Propagating normal domains in large composite superconductors

Raz Kupferman, R. G. Mints, and E. Ben-Jacob
School of Physics and Astronomy, Raymond and Beverly Sackler Faculty of Exact Sciences, Tel-Aviv University, 69978 Ramat-Aviv, Israel

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We study the nucleation and propagation of the normal zone in large composite superconductors, considering the relatively long time of current redistribution in the stabilizer. We propose a model treating the composite as an effective electrical circuit, which yields two diffusion equations for the electric current and temperature distributions along the conductor. Numerical simulations are performed to study the dynamics of propagating normal domains in cryostable conductors. We derive an analytical solution for the margins of existence and velocity of propagation of the domain. The effect of the boiling crisis on the dynamics of the normal zone is also studied.

I. INTRODUCTION

The study of the normal zone of finite size (normal domains) in superconductors has been continuously a subject of interest in the field of applied superconductivity (see, for example, the review in Ref. 1, and references therein). It is well known that in homogeneous superconductors normal domains are always unstable, so that if a normal domain nucleates, it will either expand or shrink. If inhomogeneities, such as local variations in dimensions, resistivities, etc., are present, localized stable normal domains can exist in their vicinity. Stable normal domains were also found in composite superconductors, in the presence of a transition layer with high contact resistance between the superconductor and stabilizer. In this case the domains are stable for a finite range of currents, shrinking when the current is less, while for higher currents the domain undergoes a periodic process of splitting, until a string of normal domains is formed along the conductor.

Recently, very large superconducting composites have been tested for use in energy-storage devices (see, for example, Ref. 2). Because of the large size of the stabilizer, if a normal zone nucleates, the current in this region redistributes into the stabilizer, followed by a significant decrease of the joule power and the recovery of superconductivity. However, it was found experimentally that normal domains of finite size can propagate along the conductor despite the above stabilizing mechanism.[2] The formation of these traveling domains was shown to be a result of the high joule power generated in the superconductor during the relatively long process of current redistribution between the superconductor and stabilizer.[2–10] A number of theoretical studies were performed to investigate this new effect. Huang and Eyssa[3] performed numerical simulations for the diffusion of heat and redistribution of current in the conductor in the presence of a normal domain. Their simulations showed the formation of stable traveling normal domains. For example, they compared the velocity of propagation with the experimental data,[2] obtaining a reasonable agreement. Dresner[5] proposed an analytical method to calculate the velocity of propagation of the domain if the time dependence of the joule power is known. He performed explicit calculations approximating the decay of the joule power during the process of current redistribution by an exponential term.

In a recent study,[10] we proposed a model which allows one to investigate both numerically and analytically the nucleation and propagation of the normal zone in large composite superconductors. The composite is considered as an effective electrical circuit, consisting of two connected circuits, representing the superconductor and stabilizer. This model yields two diffusion equations describing the dynamics of the temperature and current-density distributions along the conductor. Numerical simulations of these equations showed the existence of propagating domains in the cryostable regime, but the main advance was the analytical investigation which supplied explicit formulas for measurable quantities. In this paper we present a detailed study on traveling normal domains based on the above model. We describe both numerical and analytical methods. Finally, we study the effect of the transition from nuclear boiling to film boiling[11] on the propagation of the normal domain.

II. THE MODEL

In this paper we consider a rectangular conductor, consisting of a plane layer of superconducting material, electrically and thermally bonded to a stabilizing normal metal (Fig. 1). The thicknesses of the superconductor and stabilizer are denoted by d_s and d_n, respectively. The conductor carries a transport current I and is kept in thermal contact with a heat reservoir of temperature T_0.

In general, the dynamics of the normal zone in composite superconductors is determined by both temperature and current-density distributions. A complete treatment of the problem requires the solution of the heat-diffusion equation, which defines the dynamics of the temperature field, and the set of Maxwell equations, which define the dynamics of the current distribution. These equations form a set of three-dimensional and time-dependent nonlinear equations, which is too difficult for either analytical or numerical investigation. Here we exploit the existence of different characteristic scales to reduce the complexity of the problem while preserving the main physical features.

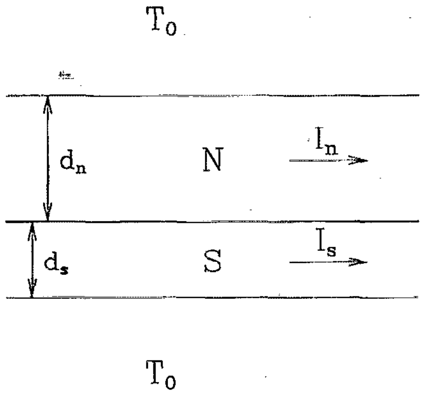

FIG. 1. Schematic structure of the composite superconductor (N, stabilizer; S, superconductor).

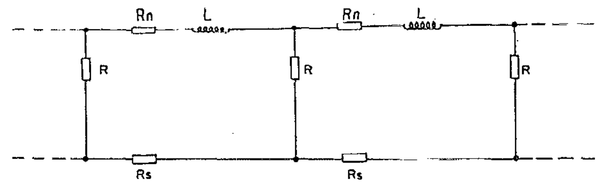

FIG. 2. Effective electrical circuit describing the current distribution in the conductor.

