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Non-uniform distribution of normal zone in composite superconductors with contact resistance

A A Akhmetov and R G Mints
Institute of High Temperatures, 127412 Moscow, USSR

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Abstract. A region of resistive domains is found in composite superconductors with thermal and electrical contact resistance between the normal metal matrix and the superconductor. The V–I characteristic of a sample with resistive domains is found. It is shown that the existence of resistive domains leads to the emergence of hysteresis upon decay and recovery of superconductivity in the presence of transport current. It is shown that the degradation of composite superconductors with contact resistance may be attributed to the formation of resistive domains.

1. Introduction

The distribution of a normal zone in composite superconductors with electrical and thermal contact resistance has been the subject of frequent studies (cf Kremlev 1967, 1980, Keilin and Ozhogina 1977). Considerably less studied is the problem of the origin, evolution and stable existence of normal zone regions of finite size. The presence of such regions (resistive domains) defines the V–I characteristics of composite superconductors and hysteresis phenomena therein upon decay and subsequent recovery of superconductivity. Of greatest interest in this case is the possibility of the stable existence of resistive domains in the regions of values of transport current I less than those of the minimum normal zone propagation current I_p. Note that such is the case, for example, in non-uniform superconductors (Mints 1979, Gurevich and Mints 1981). In uniform composite superconductors without contact resistance, resistive domains in the preset current mode are unstable (Al'tov *et al* 1975); however, they can be stabilised in the preset voltage mode or, which is the same, in a shunted sample.

As shown in a recent brief communication (Akhmetov and Mints 1982) contact resistances lead to the possibility of stable existence of resistive domains even in composite superconductors that are uniform along the sample axis, with the minimum resistive domain existence current $I_r \ll I_p$ under conditions of high contact resistance. Physically, the stability of resistive domains in composite superconductors with adequately high contact resistance is due to the fact that, under these conditions, current flows from normal metal to superconductor over a finite length l_i exceeding the size of the resistive domain, while normal metal becomes an 'external shunt' relative to the superconductor. Note that the minimum resistive domain existence current (if $I_r < I_p$), is at the same time, the superconductivity recovery current (Mints 1979).

The V–I characteristics have been found by Akhmetov and Mints (1982) in the region of transport current values $I \ll I_p$ where the length of resistive domain, $2l$, is considerably less than the characteristic scale of temperature variation in the superconductor (l_s) and normal metal (l_n), as well as l_i. The condition $l \ll l_s, l_n, l_i$ helped analytically to solve the problem of the stationary distribution of temperature in the resistive domain given an arbitrary dependence of current density in the superconductor upon temperature and electric field intensity.

In the region of transport current values where $l_s \lesssim l$, a specific V–I curve of the superconductor should be given for calculating the resistive domain characteristics and the V–I characteristic of a sample with resistive domains.

In this paper, we shall consider the stationary temperature distribution and the process of evolution of the corresponding initial heat pulse in a composite superconductor with high thermal and electrical contact resistance in the entire region of values of transport current in the sample. Using the obtained temperature distributions, the V–I characteristics of a sample with resistive domains have been plotted.

2. Basic equations

The distribution of the resistive and superconducting zones along an infinite composite superconductor is found from the heat-transfer equation describing the temperature distribution, and the continuity equation describing the electric current distribution over the conductor components. Let, for simplicity, the sample consist of three ribbons of equal width, namely, superconductor (having a thickness of d_s), interface layer (d_i) and normal metal (d_n), with $d_i \ll d_s$, d_n. Assume further that all transverse electrical and thermal resistance is concentrated in the interface layer. Then, given the parameter values characteristic of composite superconductors, the temperature of normal metal T_n and of superconductor T_s, as well as the current density in normal metal j_n and in superconductor j_s, may be regarded as uniform in the plane transverse to the sample axis (x-axis). Therefore, the temperature and electric field distribution presents a one-dimensional problem described by (Kremlev 1980, Akhmetov and Mints 1982):

$$\nu_n \frac{\partial T_n}{\partial t} = \kappa_n \frac{\partial^2 T_n}{\partial x^2} - \frac{W_0}{d_n}(T_n - T_0) + \rho_n j_n^2 + \frac{1}{2}\left(\frac{\partial j_n}{\partial x}\right)^2 \rho_i d_i d_n - \frac{\kappa_i}{d_i d_n}(T_n - T_s) \tag{1}$$

$$\nu_s \frac{\partial T_s}{\partial t} = \kappa_s \frac{\partial^2 T_s}{\partial x^2} - \frac{W_0}{d_s}(T_s - T_0) + j_s E_s + \frac{1}{2}\left(\frac{\partial j_n}{\partial x}\right)^2 \rho_i \frac{d_i d_n^2}{d_s} - \frac{\kappa_i}{d_i d_s}(T_s - T_n) \tag{2}$$

$$d_n d_i \rho_i \frac{\partial^2 j_n}{\partial x^2} - \rho_n j_n + E_s = 0. \tag{3}$$

Here and below, the subscripts s, i and n are used to denote the physical characteristics of the superconductor, interface layer and normal metal, while ν is heat capacity, κ is heat conductivity, ρ is resistivity, W_0 is coefficient of heat transfer to the coolant, T_0 is temperature of the coolant and E is electric field.

