J. Phys. D: Appl. Phys., **15** (1982) 697–704. Printed in Great Britain

Critical state stability and the training phenomenon in non-uniform superconducting composites

I L Maksimov† and R G Mints‡

† Gorky State University, Gorky 603091, USSR

‡ Institute of High Temperatures, Moscow 127412, USSR

Received 15 May 1981

Abstract. In the present work the critical state stability in non-uniform superconducting composites is investigated, taking into account the plastic yield of the material. It is shown that the critical state instability in the non-uniform region may occur considerably earlier than in the uniform part of the sample. The instability criteria have been obtained for a number of particular cases. The training phenomenon in superconducting composites is discussed as a sequence of local magnetic flux jumps and plastic strain jerks developing simultaneously

1. Introduction

Thermodynamic instabilities are well known instabilities of the critical state in superconductors (see e.g. the review paper by Mints and Rakhmanov (1977) and references therein). The thermomagnetomechanical instability—a somewhat different critical state instability—can occur provided the superconducting sample is plastically deformed (Mints 1980, Maksimov and Mints 1981a, b). In uniform superconductors both the thermomagnetic and thermomagnetomechanical instabilities appear first as a result of the development of perturbations extending over the entire sample volume (Mints and Rakhmanov 1977, Mints 1980, Maksimov and Mints 1981a, b).

In non-uniform superconductors the critical state stability threshold can be determined mainly by these sections of the sample in which the local variation of physical properties results in the stability decrease ('weak' spots). Such 'weak' spots are connected, for example, with an increase of resistivity ρ, decrease of critical current density j_c, increase of plastic strain rate $\dot{\varepsilon}$, etc. In general, a 'weak' spot occurs in the region where heat generation due to perturbations of a physical nature is larger than that in the other part of the sample.

In the present paper the critical state stability is investigated with respect to the small thermomagnetic and thermomagnetomechanical perturbations in non-uniform superconducting composites. The training phenomenon in non-uniform composites is discussed as a sequence of local thermomagnetomechanical instabilities (Mints 1980). For simplicity, a cylindrical sample with a radius R and transport current I is considered, and the external cooling is assumed to be weak.

0022-3727/82/040697 + 08 \$02.00

2. Stability criterion

Being interested in the critical state stability in composites, we shall regard the composite superconductors as an anisotropic, locally uniform medium. The physical properties of such a medium are defined by characteristics of the superconducting filaments and normal matrix, averaged over the cross-section of the sample. The applicability of such an approach to the problem of interest has been discussed in detail by Mints and Rakhmanov (1977). The heat diffusion equation, describing, in the linear approximation, the development of small temperature perturbations $\theta = T - T_0$ $(\theta \ll T_0)$ in a composite superconductor, has the form:

$$\nu \frac{\partial \theta}{\partial t} = \nabla(\kappa \nabla \theta) + j_c \boldsymbol{E} + \sigma \frac{\partial \dot{\varepsilon}}{\partial T} \theta. \tag{1}$$

Here T_0 is the cooler temperature, E is the electric field intensity, σ is the mechanical stress applied to the sample, ν and κ are the specific heat capacity and heat conductivity of the superconductor, respectively. Note that the last term in equation (1) describes the heat release due to the plastic yield of the material.

For composite superconductors (even in the case of liquid helium cooling) the typical values of parameters satisfy the inequalities:

$$(1/\tau) \ll W \ll 1 \tag{2}$$

where

$$\tau = \frac{4\pi}{c^2} \frac{\kappa}{\nu\rho} \qquad W = \frac{2W_0 R}{\kappa}.$$

Here ρ is the resistivity of composite superconductor in the flux flow regime and W_0 is the coefficient of heat transfer to the coolant. In this case (see Mints and Rakhmanov 1977):

(1) The electric field intensity E and temperature θ are connected by the relationship

$$E = \rho \left| \frac{\mathrm{d} j_c}{\mathrm{d} T} \right| \theta. \tag{3}$$

(2) The perturbations develop slowly near the instability threshold, so one can assume that in equation (1)

$$\frac{\partial \theta}{\partial t} = 0. \tag{4}$$

(3) the temperature θ is practically uniform over the cross-section of the sample, i.e. $\theta = \theta(x)$ where x is the longitudinal axis of the conductor.