It has been usually assumed that the characteristic time of current redistribution across the wire is shorter than all other characteristic time scales.[1,12] Under this assumption the current distribution is completely determined by the temperature distribution. As was mentioned in the Introduction, the relatively long time of current redistribution is the origin of the formation of traveling normal domains in large conductors. Hence current redistribution, during the nucleation and propagation of normal zones, must be considered as a time-dependent process. Our model describes the process of current redistribution in a way which preserves the simplicity of treating one-dimensional fields, while taking into account the main features of the problem.

(i) The total longitudinal flow consists of parallel flows through the superconductor and stabilizer.

(ii) A perpendicular (redistribution) current is allowed to flow from one component to the other at any point along the conductor.

(iii) Variations in the longitudinal current have a finite duration, which is of the order of the diffusion time of magnetic flux in the stabilizer. It is proportional to the ratio of the inductance and resistance of a unit length of the conductor, $\tau_m \propto \mu_0 d_n^2/\rho_n$, where ρ_n is the resistivity of the stabilizer (we have neglected the inductance of the superconductor, because of its small size and its relatively high resistivity in the normal state). For example, in Ref. 2, τ_m is estimated as approximately 0.3 s, but can attain even larger values in the range 1–10 s for larger conductors.

The process of current redistribution in the conductor is modeled by the effective electrical circuit sketched in Fig. 2. In this model each component is described by a discrete chain of resistors. The upper chain represents the stabilizer, each resistor being attributed a resistance $R_n = \rho_n \Delta x/d_n$, where Δx is an arbitrary discretization length (x is the axis along the conductor). The lower chain represents the superconductor, with $R_s = \rho_s \Delta x/d_s$. ρ_s is the resistivity of the superconductor, which depends on both local temperature and current density in the superconductor. It vanishes in the superconducting state, and it is finite above the normal transition. The two chains are linked through a third kind of resistor, $R = \gamma_R \rho_n d_n/\Delta x$, where γ_R is a numerical factor of the order of one, which depends on the geometry of the conductor. Finally, we attribute to the normal resistors an inductance $\mathscr{L} = \gamma_{\mathscr{L}} \mu_0 d_n \Delta x$, where $\gamma_{\mathscr{L}} \sim 1$ is another numerical factor. Let j_s be the current density in the superconductor and $j = I/d_s$ be the current density in the superconductor if all the current flows through it. Then the current density in the stabilizer, j_n, is given by $j_n = (j - j_s)d_s/d_n$, and the current density in the perpendicular direction, j_R, is given by $j_R = -d_s(\partial j_s/\partial x)$. We apply now Kirchhoff's laws on this circuit, obtaining thus the following equation for the current density in the superconductor:

$$\left(\frac{\gamma_{\mathscr{L}} \mu_0 d_n^2}{\rho_n}\right)\frac{\partial j_s}{\partial t} = \gamma_R d_n^2 \frac{\partial^2 j_s}{\partial x^2} - j_s\left(1 + \frac{\rho_s d_n}{\rho_n d_s}\right) + j. \quad (2.1)$$

Next, we consider the temperature distribution in the conductor. We suppose that $\kappa_s/d_s + \kappa_n/d_n \gg h$, where κ_s and κ_n are the heat conductivities of the superconductor and stabilizer, respectively, taken here as constant, and $h(T)$ is the heat-transfer coefficient. It means that the thermal relaxation time over the cross section is much shorter than the thermal relaxation time between the conductor and coolant. In this case the temperature distribution over the cross section is almost uniform, and we can consider its averaged value $T(x)$, which is a function only of the coordinate along the conductor. The temperature $T(x)$ satisfies the heat equation

$$c\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(\kappa \frac{\partial T}{\partial x}\right) - W(T) + Q(T), \quad (2.2)$$

where c is the heat capacity and κ is the heat conductivity, both average values, obtained by weighting the values in each component with their relative cross-sectional area. The term $W(T)$ is the rate of heat transfer to the coolant per unit volume, which can be written as $W(T) = h(T) \times (T - T_0)/d$, where $d = d_s + d_n$. Finally, $Q(T,j_s)$ is the rate of joule heating per unit volume, which has three contributions: from the current in the superconductor when it is in the normal state, from the current in the stabilizer, and from the perpendicular current. As a result (see also Fig. 2), $Q(T)$ is given by

$$Q(T)=\frac{1}{d}\left[d_s\rho_s j_s^2+\frac{d_s^2\rho_n}{d_n}(j-j_s)^2+\gamma_R d_n d_s^2\rho_n\left(\frac{\partial j_s}{\partial x}\right)^2\right]. \tag{2.3}$$

It is convenient to introduce the following dimensionless fields: θ, the temperature, and i_s, the current density in the superconductor:

$$\theta\equiv\frac{T-T_0}{T_c-T_0},\quad i_s\equiv\frac{j_s}{j_c}, \tag{2.4}$$

where j_c is the critical current density in the superconductor at the temperature T_0. We define L_{th}, the characteristic thermal length, and τ_{th}, the characteristic thermal relaxation time,

$$L_{\text{th}}^2\equiv\frac{(d_n+d_s)\kappa}{h_0},\quad \tau_{\text{th}}\equiv\frac{(d_n+d_s)c}{h_0}, \tag{2.5}$$

with $h_0\equiv h(T_0)$. We define L_m, the characteristic length of variations in the current distribution, and τ_m, the corresponding relaxation time,

$$L_m^2\equiv\gamma_R d_n^2,\quad \tau_m\equiv\frac{\gamma_{\mathscr{L}}\mu_0 d_n^2}{\rho_n}. \tag{2.6}$$

