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Limited flux jumps in hard superconductors

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Abstract. Limited flux jumps in superconductors are investigated under the conditions when the heating of the sample is not too high. The surface temperature rise, electric field and magnetic flux change associated with the instability development are calculated. The theory is compared with experiments, and a satisfactory agreement is found.

1. Introduction

The thermomagnetic instabilities of the critical state in hard superconductors have been the subject of investigation in a number of papers (see the review by Mints and Rakhmanov 1981). As a result, the stability criteria of the critical state have been determined in many cases of interest both experimentally and theoretically. However, the evolution of the flux jumps after initiation has not yet been systematically investigated.

It was observed in a number of experiments that the flux jumps may be of two types: full or catastrophical flux jumps, and limited or partial flux jumps. In the first case the instability results in a normal transition of the sample and the quenching of the supercurrent. In the second case the sample temperature T does not exceed the critical value T_c. The supercurrent does not become zero. The full instability can obviously occur only if the energy $\Delta\varepsilon$ dissipated into heat during the flux jump is sufficient for the heating of the sample up to $T > T_c$, i.e. (Wipf and Lubell 1965):

$$\Delta\varepsilon > V \cdot \int_{T_0}^{T_c} \nu(T)\,\mathrm{d}T \tag{1}$$

where T_0 is the initial temperature of the sample, $\nu(T)$ is the heat capacity of the conductor and V is the sample volume.

Criterion (1) is a necessary condition for the full flux jump occurrence, but not a sufficient one. For example, in any real situation there is a thermal flux from the sample into the coolant which has not been taken into account in criterion (1).

Two possible mechanisms of the instability retardation before quenching of the superconducting current may be proposed. First, the critical state stabilisation may result from nonlinear effects such as an increase of $\nu(T)$ or a decrease of critical current density $j_c(T)$, with the heating of the superconductor (Wipf 1967, Swartz and Bean 1968). Second, the perturbation growth may stop at a low value of

$$\Delta T = T - T_0 \ll T_c - T_0$$

due to the oscillating nature of the small perturbation spectrum (Mints 1978, Mints and Rakhmanov 1981). In the present paper we shall investigate the second possibility, supposing that

$$\Delta T \ll T_c - T_0.$$

The theoretical results are compared with the experiments of Akachi *et al* (1981).

2. Theory

We shall consider a flat sample of a thickness $2b$ in an external magnetic field $H_a = H_a(t)$ (figure 1). Suppose that the field $H_a(t)$ grows slowly and one may neglect the heating of

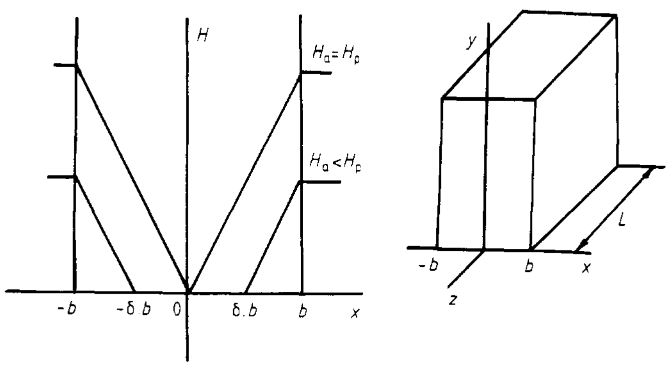

Figure 1. The sample geometry.

the sample. Moreover, let the electric field E induced in the sample by the variable magnetic field be small enough, say, less than 10^{-6} V cm^{-1}. The latter condition allows one to assume that the differential conductivity of the sample $\sigma = \sigma(E)$ is much greater than σ_f, the conductivity in the viscous flux flow mode (Campbell and Evetts 1972, Gentile *et al* 1980). Then, at $t = 0$ a small magnetic field disturbance $\Delta H \ll H_a$ is applied in the same direction as H_a. The rise time Δt of the pulse ΔH is supposed to be small compared to both the magnetic diffusion time

$$t_m = \mu_0 \sigma_f b^2$$

and the thermal diffusion time

$$t_\kappa = \nu b^2/\kappa$$

(where κ is the thermal conductivity of the conductor) and

$$\Delta H/\Delta t \gg \dot{H}_a.$$

Pulsing the external field results in an increase of the electric field, and the electric field $E(t)$ induced in the bulk of the sample approaches a value of the order of

$$E_a \sim \mu_0 \Delta H b/t_m$$

(with the rise time $\sim t_m$). If E_a is high enough, then the superconductor transfers into the flux flow mode. In the case when the magnetic field difference in the sample at $t = 0$ is larger than some critical value $H_j = H_j(\sigma_f)$, the flux jump occurs. Note that in the case of hard superconductors one may suppose that $t_\kappa \ll t_m$ and the characteristic time t_j of the instability increase obeys the inequality $t_m \ll t_j \ll t_\kappa$ (Mints and Rakhmanov 1981).