As follows from equations (1)–(3), the characteristic scales of temperature variations l_s and l_n and the transition length l_i where the current flows over from normal metal to superconductor are respectively equal to

$$l_s^2 = \frac{\kappa_s d_s}{W} \qquad l_n^2 = \frac{\kappa_n d_n}{W} \qquad l_i^2 = \frac{\rho_i}{\rho_n} d_i d_n$$

where $W = W_0 + \kappa_i/d_i$. the values of l_s, l_n and l_i in practical composite superconductors with high contact resistance ($\kappa_i \ll \kappa_s \ll \kappa_n$, $\rho_n \ll \rho_i$, $\nu_n < \nu_s$) correlate as $l_s \ll l_n \ll l_i$, with $l_i \gg l_s \ll l_n^2$, while the characteristic heat times

$$\tau_s = \nu_s d_s/W \qquad \tau_n = \nu_n d_n/W$$

satisfy the inequality $\tau_n \leqslant \tau_s$.

The current density j_s may evidently be written as

$$j_s = j - j_n d_n/d_s$$

where $j = j_s(\pm\infty)$ the current density in the superconductor at an adequate distance from the region with resistive phase. Assume further that the critical current density $j_c(T)$ may be written as

$$j_c(T) = j_0(1 - T/T_c) = j_c(1 - \theta)$$

where $j_c = j_c(T_0)$, and the dimensionless temperature

$$\theta = (T - T_0)/(T_c - T_0).$$

It follows from the foregoing that $j < j_c(T_0)$.

It is convenient to rewrite equations (1)–(3) for further use in terms of temperature θ and dimensionless current density

$$i_n = j_n d_n/j_c d_s$$

so that

$$\tau_n \dot{\theta}_n = l_n^2 \theta_n'' - \theta_n + 2\alpha i_n^2 + \alpha l_i^2 \left(\frac{di_n}{dx}\right)^2 + h\theta_s \tag{4}$$

$$\tau_s \dot{\theta}_s = l_s^2 \theta_s'' - \theta_s + 2\alpha i_s \varepsilon_s + \alpha l_i^2 \left(\frac{di_n}{dx}\right)^2 + h\theta_n \tag{5}$$

$$l_i^2 i_n'' - i_n + \varepsilon_s = 0. \tag{6}$$

Here

$$h = \frac{\kappa_i}{d_i W} = \frac{\kappa_i}{d_i W_0[1 + (\kappa_i/d_i W_0)]} \leqslant 1$$

$$i_s = \frac{j_s}{j_c} \qquad \varepsilon_s = \frac{E_s d_n}{\rho_n j_c d_s} \qquad \alpha = \frac{\rho_n j_c^2 d_s^2}{2W(T_c - T_0)d_n}.$$

The parameter α differs from the conventionally used Stekly parameter α_{St} (Al'tov *et al* 1975) in that it is normalised for the heat transfer constant

$$W = W_0 + \kappa_i/d_i$$

rather than W_0. However, if the thermal contact resistance between the superconductor and normal metal is high ($h \ll 1$), the difference between α and α_{St} is insignificant. Note that for practical composite superconductors in usual circumstances the value of h is of the order of unity ($h \leqslant 1$), but in some special cases h may be as low as 0.1 ($W_0 \cong 5\times10^{-1}$ W K^{-1} cm^{-2}, $\kappa_i \cong 10^{-2}$ W K^{-1} cm^{-1}, $d_i \cong 2\times10^{-1}$ cm).

Under conditions of arbitrary correlation between l_s and the resistive domain length $2l$, the system of equations (4)–(6) may evidently be solved only if the dependence

$\varepsilon_s = \varepsilon_s(\theta_s, i_s)$ is specified. It will be generally assumed in the present paper, with the exception of the $l \ll l_s$ case, that

$$E_s = \begin{cases} 0 & \text{at } \theta_s < 1 - i_s \\ \rho_s j_s & \text{at } \theta_s \geqslant 1 - i_s. \end{cases} \tag{7}$$

From equation (7), we find ε_s in the form

$$\varepsilon_s = (i_s/r^2)\,\eta(i_s - 1 + \theta_s) \tag{8}$$

where $\eta(x)$ is the Heaviside step function ($\eta = 0$ at $x < 0$ and $\eta = 1$ at $x \geqslant 0$), and $r^2 = \rho_n d_s/\rho_s d_n$. Given the parameter values characteristic of composite superconductors, $r \ll 1$. On substituting equation (8) in the system of equations (4)–(6), we find:

$$\tau_n \dot{\theta}_n = l_n^2 \theta_n'' - \theta_n + 2\alpha i_n^2 + \alpha l_i^2 (\mathrm{d}i_n/\mathrm{d}x)^2 + h\theta_s \tag{9}$$

$$\tau_s \dot{\theta}_s = l_s^2 \theta_s'' - \theta_s + 2\alpha \frac{(i - i_n)^2}{r^2}\eta(i - 1 + \theta_s - i_n) + \alpha l_i^2 \left(\frac{\mathrm{d}i_n}{\mathrm{d}x}\right)^2 + h\theta_n \tag{10}$$

$$l_i^2 i_n'' - i_n + \frac{i - i_n}{r^2}\eta(i - 1 + \theta_s - i_n) = 0 \tag{11}$$

where $i = j/j_c$.