Then we define the dimensionless parameters

$$\alpha\equiv\frac{d_s^2\rho_n j_c^2}{d_n h_0(T_c-T_0)},\quad \xi(\theta,i_s)\equiv\frac{\rho_s d_n}{\rho_n d_s}, \tag{2.7}$$

where ξ is the ratio of the resistances of the superconductor and stabilizer per unit length and α is the ratio of characteristic rates of joule heating and heat flux to the coolant (the Stekly parameter[13]). Finally, we use dimensionless scales of time and length. We express time in units of τ_{th} and length in units of L_{th}. Equations (2.1) and (2.2) take then the form

$$\frac{\partial\theta}{\partial t}=\frac{\partial^2\theta}{\partial x^2}-\frac{h(\theta)}{h_0}\theta+\alpha(i-i_s)^2+\xi(\theta,i_s)\alpha i_s^2+\alpha\lambda^2\left(\frac{\partial i_s}{\partial x}\right)^2, \tag{2.8}$$

$$\tau\frac{\partial i_s}{\partial t}=\lambda^2\frac{\partial^2 i_s}{\partial x^2}-[1+\xi(\theta,i_s)]i_s+i, \tag{2.9}$$

where the dimensionless parameters i, τ, and λ are given by

$$i\equiv j/j_c,\quad \tau\equiv\tau_m/\tau_{\text{th}},\quad \lambda\equiv L_m/L_{\text{th}}. \tag{2.10}$$

The stationary uniform states of Eqs. (2.8) and (2.9) are determined by setting to zero all time and space derivatives. These states are described by the roots of the equations

$$\frac{h(\theta)}{h_0}\theta=\alpha(i-i_s)^2+\xi(\theta,i_s)\alpha i_s^2,\qquad i_s=[1+\xi(\theta,\ i_s)]^{-1}i. \tag{2.11}$$

In general, Eq. (2.11) can have multiple solutions. For all currents below the critical current $(i=1)$, there is a uniform stationary solution corresponding to the homogeneous superconducting state, $\theta=0$ and $i_s=i$, as below the critical current, $\xi(0,i_s)=0$. Other states correspond to homogeneous states where the superconductor is in its normal state, and the current flows through both superconductor and stabilizer. The existence and properties of such states depend on the explicit form of the functions $h(\theta)$ and $\xi(\theta,i_s)$.

In Secs. III and IV, we consider for simplicity the case where the heat-transfer coefficient is constant, $h(\theta)=h_0$, and where the resistivity of the superconductor is a step function at the normal transition $\rho_s(j_s,T)=\rho_s\eta[j_s-j_c(T)]$, where η is the Heavyside function. We also assume that $j_c(T)=j_c[1-(T-T_0)/(T_c-T_0)]$. In dimensionless notation we define $\xi(\theta,i_s)=\xi\eta(i_s+\theta-1)$. Equations (2.8) and (2.9) take then the form

$$\frac{\partial\theta}{\partial t}=\frac{\partial^2\theta}{\partial x^2}-\theta+\alpha(i-i_s)^2+\xi\alpha i_s^2\eta(i_s+\theta-1)+\alpha\lambda^2\left(\frac{\partial i_s}{\partial x}\right)^2, \tag{2.12}$$

$$\tau\frac{\partial i_s}{\partial t}=\lambda^2\frac{\partial^2 i_s}{\partial x^2}-[1+\xi\eta(i_s+\theta-1)]i_s+i. \tag{2.13}$$

In this case there is only one steady normal state, given by $\theta=\alpha\xi i^2/(1+\xi)$ and $i_s=i/(1+\xi)$. It exists only if $j_s>j_c(T)$ (in the dimensionless notation, $i_s>1-\theta$). The minimum current of normal-zone existence, i_m, is given in this specific model by $i_m=[\sqrt{1+4\alpha\xi(\xi+1)}-1]/2\alpha\xi$. The composite is therefore cryostable ($i_m=1$) if $\alpha<1$.

To complete the presentation of the model, it is important to identify the different characteristic time and length scales of the system. Equation (2.12) has one set of characteristic scales, both defined here to be equal to one. Equation (2.13) has two sets of characteristic scales, depending whether the system is in its superconducting state ($\eta=0$) or in its normal state ($\eta=1$). In the superconducting state, current diffuses from the stabilizer to the superconductor with a characteristic length scale L_m and a relaxation time τ_m, while in the second case current diffuses into the stabilizer with a characteristic length scale $L_m/\sqrt{1+\xi}$ and relaxation time $\tau_m/(1+\xi)$. It will be shown that the existence of these different space and time scales in this problem plays an important role in the formation of the traveling normal domains.

It should be emphasized that all the properties of the system depend only on the four dimensionless parameters α, ξ, τ, and λ, as the final equations (2.12)–(2.13) include only those parameters.