In this paper we shall discuss only the sample with an adiabatically insulated surface.

Experimental conditions similar to those described above have been realised in some experiments (e.g. by Akachi *et al* 1981) and depending on the sample parameters and external conditions, flux jumps of different sizes were observed. The maximum value of the magnetic flux $\Delta\varphi$ which penetrates the sample in the case of the full flux jump may be readily found. Using Bean's critical state model

$$j_c(H, T) = j_c(H_a, T)$$

one finds:

$$\Delta\varphi < \Delta\varphi_{max} = 2bL\,H_a[1 - (1 - \delta)/2] \tag{2}$$

where L is the sample size along the z axis, $(1 - \delta)b$ is the magnetic field penetration depth (see figure 1) and j_c is the critical current density.

As has been observed experimentally by many researchers, in the vicinity of the instability threshold ($H_a = H_j$), oscillations of the electric field and temperature may occur in hard superconductors (Zebouni *et al* 1964, Neuringer and Shapira 1964, Chikaba 1970). The theoretical explanation of this effect has been proposed by Mints (1978). It has been shown that oscillations of this kind may result from the interaction of the thermal and electric disturbances in the critical state. The spectrum of the oscillations and the conditions of their existence have been investigated in detail by Mints (1978), Maksimov and Mints (1980) and Mints and Rakhmanov (1981). As was mentioned above, limited flux jumps may occur as a result of the oscillations of the small perturbations. Indeed, in this case the electric field may be written as follows (Mints 1978):

$$E(t) = E_1 + E_0 \exp(\Gamma t) \sin(\omega t + \varphi) \tag{3}$$

where E_1 and E_0 depend on x but $\partial E_1/\partial t = 0$, $\partial E_0/\partial t = 0$. At $E_1 < E_0$ (which is the case at $\Delta t \ll t_m$ in hard superconductors) the electric field E becomes close to zero in a time $t \sim \omega^{-1}$, $\sigma(E) \gg \sigma_f$ at $E \to 0$ and instability is damped by the resistive currents (Mints and Rakhmanov 1981). The increment Γ and frequency ω are functions of H_a. At $H_a = H_j$ the value Γ changes its sign ($\Gamma > 0$ at $H_a > H_j$) and at $H_a \geqslant H_{j_1}$ the frequency $\omega = 0$ and $\Gamma > 0$ and the flux jump retardation caused by oscillations of E is impossible.

Under the conditions of $\Delta T \ll T_c - T_0$ one can use the linearised thermal and Maxwell equations to find ΔT and E in the viscous flux flow mode:

$$\left.\begin{array}{ll}
\left\{\begin{array}{l}
\nu(T_0)\dfrac{\partial}{\partial t}(\Delta T) = \kappa(T_0)\dfrac{\partial^2}{\partial x^2}(\Delta T) + j_c(T_0)E \\[2ex]
\dfrac{\partial^2 E}{\partial x^2} = -\mu_0\left|\dfrac{\partial j_c}{\partial T}\right|\dfrac{\partial}{\partial t}(\Delta T) + \mu_0\sigma_f\dfrac{\partial E}{\partial t}
\end{array}\right. & |x| > \delta\cdot b \\[6ex]
\left\{\begin{array}{l}
\nu(T_0)\dfrac{\partial}{\partial t}(\Delta T) = \kappa(T_0)\dfrac{\partial^2}{\partial x^2}(\Delta T) \\[2ex]
E = 0
\end{array}\right. & |x| < \delta\cdot b.
\end{array}\right\} \tag{4}$$

The system (4) must be supplied with appropriate boundary conditions at $|x| = b$ and matching conditions at $|x| = \delta \cdot b$. First, at $|x| = \delta \cdot b$ the temperature and the thermal flux must be continuous and $E = 0$. As the surface is assumed to be adiabatically insulated, then

$$\partial(\Delta T)/\partial x = 0$$

at $|x| = b$. Further, as $\Delta t \ll t_m$, t_κ one can write the electrodynamic boundary condition in the form:

$$\frac{\partial E}{\partial x} = \mu_0 \Delta H \cdot \delta(t) \qquad |x| = b$$

where $\delta(t)$ is the delta function.