From equation (11) it follows, in particular, that if the resistive domain contains normal phase, another characteristic length l_b emerges in the problem, where

$$l_b^2 = l_i^2 \frac{r^2}{1 + r^2} \ll l_i^2.$$

Clearly, l_b defines the length over which the current flows from the superconductor in the normal state over to normal metal.

3. Stationary resistive domain

In this section, we shall consider a stationary ($\dot{\theta}_s = \dot{\theta}_n = 0$) resistive domain in a composite superconductor with contact resistance.

Let us consider first the case when the resistive domain length $2l$ is substantially smaller than the characteristic heat length l_s, i.e. $l \ll l_s$. In this case, the V–I characteristic of the resistive domain may be defined accurately at any dependence of ε_s upon θ_s and i_s (Akhmetov and Mints 1982).

Indeed, if $l \ll l_s \ll l_i$, the density of current i_s varies but slightly over the length l, and the term $2\alpha i_s \varepsilon_s$ in equation (5) may be presented as a point source of heat and written down in the form $4\alpha i_s \Phi l_i \delta(x)$ where

$$\Phi = \frac{1}{2l_i}\int_{-\infty}^{\infty} \varepsilon_s \,\mathrm{d}x = \frac{d_n}{2d_s l_i \rho_n j_c}\varphi \tag{12}$$

and

$$\varphi = \int_{-\infty}^{\infty} E_s \,\mathrm{d}x$$

is the potential difference in a sample with a resistive domain. The density of current i_s coincides in this approximation with the critical current density at the 'hot' zone boundary, i.e.

$$i_s = 1 - \theta_m \qquad \theta_m = \theta(0).$$

Analogously, in view of the inequality $l \ll l_i$, the term ε_s in equation (6) may be presented as $2l_i\Phi\delta(x)$. As a result, the system of equations (4)–(6) assumes the form

$$l_n^2\theta_n'' - \theta_n + 2\alpha i_n^2 + \alpha l_i^2(\mathrm{d}i_n/\mathrm{d}x)^2 + h\theta_s = 0 \tag{13}$$

$$l_s^2\theta_s'' - \theta_s + 4\alpha(1-\theta_m)l_i\Phi\delta(x) + \alpha l_i^2(\mathrm{d}i_n/\mathrm{d}x)^2 + h\theta_n = 0 \tag{14}$$

$$l_i^2 i_n'' - i_n + 2l_i\Phi\delta(x) = 0. \tag{15}$$

Note that equation (15) is valid at any correlation between l and l_s. Since it was derived only on the assumption that $l \ll l_i$. The stationary resistive domain is described by the solution of equations (13)–(15), satisfying the following conditions:

$$\theta_s(\pm\infty) = \theta_n(\pm\infty) = i_n(\pm\infty) = 0.$$

From considerations of symmetry, it is further clear that

$$\theta_n(x) = \theta_n(-x) \qquad \theta_s(x) = \theta_s(-x) \qquad i_n(x) = i_n(-x).$$

With the aid of equation (15), we find

$$i_n = \Phi \exp\left(-\frac{|x|}{l_i}\right). \tag{16}$$

On substituting equation (16) in equations (13) and (14), we finally obtain for θ_n and θ_s:

$$l_n^2\theta_n'' - \theta_n + 3\alpha\Phi^2 \exp\left(-\frac{2|x|}{l_i}\right) + h\theta_s = 0 \tag{17}$$

$$l_s^2\theta_s'' - \theta_s + 4\alpha(1-\theta_m)l_i\Phi\delta(x) + \alpha\Phi^2\exp\left(-\frac{2|x|}{l_i}\right) + h\theta_n = 0. \tag{18}$$

The system of linear equations (17)–(18) may readily be solved accurately. As shown by the appropriate calculation, the last two addends in equation (18) make a contribution on the order of $l_s/l_i \ll 1$ both to the V–I characteristic $\phi = \phi(i)$ and to the value $\theta_s(x)$ proper in the $|x| \lesssim l_s$ region. Thus, for determining the temperature θ_s, we find

$$l_s^2\theta_s'' - \theta_s + 4\alpha(1-\theta_m)l_i\Phi\delta(x) = 0. \tag{19}$$

The solution to equation (19) may evidently be expressed as

$$\theta_s = \theta_m \exp\left(-\frac{|x|}{l_s}\right)$$

where

$$\theta_m = \frac{(2\alpha l_i/l_s)\Phi}{1 + (2\alpha l_i/l_s)\Phi}. \tag{20}$$

In view of

$$i_s(0) + i_n(0) = 1 - \theta_m + \Phi = i$$

we find the correlation for the dependence $\phi = \phi(i)$:

$$\Phi + \frac{1}{1 + (2\alpha l_i/l_s)\Phi} = i$$

whence, to the desired accuracy:

$$\Phi = \Phi_\pm = \frac{i \pm [i^2 - i_r^2(1 - i)]^{1/2}}{2} \tag{21}$$

where

$$i_r^2 = \frac{2l_s}{\alpha l_i} \ll 1.$$

The characteristic *V–I* curve $\Phi = \Phi(i)$ is shown in figure 1. In the fixed current mode, the ascending branch of the curve (Φ_+) is apparently stable and the descending one (Φ_-) unstable. Note further that, at $i > i_r$, the *V–I* characteristic of the sample with resistive domain is practically linear, $\Phi \cong i$. In so doing, the resistance of the resistive domain will be equal to that of the normal component portion of the composite superconductor, having a length of $2l_i$. Indeed, using expression (12), we find that

