ORIGINATION AND OSCILLATIONS OF NORMAL ZONE IN SUPERCONDUCTORS

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Abstract

Dynamics of normal (N) zone regions (resistive domains) in uniform and nonuniform superconductors has been investigated both theoretically and experimentally. An analytical theory has been proposed to describe the resistive domains (RD) dynamics in nonuniform superconductors with the alternating current. The origination and localization of RD have been considered. The self-exited relaxation oscillations of the voltage were observed experimentally. The oscillations are due to the self-exited ones of the RD length when the inductance of the circuit is large enough. The superconducting sample in our experiment was made from the multifilament Nb-40%Ti cable, the characteristic values of the oscillations frequency f and the inductance L were of the order of 1 Hz and 20 μH respectively. The theory proposed is in a good agreement with the experiment.

1. Theory

The unstationary N-zone propagation considered below arises in superconductors with the alternating current, in nonuniform superconductors [1-4] etc. The problem of N-zone origination and propagation is closely associated with the problems of steady-state stabilization, energy losses and so on.

We shall consider the N-zone propagation by means of the one-dimensional heat-transfer equation. Let us write it in the following dimensionless form:

$$\dot{\theta} = \theta'' - \theta + i^2(t)[1 + F\delta(x_1)]\eta(\theta - \theta_c) \qquad (1)$$

$$\theta(x,t) = hAP[T(x,t) - T_0] / I_c^2(T_0)\rho \qquad (2)$$

where $T(x,t)$ is the temperature, $\eta(x) = 1$ at $x \geq 0$ and $\eta(x) = 0$ at $x < 0$, h is the heat transfer coefficient, T_0 is the coolant temperature, $T_c(I)$ is the critical temperature, $i = I/I_c(T_0)$, I is the transport current, $I_c(T)$ is the critical current, ρ is the specific resistivity in the normal state, and A and P are the area and the perimeter of the cross-section of the sample, d = A/P. The formula for θ_c is given by the Eq.(2) where the substitution $T(x,t) = T_c(I)$ should be made. The dimensionless time $t_1 = t/t_0$ and coordinate $x_1 = x/x_0$ are also used ($t_0 = Cd/h$, $x_0 = (dk/h)^{1/2}$, k is the heat conductivity, C is the heat capacity). The term $Fi^2\delta(x_1)$ in Eq.(1) describes the additional heat generation due to the nonuniformity [1-3]. Eq.(1) is often used for the description of N-zone both in the composite superconductors [5] and in the thin superconducting films [6].

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a) N-zone localization on the nonuniformity

Let us consider the dynamics of the N-zone localization on the nonuniformity (fig. 1). The equations describing the movement of the N-zone boundaries $D_+(t)$ and $D_-(t)$ (see the fig.1) can be obtained as follows. After Fourier transformation on x_1 the Eq.(1) becomes the first order linear differential equation for the Fourier coefficients $\theta_K(t_1)$. The solution of this equation describes the temperature distribution in the superconductor containing the RD. The conditions $\theta(D_\pm(t),t) = \theta_c$ lead to the set of two nonlinear integral equations for $D_+(t)$ and $D_-(t_1)$.

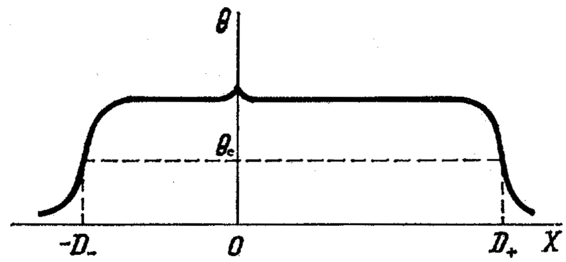

Fig. 1.