In the case of interest to us (adiabatic insulation of the surface, $t_m \ll t_\kappa$ which is true for hard superconductors) the oscillations exist only at (Maksimov and Mints 1980):

$$\tau < \tau_c = \tfrac{1}{2} + 5H_p^2/6H_a^2 - 9H_p/7H_a$$

where

$$\tau = \frac{t_m}{t_\kappa} = \mu_0 \frac{\kappa \sigma_f}{\nu} \tag{5}$$

and $H_p = b \cdot j_c$ is the field for which the flux front reaches the centre of the sample (see figure 1).

To solve the linear problem in question analytically one can use the Laplace transformation for the equations (4). After rather long but standard calculations one finds the expression for $\Delta T(x, t)$ and $E(x, t)$. For example, we have for the field $E_w = E(\pm b, t)$ and temperature $T_w = T(\pm b, t)$ at the surface of the sample:

$$\begin{aligned} E_w &= \frac{\mu_0 b \Delta H}{t_m} \cdot \frac{2H_p}{H_a} \left(\frac{\Gamma}{\omega} \sin \omega t + \cos \omega t \right) \exp(\Gamma t) \\ T_w &= T_0 + \frac{2\mu_0 H_p^2(T_0) \Delta H}{H_a \nu(T_0)} \frac{\sin(\omega t)}{\omega t_m} \cdot \exp(\Gamma t) \end{aligned} \tag{6}$$

where the values Γ and ω may be expressed as follows:

$$\begin{aligned} \Gamma &= t_\kappa^{-1} \frac{\beta_0}{\tau} \left[1 - \left(\frac{\beta_0}{\beta} \right)^{1/2} \right] \qquad \omega = t_\kappa^{-1} \frac{\beta_0}{\tau} \left\{ \tau - \left[1 - \left(\frac{\beta_0}{\beta} \right)^{1/2} \right]^2 \right\}^{1/2} \\ \beta_0 &= \frac{\pi^2}{4} \frac{H_p^2(T_0)}{H_a^2} \qquad \beta = \mu_0 \left(\frac{H_p^2}{\nu j_c} \left| \frac{\partial j_c}{\partial T} \right| \right)_{T_0}. \end{aligned} \tag{7}$$

As can be readily seen from the expressions (7), $\Gamma = 0$ at $\beta = \beta_0$ and $\Gamma > 0$, $\omega = 0$ at

$$\beta = \beta_0(1 + 2\sqrt{\tau}) \qquad (\tau \ll 1)$$

then one has:

$$H_j = \frac{\pi}{2} \left(\mu_0 \frac{\nu j_c}{|\partial j_c/\partial T|} \right)_{T_0}^{1/2} \qquad H_{j1} = H_j(1 + \sqrt{\tau}). \tag{8}$$

The plot of the function $E_w(t)$ is shown in figure 2 at different H_a. If $H_a < H_j$ then $E_w(t)$ decreases at $t > 0$ having a maximum at $t = 0$. At $H_a > H_j$ the value $E_w(t)$ increases at $t > 0$, reaches its maximum and then decreases. It follows from the equations (6) that

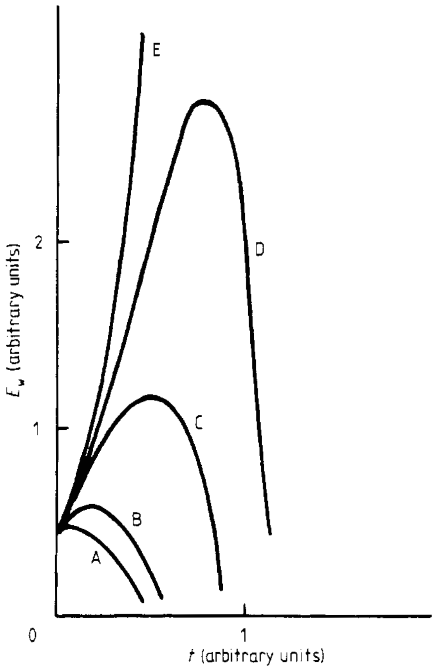

Figure 2. The curve $E_w = E_w(t)$ at different values of H_a. Curve A, $H_a < H_j$, curves B–D, $H_j < H_a < H_{j1}$, curve E, $H_a > H_{j1}$. Using data by Akachi *et al* (1981) one can attribute curve A to $\mu_0 \cdot H_a = B_a = 0.275$ T, curve B to $B_a = 0.3$ T, curve C to $B_a = 0.34$ T and curve D to $B_a = 0.36$ T.