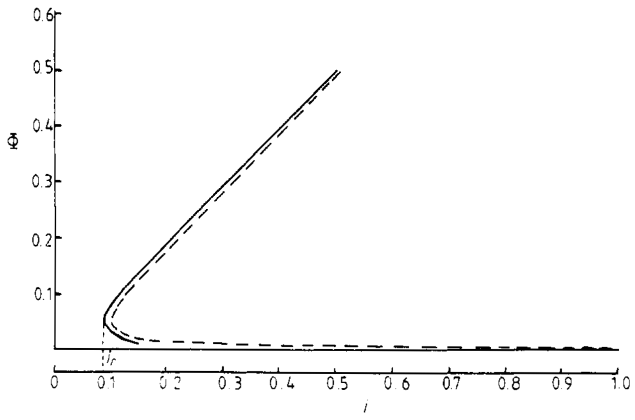

Figure 1. The curve $\Phi = \Phi(i)$ for $\alpha_{St} = 2$, $r = 0.03$, $l_i/l_s = 100$, $h = 0.1$.

$$\varphi = \frac{2l_i\rho_n}{S_n} I \tag{22}$$

where S_n is the cross-sectional area of normal metal, and I is the total current in the composite.

Physically, the emergence of a stable resistive domain in a composite superconductor with contact resistance can be readily understood based on the following considerations. Upon formation of a 'hot' zone inside the superconductor, part of the current flows out to normal metal such that the temperature in the 'hot' zone becomes self-sustaining at a level exceeding

$$\theta_c(i) = 1 - i.$$

In so doing, practically all of the current flowing in the superconducting composite passes over the normal metal portion having a length of $2l_i$. As a result, given an adequately high value of transport current, the potential difference across the sample is defined by the relationship (22), the minimum resistive domain existence current i_r proving to be substantially less than the minimum normal zone propagation current i_p. Indeed, as shown by Kremlev (1980)

$$i_p^2 \sim 1/\alpha$$

i.e.

$$\frac{i_r}{i_p} \sim \left(\frac{l_s}{l_i}\right)^{1/2} \ll 1.$$

Let us now define the range of applicability of the V–I characteristic (21). This evidently calls for the estimation of the resistive domain length l and for the observance of the inequality $l \ll l_s$. Using, for instance, the model V–I characteristic (7) of the superconductor, we can write

$$\varphi = 2l\rho_s j_s = 2\rho_s j_c(1 - \theta_m) = \Phi \frac{2d_s l_i \rho_n j_c}{d_n}.$$

From the latter relationship, we find:

$$\frac{l}{l_s} \sim \frac{\Phi}{1 - \theta_m}\frac{l_i}{l_s} r^2.$$

Then, using expression (20), we obtain the condition of applicability of all the foregoing calculations in the form

$$i_r^2 \leqslant i^2 < i_r^2 \frac{l_s}{l_i}\frac{1}{r^2}.$$

We shall now consider the case of arbitrary correlation between l and l_s, using the model V–I characteristic of the superconductor in the form of equation (7). The solution of the set of equations (9)–(11) describes a stationary resistive domain ($\dot{\theta}_s = \dot{\theta}_n = 0$) provided the conditions $\theta_n(\pm\infty) = \theta_s(\pm\infty) = i_n(\pm\infty) = 0$ are met.

Let us first assume that the addend $h\theta_n$ in equation (10) is small, then an explicit solution to equations (10) and (11) can be readily found. With the aid of the distribution of temperature $\theta_s = \theta_s(x)$ and density of current $i_n = i_n(x)$, the dependence $l = l(i)$ can be obtained through the use of the relationship

$$\theta_s(l) = 1 - i + i_n(l).$$

As a result, we find a rather bulky transcendental equation for determining the resistive domain length $l = l(i)$. The solution to this equation, given an arbitrary correlation between the constituent parameters, can only be obtained by numerical calculation. Given the dependence $l = l(i)$, the V–I characteristic of a sample with resistive domain can be readily found with the aid of the relationship

$$\Phi = \frac{i}{(1 + r^2)^2}\frac{\tanh(l/l_b)}{(l_b/l_i) + \tanh(l/l_b)} + \frac{l}{l_i}\frac{i}{(1 + r^2)}. \tag{23}$$

The numerical calculations have revealed an interesting peculiarity associated with the formation of resistive domains in composite superconductors with contact resistance, namely, that the stable (under conditions of fixed current) one-domain solution disappears at a certain value of transport current $i = i_2$. A detailed analysis of this case shows that, at $i = i_2$, the domain separates into two domains. Note that the current i_2 for composite superconductors with high contact resistance is appreciably lower than the minimum normal zone propagation current i_p.