III. RESULTS

A. Numerical simulations

In order to study the formation and propagation of normal domains, we performed numerical simulations of Eqs. (2.12) and (2.13). We observed how the temperature and current-density distributions evolve in time when the system is initially in the superconducting state, except a

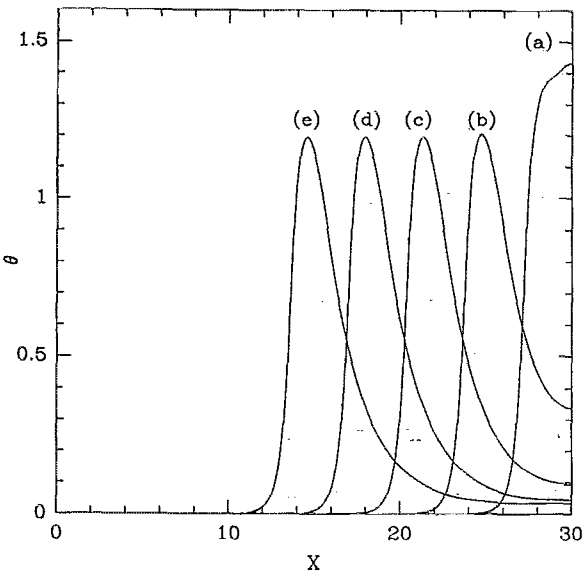

FIG. 3. Temperature distribution for $i > i_d$. The parameters are $\tau = 100$, $\xi = 100$, $\alpha = 0.5$, $\lambda = 0.3$, and $i = 0.34$, and (a) $t = 1$, (b) $t = 2$, (c) $t = 3$, (d) $t = 4$, and (e) $t = 5$.

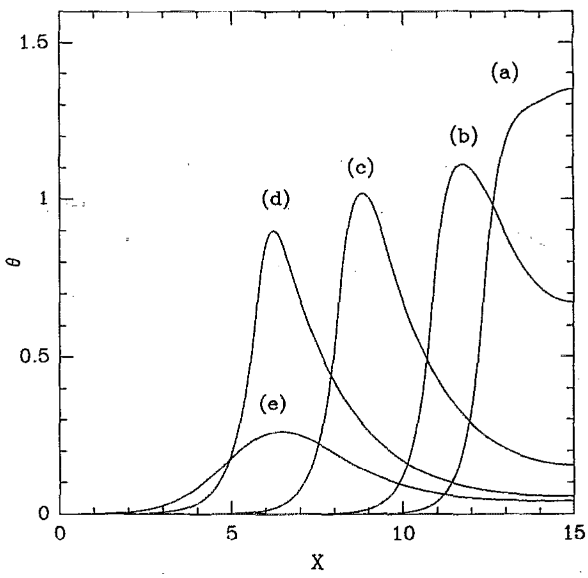

FIG. 4. Temperature distribution for $i < i_d$. The parameters are $\tau = 100$, $\xi = 100$, $\alpha = 0.5$, $\lambda = 0.3$, and $i = 0.33$, and (a) $t = 1$, (b) $t = 2$, (c) $t = 4$, (d) $t = 6$, and (e) $t = 7$.

nucleus of length $2L_{\text{th}}$, in which the temperature is raised to the critical value $\theta = 1$. The parameters were evaluated using experimental data from Refs. 1 and 2. We obtained the typical values $\xi \sim 10$–100, $\tau \sim 10$–100, and $\lambda \sim 0.1$–1. As we are interested in cryostable conductors, we consider only the case where $\alpha < 1$.

For a given set of parameters, there is a threshold current i_d, above which propagating domains are formed. For $i_d < i < 1$, the initial normal zone starts to expand during the diffusion of current out into the stabilizer. After it reached a certain length, the center of the normal zone starts to cool down, while the outer sides continue to expand (the heat generation there is maximal). As a result, superconductivity recovers at the center of the normal zone, and we find two separated normal domains traveling away in opposite directions. The system tends to a steady state where two normal domains are propagating with constant velocity, while superconductivity recovers behind.

A sequence of temperature distributions for $i > i_d$ is shown in Fig. 3. The temperature field at the front of the propagating domain reaches a steady shape after a time interval of the order of the thermal relaxation time τ_{th} (one in dimensionless units). The "tail" of the profile reaches its steady shape only after a relatively long time interval, which is of the order of the current distribution relaxation time τ_m. The velocity of propagation attains its final value much faster than the time required to obtain the steady profile. It is consistent with the well-known fact that the velocity of the normal zone is determined only by the temperature at the front of the domain.[1] In Fig. 4 we show a sequence of temperature profiles that were obtained for current below i_d, but very close to it. The initial heat release in the normal zone forms a propagating domain, whose temperature gradually decreases, until it reaches the critical temperature, where superconductivity recovers.

To study the dependence of the velocity of propagation on the various parameters, we show in Fig. 5 plots of $v(i)$ for different values of α and τ (in units of $v_{th} \equiv L_{\text{th}}/\tau_{th}$). The velocity is a monotonically increasing function of the current i, and it is finite at the threshold, in agreement with Refs. 3 and 5. The values of the threshold current i_d and the threshold velocity v_d depend both on τ and α. Above i_d the dependence of the velocity on τ becomes less significant, and it is mainly determined by α. At $i \to 1$ the velocity is relative to $\sqrt{\alpha}$. The velocity diverges as i approaches