The velocities of N-S interfaces are small ($\dot{D}_\pm \ll 1$) at $I \simeq I_p$ where I_p is the N-zone minimum propagation current [5]. In this case the equations for $D_+(t_1)$ and $D_-(t_1)$ may be reduced to the differentional ones:

$$\frac{1}{2}\dot{D}_\pm + 2\frac{\dot{I}}{I} = -\frac{V(I)}{2} + F\exp(-D_\pm) - \exp(-D_\pm - D_\mp) \qquad (3)$$

where $V(i) = 4\theta_c(i)/I^2 - 2$ is the velocity of the N-S interface in uniform superconductor at $i \simeq i_p$. Eqs.(3) are valid if $F \ll 1$, $D = D_+ + D_- \gg 1$, $|v| \ll 1$, i.e. $|I - I_p| \ll I_p$ ($v(I_p) = 0$) [5]. It is the case that will be investigated below.

Consider the localization of the N-S interface on the nonuniformity when $\theta(-\infty) = 0$, $\theta(+\infty) = i^2$, i = const. In the case one has $D_+ = \infty$ and the solution of Eqs, (3) may be written in the form:

$$D(t_1) = \ln\{2F/V + [\exp D_0 - 2F/V]\exp(-Vt_1)\} \qquad (4)$$

where $D_0 = D_-(0)$. At $D_- \gg D_c = \ln(2F/v)$ one obtains from Eq.(4) that $D_-(t_1) = D_0 - vt_1$. The velocity of the N-S interface decreases exponentially as it moves towards the nonuniformity and $D_-(t_1)$ becomes of the order of D_c ($F > 0$, $v > 0$). The localization of the N-S interface arises when the velocity v is small enough, i.e. $v \leq v_c = 2F$. Thus, the N-S interface is localized at $I_p - I \leq \delta I_b \sim F I_p$.

Consider now the localization of RD. Since the analytical solutions of Eqs.(3) are bulky functions we shall represent them graphically by the aid of the phase plane (fig.2). The time evolution of the RD is determined by the movement of the representing point along the corresponding phase track in fig.2. The initial position of the representing point depends both on the length and the position (relative to the nonuniformity) of the RD created by the external heat pulse.

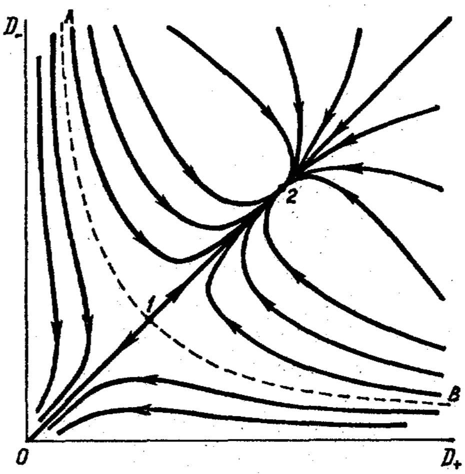

Fig. 2.

The representing point moves toward the point 2 on fig.2 if the phase track is on the right from the dashed line A1B. This situation corresponds to RD localization on the nonuniformity. The RD disappears in the opposite case. The phase tracks for this process are on the left from the curve A1B in fig. 2.

The coordinates $D_+ = D_- = D_{1,2}$ of the points 1 and 2 are given by the formula:

$$D_{1,2} = \ln\{[F \mp (F^2 - 2V)^{1/2}]/V\} \qquad (5)$$

Localized RD exists at $F > 0$, $0 \leq 2v \leq F^2$, i.e. at $I_p - I \leq \delta I_d \sim F^2 I_p$ [1-3]. Note that $\delta I_d \sim F\ \delta I_b \ll I_b$, i.e. at $F \ll 1$ the N-S interface is localized in a more wide current interval than the RD.

As it is seen from the fig.2 the localized RD is stable against an arbitrary strong perturbations if the representing point remains on the right from the dashed line A1B. The minimum energy E needed to originate the N-zone near the nonuniformity at $I \approx I_p$ is essentially less than the corresponding value of E in uniform superconductors.

b) Self-exited RD oscillations

Let us consider the self-exited oscillations of the RD. The equations for the RD length $D(t_1)$ and $I(t_1)$ may be written in the following form:

$$\dot{D}/4 + 2\dot{I}/I = -V(I)/2 + F\exp(-D/2) - \exp(-D) \qquad (6)$$

$$L\dot{I} + (D + \tau)I = \tau I_0 \qquad (7)$$

where L is the inductance of the circuit, r is the shunt resistance, L and r are normalized by $L_0 = r_0 t_0$ and r_0 respectively, $r_0 = \rho x_0/A$, I_0 is the current of the external source.