A series of computer-calculated dependences $l = l(i)$ and $\Phi = \Phi(i)$ is presented in figures 2(a, b and c) and 3(a, b and c). The dots are used to denote the current i_2 at which the one-domain solution vanishes on each of the curves. It is clear that the upper (ascending with current) branch of both dependences corresponds to resistive domains that are stable in the fixed current mode, and the lower (descending with current) branch to resistive domains that are unstable in the fixed current mode. Both branches of the curves $l = l(i)$ and $\Phi = \Phi(i)$ can evidently be observed either in the fixed current mode or upon proper shunting of the sample.

In the limiting case of $l \ll l_s$, which corresponds in the present model to the observance of the $(l_i/l_s)r^2 < 1$ condition, one can make use of the transcendental equation defining $l = l(i)$ to readily find $i_r^2, l_r = l(i_r)$ and $\Phi_r = \Phi(i_r)$ in the following form:

$$i_r^2 \cong \frac{2}{\alpha}\left(\frac{l_s}{l_i} + r^2\right) \qquad l_r \cong \frac{l_i r^2}{1 + 4(l_i/l_s)r^2} \qquad \Phi_r = \frac{i}{2 + 4(l_i/l_s)r^2}.$$

These equations describe well the numerical calculation results which coincide with those obtained earlier under arbitrary assumptions with regard to the V–I characteristics of the superconductor. Shown by the broken curve in figure 1 is the V–I characteristic of a sample with a resistive domain, calculated with the transcendental equation for the following values of parameters: $\alpha_{St} = 2$; $r = 0.03$; $l_i/l_s = 100$; $h = 0.1$. As is seen, the agreement between the numerical calculation, making use of the model V–I characteristic (7) of the superconductor, and the formula (21) appears to be rather good in the entire range of transport current values. This is due to the fact that expression (21) describes accurately the behaviour of the function $\phi = \phi(i)$ both in the vicinity of $i = i_r$ and at $i \gg i_r$ where it is formally inapplicable.

In the limiting case of $l_2 = l(i_2) \gg l_s$, which corresponds in the present model to the observance of the $r(l_i/l_s) > 1$ condition, one can make use of the transcendental equation defining $l = l(i)$ to readily find the expressions:

$$i_2^2 \cong \frac{1}{4\alpha}\left(1 + \frac{3\sqrt{3}}{2}\frac{l_s}{rl_i}\right) \tag{24}$$

$$l_2 = \frac{r}{2} l_i \ln\left(\frac{1.6 + 0.5\,(l_s/rl_i)}{0.4 - 0.5\,(l_s/rl_i)}\right). \tag{25}$$

Equations (24) and (25) describe well the numerical calculation results; in particular, it follows from equation (25) that the value of l_2 under given conditions is independent of α. This is also well seen in figure 2.

Note further that the division of the resistive domain apparently takes place when a substantial current variation in the superconductor over the domain length becomes possible. In the case under consideration, this length is represented by the transition length l_b over which the current from the superconductor in the normal state flows over to normal metal. Indeed, as shown by equation (25), $l_2 \sim l_b$. The problem of separation of the resistive domain into two and, subsequently, more domains will be discussed in a separate paper.