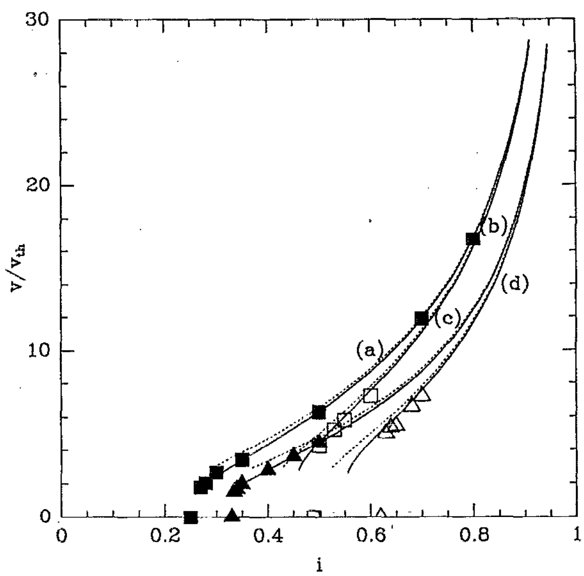

FIG. 5. Velocity in units of v_{th} vs current. The dots represent the results obtained in the numerical simulations, the solid lines are the solutions of Eq. (3.10), and the dashed lines represent formula (3.11). The parameters are $\xi = 100$, (a) $\tau = 100$, and $\alpha = 0.9$, (b) $\tau = 10$ and $\alpha = 0.9$, (c) $\tau = 100$ and $\alpha = 0.5$, and (d) $\tau = 10$ and $\alpha = 0.5$.

the critical value one (this fact is due to the specific I-V characteristic used here[1]).

B. Analytical solution of the steady state

For $i > i_d$ the temperature and current-density distributions of the steady state are the stationary solutions of Eqs. (2.12) and (2.13), with $i_s = i_s(x + vt)$ and $\theta = \theta(x + vt)$, which correspond to a frame of reference moving along the conductor with velocity v:

$$0 = \frac{\partial^2 \theta}{\partial x^2} - v \frac{\partial \theta}{\partial x} - \theta + \alpha(i - i_s)^2 + \xi \alpha i_s^2 \eta(i_s + \theta - 1) + \alpha\lambda^2 \left(\frac{\partial i_s}{\partial x}\right)^2, \tag{3.1}$$

$$0 = \lambda^2 \frac{\partial^2 i_s}{\partial x^2} - v\tau \frac{\partial i_s}{\partial x} - [1 + \xi\eta(i_s + \theta - 1)] i_s + i, \tag{3.2}$$

where v still has to be determined. We define $x = 0$ to be the point where the normal transition occurs and $x = l$ to be the point where superconductivity recovers. Equation (3.2) is nonlinear, but it can be solved in the three regions $x < 0$ ($\eta = 0$), $0 \leqslant x \leqslant l$ ($\eta = 1$), and $l \leqslant x$ ($\eta = 0$). In each region it becomes a linear equation with constant coefficients. The explicit expressions for $i_s(x)$ can be then substituted into Eq. (3.1), yielding a linear equation for $\theta(x)$. Finally, the boundary and matching conditions form a closed set of equations for the integration constants v and l.

A considerable simplification of this procedure can be obtained by ignoring the recovery of superconductivity, thus performing the above procedure only in the two regions $x < 0$ and $x > 0$. As was shown in Sec. III, the recovery of superconductivity occurs far behind the propagating front and hence does not affect the propagation. This approximation is found to be justified for most values of current i and breaks down only close to the threshold current.

We start with Eq. (3.2). The boundary conditions at infinity are

$$i_s(-\infty) = i, \quad i_s(\infty) = i/(1 + \xi). \tag{3.3}$$

The solution is given by

$$i_s(x) = \begin{cases} i - Ae^{k_+ x}, & x < 0, \\ i/(1+\xi) + Be^{-k_- x}, & x > 0, \end{cases} \tag{3.4}$$

where

$$k_+ \equiv \frac{v\tau + \sqrt{(v\tau)^2 + 4\lambda^2}}{2\lambda^2}, \tag{3.5}$$

and

$$k_- \equiv \frac{-v\tau + \sqrt{(v\tau)^2 + 4\lambda^2(1+\xi)}}{2\lambda^2}. \tag{3.6}$$

Substituting the current distribution (3.4) into Eq. (3.1) with the boundary conditions, at infinity,

$$\theta(-\infty) = 0, \quad \theta(\infty) = \alpha\xi i^2/(1 + \xi), \tag{3.7}$$

we obtain

$$\theta(x) = \begin{cases} Ce^{\omega_+ x} - \dfrac{A^2(\alpha + \alpha\lambda^2 k_+^2)}{4k_+^2 - 2vk_+ - 1} e^{2k_+ x}, \quad x < 0, \\ De^{-\omega_- x} + \dfrac{\alpha\xi i^2}{1+\xi} - \dfrac{B^2(\alpha + \alpha\xi + \alpha\lambda^2 k_-^2)}{4k_-^2 + 2vk_- - 1} \\ \quad \times e^{-2k_- x}, \quad x > 0, \end{cases} \tag{3.8}$$

where

$$\omega_\pm \equiv \frac{\pm v + \sqrt{v^2 + 4}}{2}. \tag{3.9}$$

The four unknown constants A, B, C, D and velocity v are determined by four matching conditions and by the requirement of self-consistency at the transition point $i_s(0) = 1 - \theta(0)$. A closed equation is obtained for the velocity:

$$\begin{aligned}(1 - i)(\omega_+ + \omega_-) &= -\frac{(\omega_+ - 2k_+)k_-^2}{(k_- + k_+)^2} \frac{\xi^2 i^2}{(1+\xi)^2} \frac{\alpha + \alpha\lambda^2 k_+^2}{4k_+^2 - 2vk_+ - 1} \\ &\quad - \frac{k_-(\omega_+ + \omega_-)}{k_- + k_+} \frac{\xi i}{1+\xi} - \frac{(\omega_- - 2k_-)k_+^2}{(k_- + k_+)^2} \\ &\quad \times \frac{\xi^2 i^2}{(1+\xi)^2} \frac{\alpha(1+\xi) + \alpha\lambda^2 k_-^2}{4k_-^2 + 2vk_- - 1} + \omega_- \frac{\alpha\xi i^2}{1+\xi}.\end{aligned} \tag{3.10}$$