In the case $L > L_c$ the localized RD becomes unstable against the small perturbations δD and δI with the frequency ω. The expression for L_c may be obtained by the aid of the Eqs.(6),(7):

$$L_c = (\tau I_0/4I_p - 2)\exp D_0/[1 - F/2 \cdot \exp(D_0/2)] \qquad (8)$$

where $D_0 = (I_0/I_p - 1)r$ is the "stationary" RD length at $F \ll 1$ ($D_0 \gg 1$ if $r \sim 1$ and $I_0 \gg I_p$). The formula for ω is given by:

$$\omega^2 = \frac{2I}{L_c}\left|\frac{dV}{dI}\right|_{I=I_p} \qquad (9)$$

Thus, the RD may be in the pulsing regime with the frequency $\omega^{-1} \sim L^{-1/2} \ll 1$ if $L > L_c$, $D_0 > D_c = 2\ln(2/F)$.

For $L \leq L_c$ RD is stable against the current perturbations only if their amplitude δI is small enough ($\delta I < \Delta I$). ΔI may be written in the form:

$$\left(\frac{\Delta I}{I_p}\right)^2 = \frac{8}{\omega^2 L_c}\left(1 - \frac{L}{L_c}\right) \qquad (10)$$

where L_c and ω are given by the formulae (8),(9). At $I_0 > I_c$ and $L > L_c$ any stationary state of the superconductor is unstable. In this case the self-exited current oscillations with finite amplitude arise [7]. The oscillations may be represented as follows.

First RD originates in the "weak" point ($\tilde{I}_c < I_c$) then it expands into the length D_m and finally disappears. This process is a periodical one with the period determined by the time of the current relaxation $t_c = L/r$. The time of the RD existance is of the order of $\omega^{-1} \sim L^{1/2} \sim t_c$. In this case the formula for the oscillations frequency f may be written in the form:

$$f = \frac{\tau}{L}\ln^{-1}\left(\frac{I_0}{I_0 - \tilde{I}_c}\right) \qquad (12)$$

The dependence of D_m on I_0 may be obtained in the limit $D_0^2 \ll L$ by integrating of the Eqs.(6),(7). Assuming that $i_p D_m \gg i_0 r$ and neglecting the term $\exp(-D) \ll 1$ one finds:

$$D_m = 2\left\{\ln\left(\frac{I_c}{I_p}\right) - \frac{1}{\alpha}\left(\frac{I_c}{I_p} - 1\right)^2\right\}^{1/2} \cdot (2L)^{1/2} + i_0 \tau g(\alpha) \quad (13)$$

where $\alpha = \rho j_c^2 d/(T_c - T_0)h$ is Stekly's parameter, $g(\alpha)$ is some slowly changing function which is of the order of unity. As the second term in Eq.(13) is negligible then the maximum voltage along RD $U_m \simeq i_p D_m$ is practically independent on I_0. In the opposite case $L \sim D_0^2$ one obtains $D_m \sim D_0$ [7].

The condition $L > L_c(I_0)$ is not valid at $I_0 > I_k$ ($L = L_c(I_k)$) because $L_c(I_0)$ increases exponentially with I_0. Thus, the self-exited oscillations may disappear at the current I_0 is large enough. Note that RD stability may be violated by the strong perturbations even though $L < L_c$ (see for example Eq.(10)). This effect may be responsible for some hysteresis phenomena accompaning the oscillation growth.

2. Experiment

The superconducting sample was made from the multifilament cable containing 6 Nb-40%Ti filaments (⌀ 69 μm) in Cu matrix (the ratio Nb-40%Ti/Cu is about of 32%).