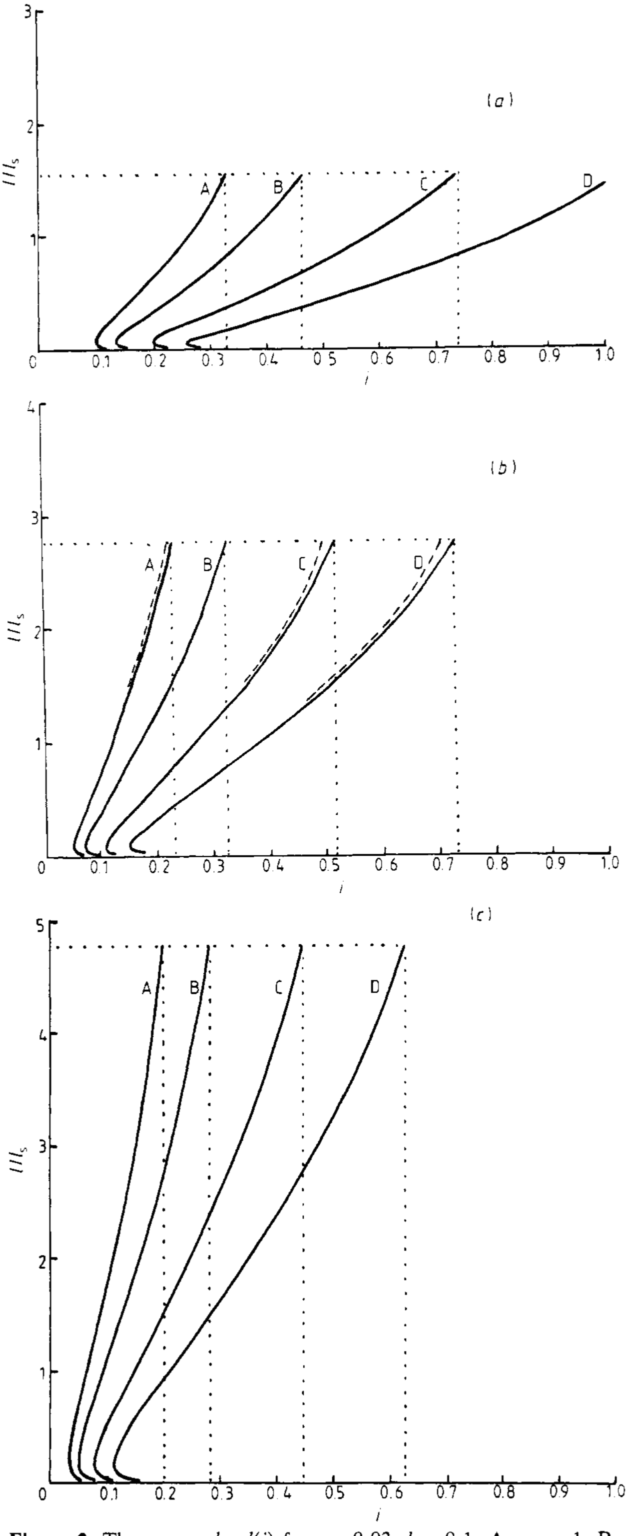

Figure 2. The curves $l = l(i)$ for $r = 0.03$, $h = 0.1$; A, $\alpha_{St} = 1$; B, $\alpha_{St} = 2$; C, $\alpha_{St} = 5$; D; $\alpha_{St} = 10$; (*a*) $l_i/l_s = 50$; (*b*) $l_i/l_s = 100$; (*c*) $l_i/l_s = 200$.

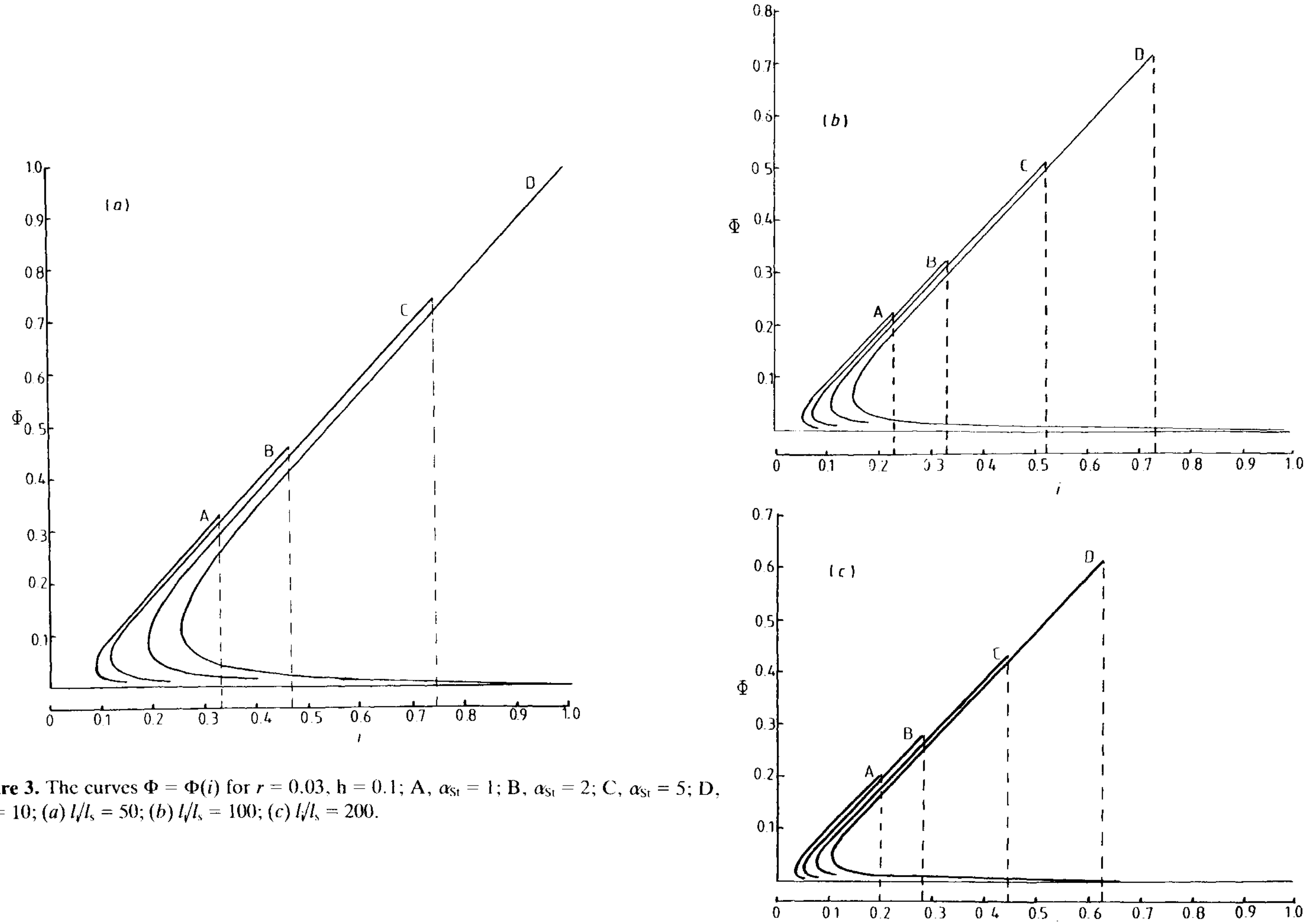

Figure 3. The curves $\Phi = \Phi(i)$ for $r = 0.03$, $h = 0.1$; A, $\alpha_{St} = 1$; B, $\alpha_{St} = 2$; C, $\alpha_{St} = 5$; D, $\alpha_{St} = 10$; (*a*) $l_i/l_s = 50$; (*b*) $l_i/l_s = 100$; (*c*) $l_i/l_s = 200$.