Calculations of $v(i)$ show, in agreement with the dynamical simulations, that for any given set of parameters, Eq. (3.10) has solutions of v only if i is larger than a threshold value i_d. For $i_d < i < 1$ there are two roots, where only the largest corresponds to a stable solution (the second root is a decreasing function of i). At the threshold there is only one root v_d, which has a finite value.

As stated before, the relevant range of parameters in the case of large cryostable conductors is $\xi \gg 1$, $1 < \tau < \xi$, and $\lambda < 1$. The computations show that in this regime of parameters, the right-hand side in Eq. (3.10) is dominated by the third term. Expanding the leading terms in powers of $1/v$, we find

$$v(i) \simeq \sqrt{\frac{\alpha\xi i^2}{1 - i} - \frac{2\xi}{\tau}} \quad (v > 1). \tag{3.11}$$

Figure 5 shows a comparison of the velocity obtained by the numerical simulations with the velocity obtained by the analytical solution. For large values of τ ($\tau = 100$), the roots of the implicit Eq. (3.10) give the velocity with an extremely high degree of accuracy in the entire range of currents. In particular, it gives with excellent agreement the threshold current i_d and the corresponding velocity v_d. For smaller values of τ ($\tau = 10$), the velocity calculated by (3.10) is still very close to the exact value, deviating from it in less than 5% for $i > i_d$, while the threshold current is about 10% less than the value obtained in the numerical simulations. This discrepancy is easily explained by the fact that when τ is smaller, the length of the domain decreases, and the recovery of superconductivity affects

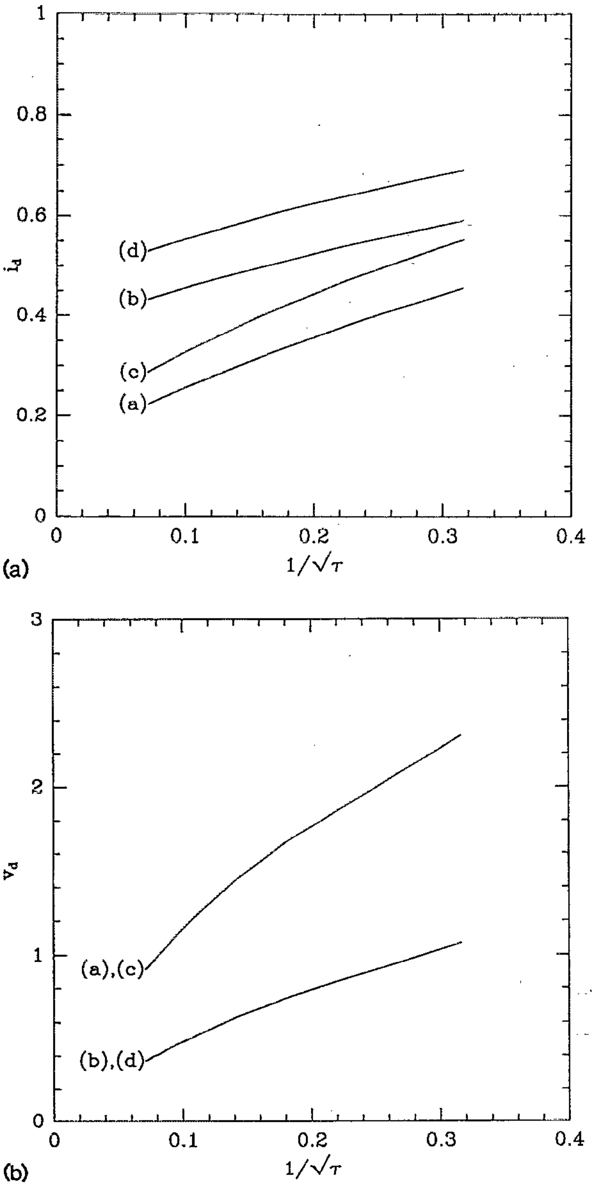

FIG. 6. (a) The threshold current i_d vs $\tau^{-1/2}$ for (a) $\xi = 100$ and $\alpha = 0.9$, (b) $\xi = 10$ and $\alpha = 0.9$, (c) $\xi = 100$ and $\alpha = 0.5$, and (d) $\xi = 10$ and $\alpha = 0.5$. (b) The threshold velocity v_d vs $\tau^{-1/2}$ for (a) $\xi = 100$ and $\alpha = 0.9$, (b) $\xi = 10$ and $\alpha = 0.9$, (c) $\xi = 100$ and $\alpha = 0.5$, and (d) $\xi = 10$ and $\alpha = 0.5$.

more significantly the velocity. Equation (3.11) fits the roots of Eq. (3.10) for large values of i. As this formula was obtained by expanding the implicit equation in powers of $1/v$, it is accurate only if the velocity is sufficiently large. For $v > 4$ it is accurate within a few percent.