The sample was wounded on the textolite tube ⌀ 2,5 cm, and had the following parameters: $d = 7{,}5 \cdot 10^{-3}$ cm, the total length l = 32 cm, I_c = 108 A, I_p = 48 A (T_0 = 4,2°K, B = 0,5 T), T_c(°K) = 9,37 - 0,52B(T), $C = (1{,}23T + 0{,}114T^3) \cdot 10^{-4}$ J/cm^3°K, k=1,36 W/cm°K, the specific resistance per unit length $R_0 = 3{,}5 \cdot 10^{-5}\,\Omega$/cm the total contact resistance of the sample ends R_b = $= 5{,}05 \cdot 10^{-6}\,\Omega$. The experiments were performed at the different magnetic fields $0 < B < 4{,}5$T, and the currents $0 < I_0 < 250$A. The electric circuit contained the superconducting sample (L_s = 0,4 μH), the shunt ($r = 12{,}3 \cdot 10^{-6}\,\Omega$) and the superconducting coil (L = 19,5 μH). The accuracy of the measurements discussed below was about 10%.

The voltage along the sample was measured simultaneously by the aid of 62 potential contacts to determine both the length and the position of the RD. (the distance between the contacts was 0,5 cm).

At $I_0 > I_c$ the pulsing voltage arised on some sections of the sample 3-6 cm long. It is interesting to note that the number of these sections depends on the external magnetic field B. At low magnetic fields ($0 < B < 1$T) there exists one RD in the sample but at high magnetic fields ($2T < B < 4T$) there exist 2-4 RD. In the latter case for example one RD arose on the end of the sample and two others arose inside the sample.

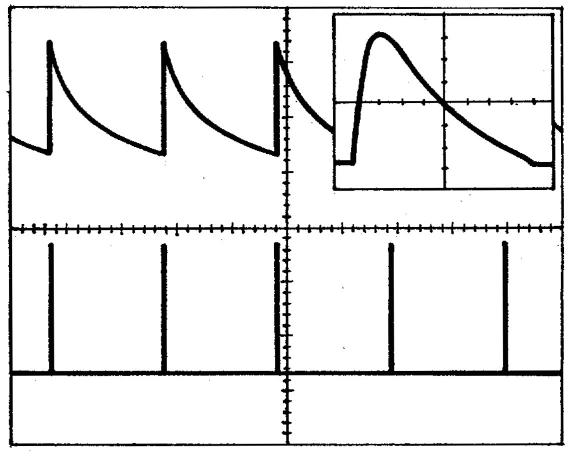

Fig. 3.

The observed oscillograms of the voltage applied to the sample (lower curve) and to the shunt (upper curve) are shown in fig.3. The experimental dependence of f on I_0 is described by the Eq.(11) with the accuracy 3-5%. The oscillogram of one voltage pulse is shown in the insertion in fig.3.

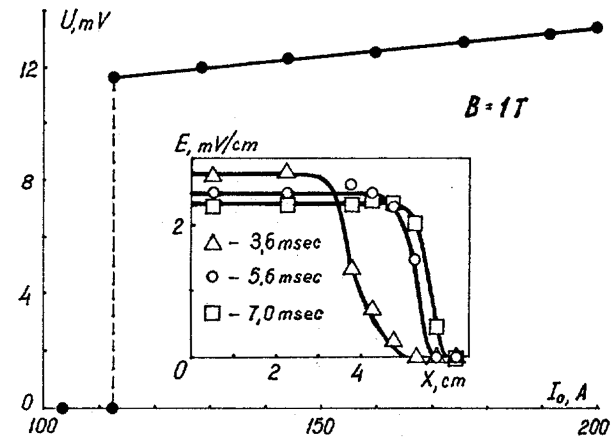

Fig. 4.

The dependence of U_m on I_0 is shown in fig.4. The shunt resistance in our experiments is negligible compared with the maximum RD resistance $R_m \simeq R_0 D_m$. In this case the Eq.(12) describes the experimental data with the accuracy about 30% (the value U_m should be replaced by $2U_m$ to apply the Eq.(12) in the case when the RD arises at the sample end). The distributions of the electric field along the RD arised at the sample and are shown in the insertion in fig.4 for different moments (B = 1 T).

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