4. Dynamics of resistive domain formation

Let us now consider the process of evolution of the initial heat pulse leading to the formation of a stationary resistive domain in a composite superconductor with high contact resistance. To this end, one should apparently solve numerically the set of non-stationary heat-transfer equations (9)–(10), given some initial perturbation. For

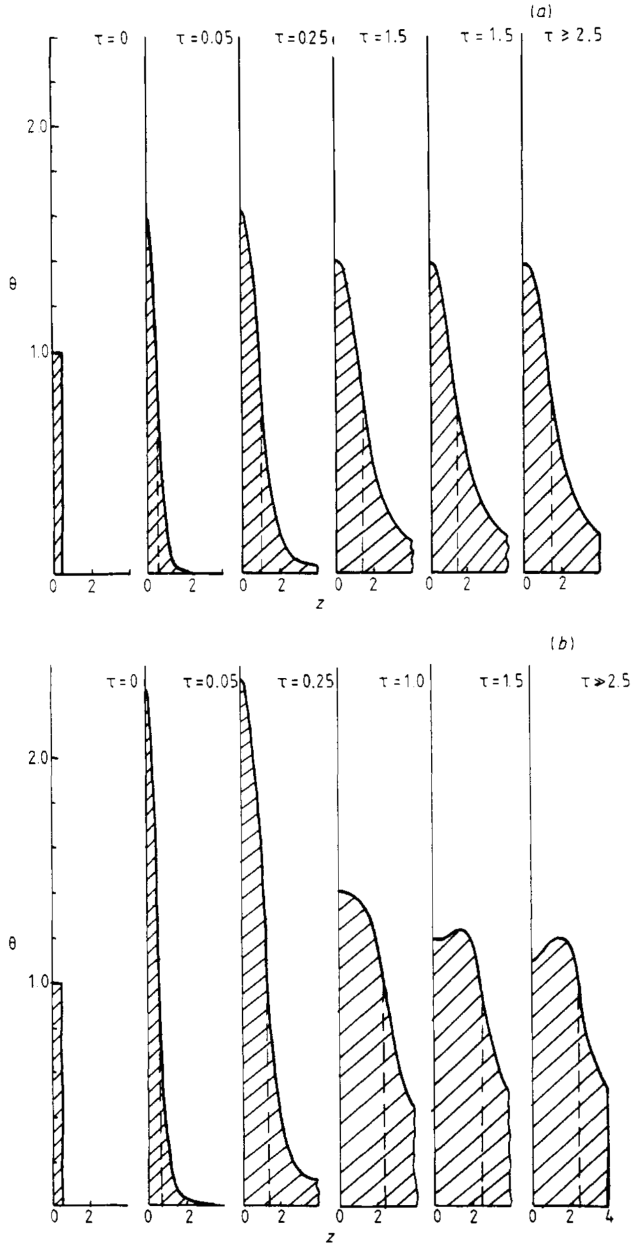

Figure 4. The computer-calculated temperature distribution $\theta_s = \theta_s(z)$ for various values of time τ for $\alpha_{St} = 2$, $r = 0.03$, $l_t/l_s = 100$, $h = 0.1$; (*a*) $i = 0.3$; (*b*) $i = 0.5$.

the following calculations we have taken the initial condition of a rectangular heat pulse concentrated in the superconductor, having a length of $l = l_s$ and amplitude of $\theta_0 = 1$. In solving equations (9) and (10), the current distribution at every moment of time is found with the aid of equation (11). This procedure helps to study the dynamics of resistive domain formation and to evaluate the heat interference of both components of the composite superconductor, i.e. to take account of the addends $h\theta_s$ and $h\theta_n$ in equations (9) and (10).

First, assume that the addend $h\theta_n$ in equation (10) is small and can be neglected. In this case, the process of resistive domain formation is described by equation (10). Presented in figure 4(a, b) is the result of numerical calculation of temperature distribution $\theta_s(z)$ in the $z = x/l_s \geqslant 0$ region for various values of time. The resistive domain length l found from the relationship

$$\theta_s(l) = 1 - i + i_n(l)$$

is shown in the figure by a dashed line.

The process of evolution of the initial heat pulse towards the stationary state takes a time $\tau \sim 1 (\tau = t/t_s)$. The values of temperature distribution θ_s, obtained at $\tau \gg 1$, coincide to a high accuracy with analogous values calculated from the stationary equations. The calculation whose results are presented in figure 4(b) has been performed at a current value of $i = 0.5$ that is close to the value of $i = i_2 = 0.52$. Therefore, a characteristic dip is observed in the curve $\theta_s = \theta_s(z)$ (at $\tau > 2.5$) in the vicinity of the $z = 0$ point, prior to the resistive domain separation into two parts.