E_w becomes equal to zero at

$$t = t_1 = \omega^{-1} \tan^{-1}(\omega/\Gamma)$$

or, in other words, at $t \simeq t_1$, E becomes small, $\sigma(E) \gg \sigma_f$ and instability is damped by the resistive currents. If $H_a > H_{j1}$ then $\omega = 0$, $\Gamma > 0$, $E(t)$ increases monotonically and in this case instability retardation could not be described in terms of the present theory.

Now, one can readily find the magnetic flux $\Delta\varphi$ penetrating the sample during the flux jump:

$$\Delta\varphi = 2L \int_0^{t_1} E_w(t)\,\mathrm{d}t = \frac{16}{\pi^2} L \cdot b \frac{B_a \Delta H}{H_p \sqrt{\tau}} \exp\left[\frac{\Gamma}{\omega}\left(\pi - \tan^{-1}\frac{\omega}{\Gamma}\right)\right]$$
$$B_a = \mu_0 H_a. \qquad (9)$$

The expression (9) is obviously valid only at $\Delta\varphi < \Delta\varphi_{max}$. The surface temperature T_w reaches its maximum T_m at $t = t_1$. Then, using equation (6) we have

$$\Delta T_m = T_m - T_0 = \frac{8 B_a \Delta H}{\pi^2 \nu(T_0) \sqrt{\tau}} \exp\left[\frac{\Gamma}{\omega}\left(\pi - \tan^{-1}\frac{\omega}{\Gamma}\right)\right]. \qquad (10)$$

The dependences of $\Delta\varphi$ and ΔT_m on B_a are shown in figures 3 and 4 by solid and broken curves at different values of T_0. The solid line indicates the region of applicability of the linear theory (formulae (9) and (10)). The dashed line is the extrapolation of this result to the region of higher ΔT where the nonlinear effects are to be taken into account. The numerical parameters used to plot these curves are discussed in § 3.

The value $\Delta\varphi_{\max}(B_a)$ obtained from formula (2) is shown by the chain curve in figure 3. In figure 4 the chain curve depicts the evaluation of the maximum value of ΔT_m at full flux jump found by means of the energy conservation law:

$$\Delta\varepsilon = \int_{T_0}^{T_m} \nu(T)\,\mathrm{d}T.$$

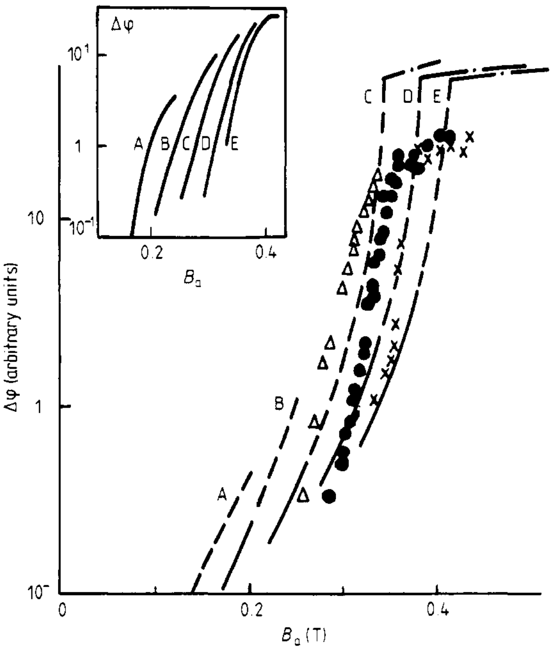

Figure 3. The dependence of $\Delta\varphi$ on H_a at different values of T_0: theory and experiment by Akachi *et al* (1981). The experimental curves for the total temperature range are shown in the insert. The theoretical curve A corresponds to $T_0 = 1.69$ K, curve B to $T_0 = 2.88$ K, curve C to $T_0 = 3.5$ K, curve D to $T_0 = 4.5$ K, curve E to $T_0 = 5.16$ K. Experimental data at $T_0 = 1.69$ K are shown by curve A in the insert, $T_0 = 2.88$ K by curve B, $T_0 = 3.5$ K by curve C and $\triangle$, $T_0 = 4.5$ K by curve D and ●, and $T_0 = 5.16$ K by curve E and $\times$.