As mentioned before, in this model all the properties of the system depend on the four dimensionless parameters α, ξ, τ, and λ. To obtain the dependence on the real physical parameters, it is necessary to use the definitions (2.5)–(2.10). The dependence of the threshold current on the parameters α, ξ, and τ is demonstrated in Fig. 6(a). The threshold current i_d is approximately linear with respect to $\tau^{-1/2}$, with a maximum deviation less than 3%. It is a decreasing function of both α and ξ. Note that the equation

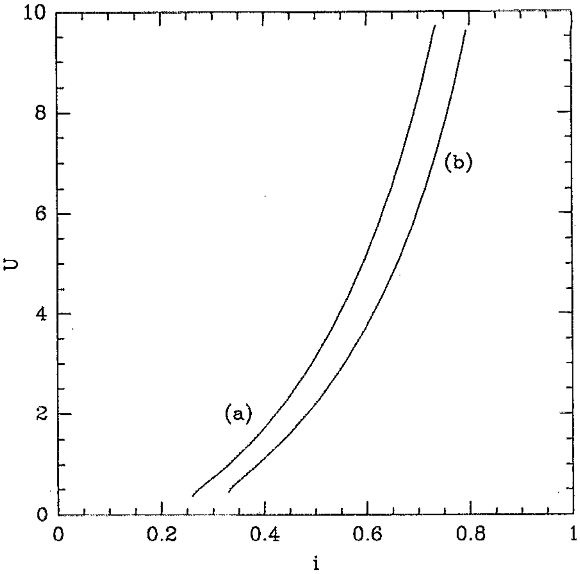

FIG. 7. V-I characteristic of a large composite superconductor in the presence of a traveling normal domain (U is in units of $\rho_s j_c L_{\rm th}$). The parameters are $\tau = 100.0$, $\xi = 100$, and $\lambda = 0.3$, and (a) $\alpha = 0.9$ and (b) $\alpha = 0.5$.

$i_d(\alpha,\xi,\tau,\lambda) = 1$, defines the margins of stability. The threshold velocity v_d as a function of α, ξ, and τ is shown in Fig. 6(b). We find that v_d is an increasing function of α, a decreasing function of τ, and does not depend on the parameter ξ.

In the presence of a normal domain, there is a potential drop on the conductor. The voltage U is equal to

$$U = j_c \rho_s \int dx\, i_s(x). \tag{3.12}$$

Using Eq. (3.4)–(3.10), we obtain the plots $U(i)$ shown in Fig. 7. It follows from the above discussion that the V-I characteristic starts at $i = i_d$, with a finite threshold voltage. The curves $U(i)$ are strongly nonlinear, with a diverging derivative at $i \to i_d$.

IV. THE BOILING CRISIS

One conclusion arising from the above results is that despite the existence of a stabilizing mechanism, a region of high temperature can propagate along the conductor. It is well known that as the temperature gradient between the conductor and coolant becomes large, the heat flux to the coolant is strongly affected by the so-called boiling crisis—a transition from the nuclear boiling to the film boiling regime.[11] To study this effect, we will assume that $h(\theta)$ is a step function, given by

$$\frac{h(\theta)}{h_0} = \begin{cases} 1, & \theta < \theta_{bc}, \\ m, & \theta > \theta_{bc}. \end{cases} \tag{4.1}$$

Experimental data for liquid helium at 4.2 K give typical values $\theta_{bc} \sim 0.2$ and $m \sim \frac{1}{40}$.

The stationary homogeneous states of this system are the solutions of Eq. (2.11), substituting the heat-transfer

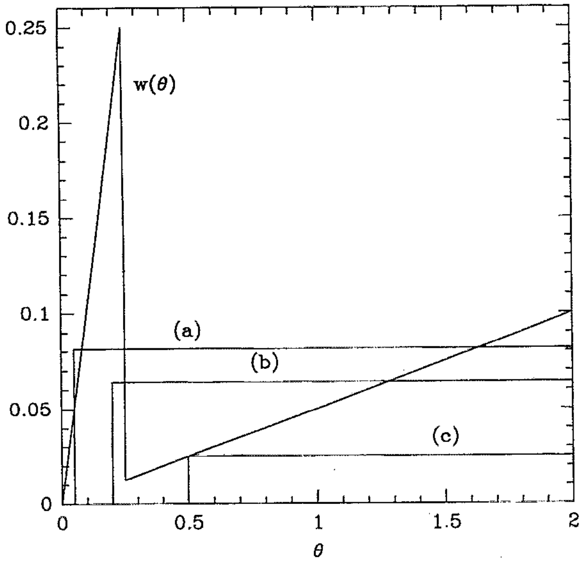

FIG. 8. Graphical solution of the uniform stationary states in the presence of the boiling crisis. The parameters are $\alpha = 0.1$, $m = 0.05$, and $\theta_{bc} = 0.25$. The number of states are (a) two, (b) three, and (c) five.

coefficient given by (4.1). A graphical solution of these equations is shown in Fig. 8, where the states of the system correspond to the intersections of the dimensionless heat flux $w(\theta) = [h(\theta)/h_0]\theta$ and the dimensionless joule power $q(\theta) \simeq \alpha i^2 \eta(\theta + i_s - 1)$ [see Eq. (2.8)]. As a result of the N-shaped function $w(\theta)$, there are ranges of parameters where there are three locally stable states. A conductor will be cryostable if the only intersection is $\theta = 0$.[1] This condition is satisfied for all $i < 1$ if $\alpha < m\theta_{bc}$.