The equation to determine $\theta = \theta(x)$ can be easily obtained by integrating equation (1) over the cross-section and using the relations (3) and (4). As a result, one can find the equation of interest in the following form:

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\kappa \frac{\mathrm{d}\theta}{\mathrm{d}x}\right) + \left(\frac{\rho I}{\pi R^2}\left|\frac{\mathrm{d} j_c}{\mathrm{d}T}\right| + \sigma \frac{\partial \dot{\varepsilon}}{\partial T} - \frac{2W_0}{R}\right)\theta = 0. \tag{5}$$

Thus, the critical state stability threshold in superconducting composites is determined from the condition of existence of a non-trivial solution of equation (5). Note that in the non-uniform superconductors all the parameters of equation (5) may depend on the coordinate x.

The characteristic dimension L of the spatial variation of temperature θ, as seen from equation (5), can be evaluated as

$$L \geqslant L_0 \tag{6}$$

where

$$L_0 = R/W^{1/2} \gg R$$

is the characteristic thermal length. We shall further assume the non-uniformity to be localised in the layer $|x| \leqslant l \ll L$ (local non-uniformity) and the physical properties of superconductor at $|x| < l$ to be symmetrical relative to the x axis. Then equation (5) can be written (with accuracy $l/L \ll 1$) in the following form:

$$L_0^2\theta'' + (\tilde{\alpha} + \tilde{\beta} - 1)\theta + \Phi \cdot \theta \cdot \delta(x) = 0 \tag{7}$$

where

$$\tilde{\alpha} = \frac{\sigma R}{2W_0}\frac{\partial \dot{\varepsilon}}{\partial T} \qquad \tilde{\beta} = \frac{\rho I}{2\pi R W_0}\left|\frac{\mathrm{d}j_c}{\mathrm{d}T}\right|.$$

Here and below the parameters relating to the non-uniformity region ($|x| < l$) are designated as $\varphi(x)$, while the analogous parameters relating to the uniform part of the sample ($|x| > l$) are designated as φ. The value of Φ (to be defined below) characterises the power of the additional local heating due to the presence of the perturbation of temperature θ in the non-uniform region. Naturally, the instability is mainly localised near the cross-section $x = 0$. Away from this region the perturbations should be absent. Therefore a non-trivial solution of equation (7), decreasing at infinity, describes the instability with an extremely small increment (see condition (4)). By integrating equation (7) from -0 to $+0$ one can obtain the boundary condition for $\theta = \theta(x)$ at $x = \pm 0$:

$$\frac{\mathrm{d}}{\mathrm{d}x}(\ln \theta)\bigg|_{x=\pm 0} = \mp \frac{\Phi}{2L_0^2}. \tag{8}$$

In the $|x| > l$ region the solution, decreasing at infinity, as follows from (7), has the form:

$$\theta = \theta_0 \exp\left(-\frac{|x|}{L_0}(1 - \tilde{\alpha} - \tilde{\beta})^{1/2}\right). \tag{9}$$

Substituting the expression (9) into the condition (8), one can find the critical state stability criterion for uniform superconducting composites:

$$\Phi^2/4L_0^2 < 1 - \tilde{\alpha} - \tilde{\beta}. \tag{10}$$

It follows from condition (10) that, if $\Phi \sim L_0$, the variation of the stability criterion is considerable as compared with the case of a uniform composite superconductor ($\Phi = 0$). Physically, in this case, the heat released on the non-uniformity is of the order of the heat released over the characteristic space length L_0. Note further that the scale of spatial variation of temperature θ at $|x| > l$ is:

$$L = L_0/(1 - \tilde{\alpha} - \tilde{\beta})^{1/2}. \tag{11}$$

Let us now derive the expression for Φ. To this end one should solve equation (5) in the $|x| < l$ region and join the solution with the analogous one in the $|x| > l$ region at the boundary $|x| = l$. At $|x| < l$ one can assume that

$$\theta_m - \theta_l \ll \theta_m$$

where $\theta_m = \theta(0)$, $\theta_l = \theta(l)$. Then it follows from (5):

$$\theta = \theta_m\left(1 + \int_0^x \frac{dx'}{\kappa(x')} \int_0^{x'} f(x'')\, dx''\right) \tag{12}$$

where

$$f(x) = \frac{2W_0(x)}{R(x)} - \frac{I\rho(x)}{\pi R^2(x)} \left|\frac{dj_c(x)}{dT}\right| - \sigma\frac{\partial \dot{\varepsilon}(x)}{\partial T}. \tag{13}$$

The condition of applicability of equation (12) evidently follows from the inequality $\theta_m - \theta_l \ll \theta_m$, which is equivalent to

$$\left|\int_0^l \frac{dx}{\kappa(x)} \int_0^x f(x')\, dx'\right| \ll 1. \tag{14}$$