To find the upper limit for T_m one can suppose that $\Delta\varepsilon$ is the difference between the Poynting energy flow during the full flux jump and the change in the internal energy of the magnetic field at $T \geqslant T_c$ and $T = T_0$:

$$\Delta\varepsilon = H_a \cdot \Delta\varphi_{\max} - \frac{\mu_0}{2}\int_V \mathrm{d}V\,[H_a^2 - H^2(x)]\,\mathrm{d}V.$$

Assuming that $\nu(T) \sim T^3$ one can find T_m. This result may be correct only at $T_m \geqslant T_c$. If $T_m < T_c$ then it presents only an upper limit for T_m.

In many papers the flux jump field is determined as the value of H_a at which $\Gamma = 0$. For example, one can readily see that it is the case in the so-called adiabatic ($\tau = 0$) approximation (Wipf 1967, Swartz and Bean 1968). In other papers (e.g. Kremlev 1973, Mints and Rakhmanov 1975) the stability criterion is defined as $H_a < H_{j1}$; at $H_a > H_{j1}$, the value $\omega = 0$, $\Gamma > 0$ and in the region of low ΔT and E the flux jump could not be

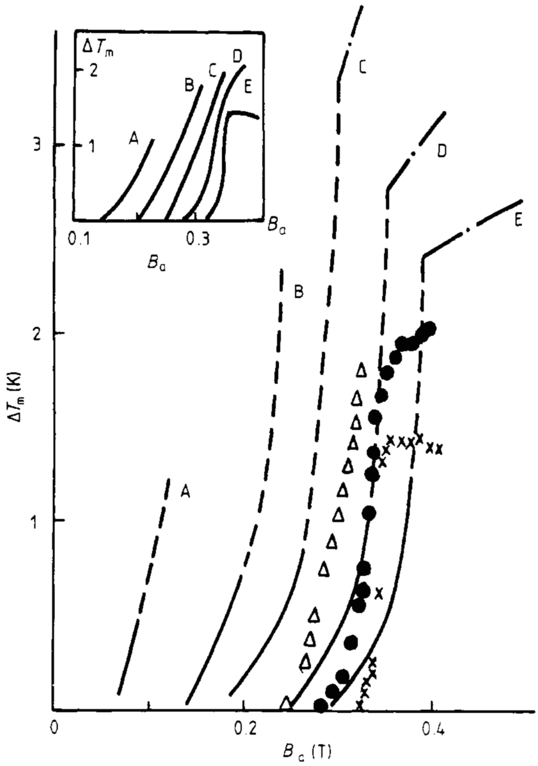

Figure 4. The dependence $\Delta T_m = T_m(H_a)$ at different values of T_0: theory and experiment by Akachi *et al* (1981). The experimental data for the total temperature range are shown in the insert. The labels on the curves are the same as in figure 3.

stopped. Analysis of the experimental papers shows that the flux jump field H_j' determined here is usually contained within the limits $H_j < H_j' < H_{j1}$. The ratio $(H_{j1} - H_j)/H_j < 1$ at $\tau \ll 1$. In the case under consideration $(H_{j1} - H_j)/H_j = \sqrt{\tau}$ and at $\tau = 0.1$ this ratio is of the order of 30%. However, one may need in a more accurate determination of the 'basic' values H_j or H_{j1} to compare the theory and experiment. For example, the value H_j' close to H_j was found by Akachi *et al* (1981) in their experiments.

3. Comparison of theory with experiment

Now we shall compare the theory discussed above with the experiment by Akachi *et al* (1981). In these experiments the limited flux jumps were studied in Nb–Ti cylindrical samples placed in the parallel magnetic field. To calculate ΔT_m and $\Delta\varphi$ theoretically one needs to know ΔH, H_p, $j_c(T_0)$, $\nu(T_0)$ and $\tau = t_m/t_\kappa$. All of these values except τ may be found using the data presented in the paper (Akachi *et al* 1981). Thus, τ is the only uncertain parameter of the problem.

