k_- + k_+)^2} \frac{\xi^2 i^2}{(1+\xi)^2} \frac{\alpha + \alpha\lambda^2 k_+^2}{4k_+^2 - 2\nu k_+ - 1}$$

$$-\frac{k_-(\omega_+ + \omega_-)}{k_- + k_+} \frac{\xi i}{1+\xi}$$

$$-\frac{(\omega_- - 2k_-)k_+^2}{(k_- + k_+)^2} \frac{\xi^2 i^2}{(1+\xi)^2} \frac{\alpha(1+\xi) + \alpha\lambda^2 k_-^2}{4k_-^2 + 2\nu k_- - 1}$$

$$+ \omega_- \frac{\alpha\xi i^2}{1+\xi} \tag{11}$$

where

$$\omega_\pm \equiv \frac{\pm\nu + \sqrt{\nu^2 + 4}}{2} \tag{12}$$

and

$$k_+ \equiv \frac{\nu\tau + \sqrt{(\nu\tau)^2 + 4\lambda^2}}{2\lambda^2}$$

$$k_- \equiv \frac{-\nu\tau + \sqrt{(\nu\tau)^2 + 4\lambda^2(1+\xi)}}{2\lambda^2} \tag{13}$$

In the relevant limit $\xi \gg 1$ and $\xi > \tau_m/\tau_{th}$, the solution of this equation is

$$v(i) \simeq \frac{L_{th}}{\tau_{th}} \sqrt{\frac{\alpha\xi i^2}{1-i} - \frac{2\xi\tau_{th}}{\tau_m}}, \quad v \gg 1 \tag{14}$$

In *Figure 4* we present numerical results of the velocity as a function of current for different values of α and $\tau = \tau_m/\tau_{th}$, together with the results of the analytical approximation (14). Comparison of the analytical approximation with the numerical results shows that the correspondence between them is very good.

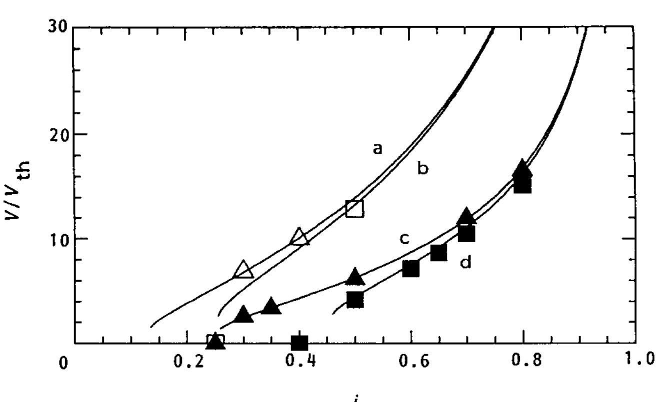

Figure 4 Velocity in units of $v_{th} = L_{th}/t_{th}$ as a function of current: (a) $\tau = 100.0$, $\alpha = 4.0$; (b) $\tau = 10.0$, $\alpha = 4.0$; (c) $\tau = 100.0$, $\alpha = 0.9$; (d) $\tau = 10.0$, $\alpha = 0.9$

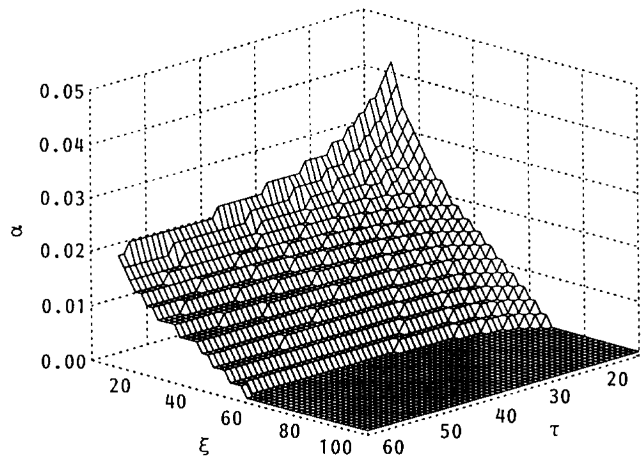

Figure 5 The margins of stability presented as a surface $\alpha = \alpha(\xi, \tau)$ for $\lambda = 0.3$

As the stability of the superconducting state is defined by the four dimensionless parameters α, ξ, τ and λ, the margins of stability can be presented as a surface $F(\alpha, \xi, \tau, \lambda) = 0$. In *Figure 5* we show the margins of stability in the form $\alpha = \alpha(\xi, \tau)$ for a fixed value of λ. This surface was obtained from Equation (11), taking the limit $i_d = 0.95$ as the stability criterion. In the step model, the velocity diverges at $i \rightarrow 1$, and $i = 0.95$ was taken as a cutoff value.

Discussion and summary

The above results can be explained qualitatively by the following arguments. When a part of the superconductor undergoes a normal transition, the current is still confined in the superconductor during a time interval of order τ_m/ξ. The Joule power is then higher than the Joule power when the current is redistributed across the conductor, by a factor of ξ. This initial heat release produces the 'hot' region observed at the front of the normal zone, and causes expansion of the normal zone. After the current is redistributed in the stabilizer, the conductor cools down towards a stable state; in particular, if $i < i_m$, superconductivity recovers. The propagation of the normal zone is thus completely determined by a segment of length $v\tau_m/\xi$ at the front, which behaves like a temporary unstabilized superconductor. The effective Stekly parameter associated with this temporary unstabilized superconductor is equal to $\alpha\xi$. Since the excess of heat at the front should be high enough to sustain a propagating resistive domain, v cannot be too small. As the dependence $v = v(i)$ is defined by the nearest vicinity of the front, the point $i = i_d$ is a regular point on the curve $v = v(i)$ even in the case of a non-cryostable conductor. The temperature distribution has three characteristic parts. The length of each part is determined by the product of the velocity of propagation of the normal zone, and the characteristic relaxation time of the current redistribution in this region. The first part is a region of length $v\tau_m/\xi$ behind the transition point, where current is diffusing into the stabilizer. The second part is a region of length $v\tau_{th}$ where the current flows mostly through the stabilizer, and the temperature is decreasing towards the transition point. Behind the normal zone, there is a region of length $v\tau_m$, which is in the superconducting state, and where current diffuses back into the superconductor.

To summarize, a system of two diffusion equations has been proposed modelling the nucleation and propagation of a normal zone in a composite superconductor for a relatively long time of current redistribution in the stabilizer. This model contains all the main physical features of current and temperature dynamics. The propagation of normal domains has been investigated. They exist if the current exceeds a threshold value. An analytical solution has been found for the temperature and the current density distributions in the steady state, for both normal domains and superconducting-to-normal switching waves, as well as an explicit expression for the velocity of propagation.

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