At $|x| > l$ the solution of equation (5) can be represented in the following form:

$$\theta = \theta_l \exp\left(-\frac{(|x| - l)}{L_0}(1 - \tilde{\alpha} - \tilde{\beta})^{1/2}\right). \tag{15}$$

The temperature and heat flux are continuous at $|x| = l$. Then, joining the solutions (12) and (15) at $|x| = l$, one can obtain the expression for Φ^2 with the required accuracy:

$$\Phi^2 = \left(\frac{L_0^2}{\kappa}\int_{-l}^{l} f(x)\, dx\right)^2 - 8L_0^2(1 - \tilde{\alpha} - \tilde{\beta}) \int_0^l \frac{dx}{\kappa(x)} \int_0^x f(x')\, dx'. \tag{16}$$

Note that in a uniform sample $\Phi \equiv 0$ and the criterion (10) coincides with an appropriate criterion obtained for the uniform case (Mints and Rakhmanov 1977, Mints 1980):

$$\tilde{\alpha} + \tilde{\beta} < 1. \tag{17}$$

3. Effect of non-uniformity on critical state stability

The criterion (10) defines on the plane of parameters $\tilde{\alpha}$, $\tilde{\beta}$ a certain curve $\tilde{\beta} = \tilde{\beta}_c(\alpha)$ determining the stability region. Evidently, the system is stable if $\tilde{\beta} < \tilde{\beta}_c(\tilde{\alpha})$. The actual form of the function $\tilde{\beta} = \tilde{\beta}_c(\tilde{\alpha})$ depends largely on the nature of non-uniformity. We shall consider here a number of examples corresponding to different physical situations and illustrating the application of the criterion (10) obtained.

3.1 Uniform plastic yield of the material ($\tilde{\alpha}$ = const)

In this case a local violation of stability can only be connected with the non-uniformity of the critical state parameters and the thermal characteristics of the sample.

Let us consider first the case of a local absence of heat removal, i.e. $W_0(x) = 0$ at $|x| < l$. Then, it follows from equation (16):

$$\Phi^2 = 4l^2(\tilde{\alpha} + \tilde{\beta}). \tag{18}$$

Substituting this expression for Φ^2 into condition (10), one can obtain the stability criterion in the form:

$$\tilde{\alpha} + \tilde{\beta} < 1 - (l/L_0)^2. \tag{19}$$

The condition of applicability of expressions (18) and (19) is limited by the inequality $l \ll L_0$. It is seen that, if the non-uniformity is caused by a local deterioration of external heat removal, the stability criterion in a non-uniform sample remains practically the same as in the uniform one.

Now, let the non-uniformity be connected with a local increase of resistivity. We shall consider here two possibilities. In the first case κ = const and in the second case κ is related to ρ by means of the relationship $\kappa\rho = \varphi(T)$ (for example, by the Wiedemann–Franz law). Accordingly, one can obtain with the aid of equation (16):

$$\Phi^2 = l^2\tilde{\beta}^2R_0^2 - 8l^2\tilde{\beta}(1 - \tilde{\alpha} - \tilde{\beta})R_1 \tag{20}$$

$$\Phi^2 = l^2\tilde{\beta}(1 - \tilde{\alpha})R_0^2 - 8l^2(1 - \tilde{\alpha})(1 - \tilde{\alpha} - \tilde{\beta})R_1. \tag{21}$$

Here

$$R_0 = \int_{-l}^{l} \left(\frac{\rho(x)}{\rho} - 1\right)\frac{\mathrm{d}x}{l} \tag{22}$$

$$R_1 = \int_{-l}^{l} \left(\frac{\rho(x)}{\rho} - 1\right)\frac{x}{l}\frac{\mathrm{d}x}{l}. \tag{23}$$

It follows from a thorough analysis of the expressions (20) and (21) that a significant variation of the critical state stability criterion in non-uniform composites is possible only if $R_0 \sim R_1 \gg 1$. Then, the first term is the main one both in the expressions (20) and (21), as will be assumed further. Substituting equations (20) and (21) into condition (10) one can find the stability criterion in the following form:

$$\kappa = \text{const} \qquad \tilde{\beta} < \tilde{\beta}_c(\tilde{\alpha}) = \frac{[1 + 4\Gamma^2(1 - \tilde{\alpha})]^{1/2} - 1}{2\Gamma^2} \tag{24}$$

$$\kappa(x)\rho(x) = \text{const} \qquad \tilde{\beta} < \tilde{\beta}_c(\tilde{\alpha}) = \frac{1 - \tilde{\alpha}}{1 + \Gamma^2(1 - \tilde{\alpha})} \tag{25}$$

where

$$\Gamma = \frac{R_0 l}{2L_0}$$

The parameter Γ characterises here the ratio between the additional resistance of non-uniform sample section and the resistance of uniform section with the length $2L_0$.