Unfortunately, the precise analytical solution to the problem could not be found in the case of a cylinder placed in a magnetic field parallel to its axis. However, it can be shown that at $\tau \ll 1$ the values Γ and ω have the form (7), but the parameter β_0 is defined by the equation (Mints and Rakhmanov 1975)

$$N_1(\delta \cdot \beta_0^{1/2}) \cdot J_0(\beta_0^{1/2}) = J_1(\delta \cdot \beta_0^{1/2}) N_0(\beta_0^{1/2}) \tag{11}$$

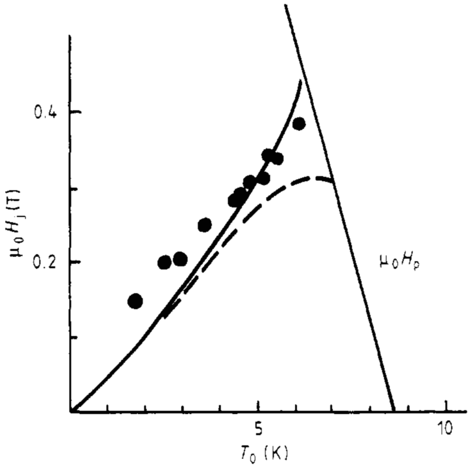

Figure 5. The field $H_j = H_j(T_0)$. Solid curve, theory for the cylindrical sample; dashed curve, theory for the flat geometry; circles and line $\mu_0 H_p$, experimental data by Akachi *et al* (1981).

where J_0, J_1, N_0, N_1 are zero and first-order Bessel functions of the first and second kind. We could not, however, find the precise numerical pre-exponential factors in the expressions (9) and (10) for $\Delta\varphi$ and ΔT_m. However, their accurate values are not of extreme importance and we shall use the expressions found for the plane geometry.

The theoretical curve $H_j = H_j(T_0)$ found by means of equation (11) is shown by the solid curve in figure 5. The same value obtained for the plane geometry is shown by the dashed line. The difference between them is not more than 30%. The experimental data of Akachi *et al* (1981) are shown by circles. The value $H_p(T_0)$ measured by the same authors is plotted by the solid curve. As is well known, if $H_j > H_p$ the flux jumps could not be observed. Note that the agreement between the theory and the experiment is rather good at $T_0 \geqslant 4$ K. The discrepancy becomes greater with decrease of T_0, but it does not exceed $\sim 40\%$ even at $T_0 = 1.6$ K. The difference between theory and experiment may be for one of the following reasons. First, the value $H_p(T_0)$ was measured only at $T \geqslant 4$ K, where

$$H_p(T_0) \sim 1 - T_0/T_c.$$

The field H_p at $T_0 < 4$ K was found by linear extrapolation. Second, the parameter

$$\tau \sim \kappa/\nu \sim T^{-p}$$

where $p > 0$, and τ increases with decreasing T_0. So, at low T_0 the approximation $\tau \ll 1$ may be not as good as at $T_0 \geqslant 4$ K.

The theoretical curves $\Delta\varphi = \Delta\varphi(B_a)$ and $\Delta T_m = T_m(B_a)$ shown in figures 3 and 4 were calculated using the value τ chosen in the form

$$\tau = 0.1 \left(\frac{4.5\ \mathrm{K}}{T_0}\right)^2$$

that is the dependences $\nu \sim T_0^3$, $\kappa \sim T_0$ and $\sigma_f =$ constant were assumed. The values $\Delta\varphi$ and ΔT_m measured by Akachi *et al* (1981) are shown in figures 3 and 4 and in the inserts in these figures. As it follows from the figures, the theory and experiment are in qualitative agreement. The qualitative tendency of their behaviour is analogous even at $\Delta\varphi \simeq \Delta\varphi_{max}$. The measured and calculated values $\Delta\varphi$ and ΔT_m coincide within an order of magnitude. Note that changing the value of τ by a factor of ~ 2 does not change the results drastically.

The curves $E_w = E_w(t)$ shown in figure 2 are in many features analogous to the measured ones by Akachi *et al* (1981). To compare these curves quantitatively one has to know $\kappa(T_0)$ and σ_f. We may suppose that $\kappa = 10^{-4}$–10^{-3} J cm^{-1} s^{-1} K^{-1} and $\tau = 0.1$ at $T = 4.5$ K, then the duration of the limited instability $\sim t_1$ is of the order of 10^{-2} s, which is in agreement with the experiment.

4. Conclusions

The limited flux jumps in hard superconductors are theoretically investigated. The temperature rise, the electric field and the magnetic flux that penetrates the sample are found in the parameter region where the limited instabilities result from the oscillating nature of the small perturbation spectrum. The theory is in a good qualitative agreement with the experiment.

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