Figure 9 shows a typical sequence of temperature-field profiles during the formation of a propagating domain in a cryostable conductor. The horizontal dashed line represents the temperature at which the boiling transition occurs. In the front of the domain, the temperature is usually above θ_{bc}; hence this region is hardly affected by this transition. As a result, the propagation of the domain is as in the case of a composite superconductor with an effective Stekly parameter α/m corresponding to the film-boiling regime. Behind the propagating front, we see two distinct regions in which the temperature decays at different rates. In the region where the temperature is above θ_{bc}, the decay is moderate as a result of the small value of $h(\theta)$. When the temperature crosses the transition point, the sharp increase of h causes the drastic change in the cooling rate.

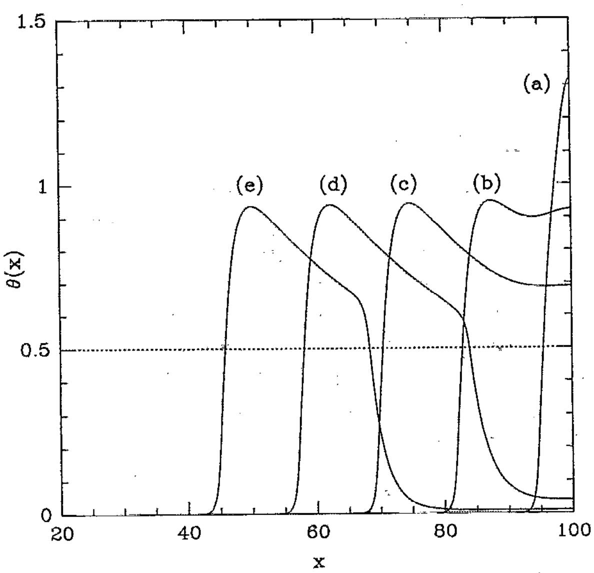

FIG. 9. Temperature distribution in the presence of the boiling crisis. The parameters are $\tau = 200$, $\xi = 100$, $\alpha = 0.02$, $\lambda = 1.2$, $i = 0.80$, $m = 0.02$, and $\theta_{bc} = 0.5$, and (a) $t = 1$, (b) $t = 3$, (c) $t = 5$, (d) $t = 7$, and (e) $t = 9$.

V. DISCUSSION

Let us now discuss the physical mechanism of normal-zone propagation in large composite superconductors. Imagine that a part of the superconductor undergoes to the normal state. The current starts to redistribute between the superconductor and stabilizer by diffusion, a process which has a characteristic duration of τ_m/ξ. After the redistribution of current is complete, the conductor cools down during a time period of the order of the thermal relaxation time $\tau_{\rm th}$. When the temperature crosses the critical temperature, the superconducting state is recovered, and the current rediffuses back to the superconductor during a time period of the order of τ_m.

As the redistribution of current requires a finite duration, the stabilizing mechanism suffers an effective delay time τ_m/ξ. The joule power in the normal zone during this time interval is consequently high as in the case of a unstabilized superconductor. The effective Stekly parameter $\tilde{\alpha}$ associated with this temporarily unstabilized superconductor is determined by the resistivity of the superconductor in the normal state, ρ_s, given by $\tilde{\alpha} = \rho_s j_c^2 d_s / h_0(T - T_0) = \alpha\xi$. We note that in most cases of practical interest (where $\alpha \ll 1$), $\alpha\xi \gg 1$. In the case of an unstabilized conductor, the normal zone expands with constant velocity v if the current exceeds the minimum normal-zone propagating current $\tilde{i}_p$. In this range of parameters, $\tilde{i}_p$ is given by $\tilde{i}_p \sim \sqrt{2/\tilde{\alpha}}$ and the velocity v is given by the approximate expression[1,12]

$$v \simeq v_{\rm th}\sqrt{\alpha\xi i^2/(1-i)}, \quad v(i) \gg v_{\rm th}.$$

This expression coincides with formula (3.11) when $\tau/\xi \gg 1$, as in this limit the delay time of the stabilizing mechanism becomes very long.

The normal-zone expansion, accompanied with the local recovery of superconductivity at each point, is the origin of the formation of traveling normal domains of finite size. As it is seen in Fig. 3, the temperature distribution has three characteristic parts. The length of each part is determined by the product of the velocity of propagation of the normal zone and the characteristic relaxation time of the current redistribution in this region. The first part is a region of length $v\tau_m/\xi$ behind the transition point, where current is diffusing into the stabilizer. In this region the heat generation is the highest, and temperature distribution there determines the velocity of propagation of the normal zone. The second part is a region of length $v\tau_{th}$, where the current flows mostly through the stabilizer and the temperature is decreasing toward the transition point. Behind the normal zone, there is a region of length $v\tau_m$, which is in the superconducting state and where current diffuses back into the superconductor.

To conclude, we proposed a system of two diffusion equations modeling the nucleation and propagation of the normal zone in large composite superconductors. This model contains all the main physical features of current and temperature dynamics. We investigated the propagation of normal domains and showed that they exist if the current exceeds a threshold value. We found an analytical solution for the temperature and current-density distributions for the stationary normal domains, as well as an explicit expression for the velocity of propagation.

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