As follows from the condition (14), the applicability of criteria (24) and (25) is limited by the inequalities respectively:

$$\Gamma(l/L_0) \ll 1 \qquad \Gamma^2 < 1.$$

The function $\tilde{\beta} = \tilde{\beta}_c(\tilde{\alpha})$ for $\Gamma \lessdot 1$ is shown schematically on the figure 1 (curve B). It is seen from criteria (24), (25), that in the case when the nonuniformity is caused by a local increase of resistivity, the criterion of critical state stability may differ essentially from an analogous criterion in a uniform sample. This situation takes place provided that the additional resistance of non-uniform section is of the order of resistance of a composite superconductor with the length of $2L_0$.

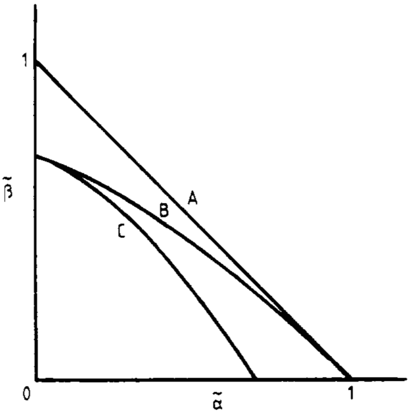

Figure 1. The function $\tilde{\beta} = \tilde{\beta}_c(\tilde{\alpha})$ for different Γ, Γ_1. Curves: A, $\Gamma = \Gamma_1 = 0$; B, $\Gamma \lesssim 1$, $\Gamma_1 = 0$; C, $\Gamma \sim \Gamma_1 \leqslant 1$.

3.2. Non-uniform plastic yield

Consider now the case when the plastic yield of composite superconductor is non-uniform along the x axis. For definiteness, let us assume that the plastic strain is concentrated in bands of thickness l, disposed through a distance d from each other, with $l \ll L \ll d$. Hence the local rate of plastic strain $\dot{\varepsilon}_{\text{loc}}$ is relatively high

$$\dot{\varepsilon}_{\text{loc}} \sim \frac{d}{l}\,\bar{\dot{\varepsilon}} \gg \bar{\dot{\varepsilon}}$$

($\bar{\dot{\varepsilon}}$ is the plastic strain rate averaged along the sample). Correspondingly, the local heat release due to plastic strain is relatively high, too. The condition $L \ll d$ enables one to consider the stability of the critical state and plastic yield independently in the vicinity of each one of the bands where the plastic yield is concentrated.

Let us first discuss the case when only the plastic yield of the material is non-uniform. With the aid of equation (16) one can easily obtain the expression for Φ^2 in the following form:

$$\Phi^2 = l^2\tilde{\alpha}^2A_0^2 - 8l^2\tilde{\alpha}(1 - \tilde{\alpha} - \tilde{\beta})A_1. \tag{26}$$

Here

$$A_0 = \int_{-1}^{l}\left[\frac{\partial\dot{\varepsilon}(x)/\partial T}{\partial\bar{\dot{\varepsilon}}/\partial T} - 1\right]\frac{\mathrm{d}x}{l}. \tag{27}$$

$$A_1 = \int_0^{l}\left[\frac{\partial\dot{\varepsilon}(x)/\partial T}{\partial\bar{\dot{\varepsilon}}/\partial T} - 1\right]\frac{x}{l}\frac{\mathrm{d}x}{l}. \tag{28}$$

As in the previous case (see § 3.1) it follows from expressions (10) and (26) that an appreciable variation of the critical state stability threshold in a non-uniform composite is possible only if $A_0 \sim A_1 \gg 1$. Hence the first term in the expression (26) is the main one and the stability criterion has the form:

$$\tilde{\beta} < \tilde{\beta}_c(\tilde{\alpha}) = 1 - \tilde{\alpha} - \tilde{\alpha}^2\Gamma_1^2 \tag{29}$$

where

$$\Gamma_1 = A_0 \frac{l}{2L_0}.$$

Physically, the parameter Γ_1 characterises the ratio between the rates of plastic strain in the non-uniform part with the length $2l$ and in uniform section of the sample with the length $2L$. By the order of magnitude parameter Γ_1 can be evaluated as $\Gamma_1 \sim d/L_0 \gg 1$ (see Mints and Petukhov 1980). From the inequality (14) it follows that the criterion is valid provided $\Gamma_1 l/L_0 \ll 1$.