In order to evaluate the heat interference between the normal and superconducting components of a composite superconductor with adequately high contact resistance, we have solved numerically the full set of non-stationary heat-transfer equations (9)–(10). The result of the calculation is shown by the broken curves in figure 2(b), at $h = 0.1$ and $l_i/l_n = 2$. It is seen that, by taking into account the normal metal heating, the size of resistive domain is somewhat increased. This, however, has practically no effect upon the V–I characteristic of a sample with a resistive domain. Figure 5 shows the calculated

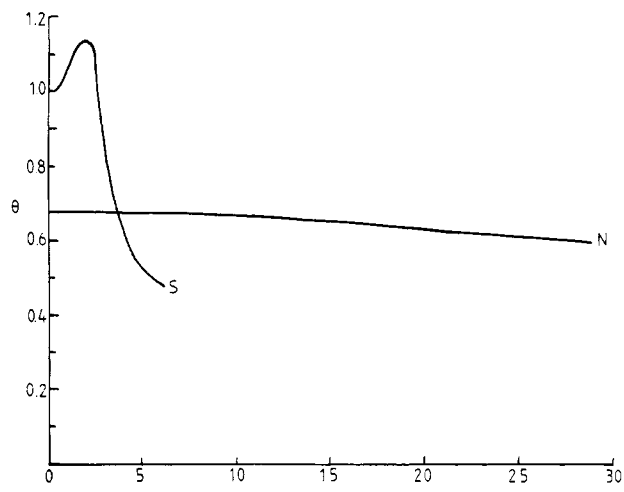

Figure 5. The computer-calculated temperature distributions $\theta_s = \theta_s(z)$ and $\theta_n = \theta_n(z)$ for $\alpha_{St} = 2$, $r = 0.03$, $l_i/l_s = 100$, $i = 0.5$, $h = 0.1$, $l_i/l_n = 2$.

stationary temperature distribution in the superconductor, $\theta_s = \theta_s(z)$, and in normal metal, $\theta_n = \theta_n(z)$. The $\theta_s = \theta_s(z)$ curve clearly exhibits a dip in the vicinity of the $z = 0$ point, prior to the division of the resistive domain ($i_2 = 0.52$). Figure 6 shows the results of calculation of the superconductor and normal metal temperatures at the centre of the resistive domain as a function of transport current i, given the same values of parameters as in figure 5. The temperature of the normal metal is seen to increase monotonically with current, while the superconductor temperature begins to decrease, starting from some current value, and leading to the division of a resistive domain.

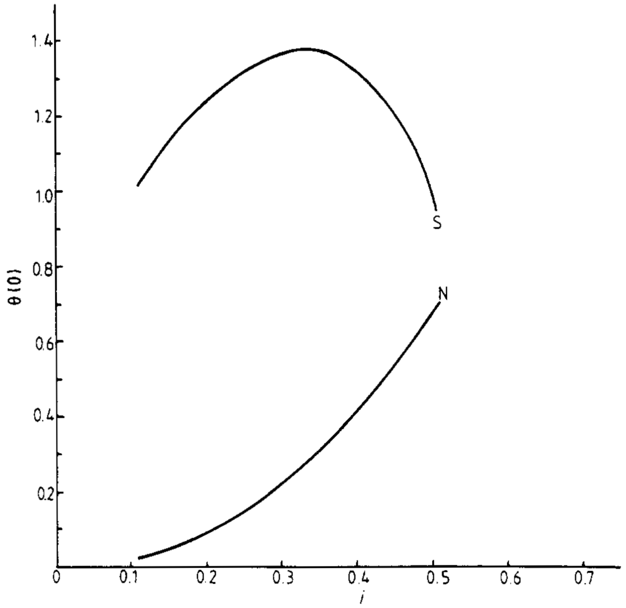

Figure 6. The computer-calculated temperatures $\theta_s(0)$ and $\theta_n(0)$ for $\alpha_{St} = 2$, $r = 0.03$, $l_i/l_s = 100$, $i = 0.5$, $h = 0.1$.

5. Discussion of the results and conclusions

The present paper shows the possibility of stable resistive domain formation in composite superconductors with high thermal and electrical contact resistance between normal metal and superconductor ($h \ll 1$, $l_s/l_i \ll 1$). Note that the minimum resistive domain existence current i_r turns out to be considerably lower than the minimum normal zone propagation current i_p. Apparently, the current i_r also serves as the superconductivity recovery current if the transition to superconductive state from normal state occurs while the transport current in the sample decreases.

The existence of resistive domains is responsible for the presence of hysteresis phenomena upon superconductivity decay and recovery by current. It is due to the fact that the superconductivity decay current, in the absence of strong local heat release, will always be higher than the minimum resistive domain existence current.

The V–I characteristic of a sample with resistive domains has been found; it has been shown that the V–I characteristic can be described, in a wide range of parameters, using expression (21). It is worthy of note that the V–I characteristic comprises both ascending and descending (with an increase of current) portions. The presence of portions with negative differential resistance permits the use of a sample with resistive domains as an active circuit element.

The formation of stable resistive domains in composite superconductors with adequately high contact resistance may be responsible for the degradation of those superconductors, inasmuch as an excited resistive domain cannot be eliminated at currents exceeding i_r.

The heat interference between normal metal and superconductor has little effect upon the properties and conditions of formation of resistive domains if both components of the composite are cooled down sufficiently independently of each other and $h \ll 1$.

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