The criterion (29) shows that the non-uniformity of the plastic yield of the material leads to a considerable variation of the stability criterion both of critical state and plastic yield compared to the uniform case (Mints 1980, Petukhov and Estrin 1975, Mints and Petukhov 1980).

Let us consider now the case when the non-uniformity of plastic yield is accompanied by a resistivity increase in the $|x| < l$ region. We shall assume for simplicity that $\kappa = \text{const}$ and $R_0, A_0 \gg 1$. Then, it follows from equation (16) that:

$$\Phi^2 = l^2(\tilde{\alpha}A_0 + \tilde{\beta}R_0)^2. \tag{30}$$

Substituting the relation (30) in condition (10) one can obtain the critical state stability criterion in non-uniform superconducting composites to have the form:

$$\tilde{\beta} < \tilde{\beta}_c(\tilde{\alpha}) = \frac{[1 + 4\Gamma^2(1 - \tilde{\alpha}) + 4\Gamma\Gamma_1\tilde{\alpha}]^{1/2} - 1 - 2\Gamma\Gamma_1\tilde{\alpha}}{2\Gamma^2}. \tag{31}$$

The function $\tilde{\beta} = \tilde{\beta}_c(\tilde{\alpha})$ for the present case is shown in figure 1, curve C. It follows from the comparison of the expressions (24) and (31) that the presence of plastic yield non-uniformity results in the considerable decrease of the critical state stability.

4. The training phenomenon

As was shown in the present work, thermomagnetomechanical instabilities can exist in the critical state of non-uniform superconductors. This fact allows us to understand the training phenomenon in superconductors as a process of successive strain hardening of the material (Mints 1980). Under conditions of non-uniform plastic yield, the thermomagnetomechanical instability appears first in the vicinity of a certain non-uniformity with the least stable critical state. This, as seen from the criterion (10), corresponds to a non-uniformity with the maximum value of the parameter Φ. A simultaneous development of a magnetic flux jump and a plastic strain jerk is accompanied by intense local heating, thermal softening of the material and results in strain hardening in that place where the instability occurred. At the next cycle of mechanical loading, this process will apparently recur at a higher σ and in the vicinity of another non-uniformity corresponding to a somewhat smaller Φ. Therefore, successive strain hardening of 'weak' spots distributed at random along the sample axis leads to a relatively strong training of composite superconductors, especially on the initial part of the training curve. This conclusion is in good agreement with the experimental investigation of training in 'short' samples (Pasztor and Schmidt 1978). Note that a random distribution along the sample of 'hot' regions, where instability occurs, was observed directly by Pasztor and Schmidt (1978).

Beginning from some training step, the non-uniformities remain in the sample corresponding only to relatively low values of the parameter Φ. In its turn, this leads to an increase of the characteristic thermal length L. Indeed, as follows from expression (11) and criterion (10), at the stability threshold L can be estimated as

$$L = \frac{L_0}{(1 - \bar{\alpha} - \bar{\beta})^{1/2}} = \frac{2L_0^2}{\Phi}. \tag{32}$$

With a decrease of Φ it may occur that

$$L \geqslant d$$

i.e. the plastic yield of the material becomes thermally uniform. The stability of the critical state and plastic yield in this case can be studied in a framework of the theory developed by Mints (1980), Maksimov and Mints (1981a, b) for uniform superconductors.

5. Conclusions

The critical state stability in non-uniform superconducting composites has been studied in the presence of the plastic yield of the material. It is shown that the thermomagnetic and thermomagnetomechanical instabilities in the vicinity of 'weak' spots occur earlier than in the uniform section of sample. The stability criterion has been found in the case of both uniform and non-uniform yield. The stability criteria obtained enable us to explain the initial part of the superconductor training curve by strain hardening of 'weak' spots.

References

Maksimov I L and Mints R G 1981a *J. Phys. D: Appl. Phys.* **14** 267–82
—— 1981b *IEEE Trans. Magn.* **MAG-17** 228–32
Mints R G 1980 *J. Phys. D: Appl. Phys.* **13** 847–51
Mints R G and Petukhov B V 1980 *Sov. Phys.–Solid State* **22** 1085–90
Mints R G and Rakhmanov A L 1977 *Usp. Fiz. Nauk* (*Sov. Phys.–Uspekhi*) **121** 499–524
Pasztor G and Schmidt C 1978 *J. Appl. Phys.* **49** 886–99
Petukhov B V and Estrin Yu Z 1978 *Sov. Phys.–Solid State* **17** 2041–5