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CRITICAL STATE STABILITY AND TRAINING PHENOMENON

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The critical state stability in hard and composite superconductors has been studied under conditions of plastic yield of the material. There has been found the criterion of critical state stability with respect to the jointly developing magnetic flux jump and plastic strain jerk. Based on the obtained results, the authors have offered an interpretation of the training phenomenon in superconductors.

1. Introduction

The critical state stability in hard and composite superconductors with respect to magnetic flux jumps has been thoroughly studied both theoretically and experimentally. The problem of the critical state stability under conditions of plastic yield of the material still remains relatively inadequately studied. It has been shown experimentally[1-3] that a mechanical loading of samples (as a result of the effect of external or ponderomotive forces) under certain conditions causes the stability threshold to shift considerably while the mechanical properties of superconductors are largely responsible for the existence of training and degradation in superconductors.

The present paper deals with the stability of distribution of current I and magnetic field H in the presence of mechanical stress σ causing plastic yield of the material. There has been found the criterion of stability of critical state and plastic yield in a plane-parallel plate having a thickness of 2b (current distribution shown in Fign 1) relative to thermomagnetomechanical instability (jointly developing magnetic flux jump and plastic strain jerk).

A successive quantitative solution to the problem set can be found with the aid of the technique suggested by the author of Ref.[4] using an exact expression for heat release due to plastic strain[5]. For the sake of brevity, we shall restrict ourselves to the qualitative analysis, which helps obtain basic results, while the exact calculation remains to be described in another communication[6].

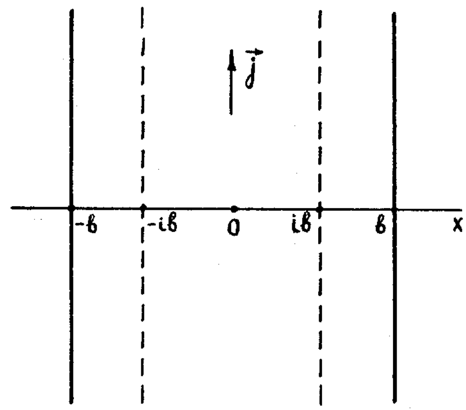

Fig. 1

2. Qualitative theory

The interaction of magnetic flux jumps and plastic strain jerks, due to the heat origin of these two processes, only appears to be effective if the characteristic times of development of both instabilities, t_j and t_ε , are of the same order of magnitude, i.e., $t_j \sim t_\varepsilon$. As a rule, plastic strain jerks develop slowly as compared to the characteristic time of temperature relaxation in the sample, $t_\varkappa$ $(t_\varepsilon \gg t_\varkappa)$[5,7]. This means that magnetic flux jumps and plastic strain jerks may effectively initiate each other only in case $t_j \gg t_\varkappa$. The characteristic time of magnetic flux jump development, t_j, depends mainly upon the intensity of external heat transfer and the value of parameter $\tau = D_t/D_m$ defining the ratio of the coefficients of heat diffusion ($D_t = \varkappa/\nu$) and magnetic flux diffusion ($D_m = \rho c^2/4\pi$). Here, $\varkappa$ and ν denote the heat conductivity and heat capacity of the superconductor, respectively, ρ is the electrical resistivity of the superconductor in the viscous-flow mode of the magnetic flux. If $\tau \ll 1$ (hard superconductors), the condition $t_j \gg t_\varkappa$ is only realized under conditions of weak external heat transfer whereas, at $\tau \gg 1$ (composite superconductors), we have $t_j \gg t_\varkappa$ irrespective of the value of external heat transfer[8,9]. In both bases listed above, it appears possible to obtain the sought-for criteria of stability of critical state and plastic yield of the material on the basis of simple physical considerations.

2.1. Adiabatic boundary conditions ($\tau \ll 1$)

Consider now, from the qualitative standpoint, the development of thermomagnetomechanical perturbations in hard superconductors ($\tau \ll 1$) in the case of $t_j \gg t_{\ae}$.

Let a fluctuation or external influence cause the temperature in superconductor to increase by a value θ_0 , i.e., priming amount of heat $Q_0 = \nu\theta_0$ has been applied to the sample. In the course of re-distribution of current, electric field and plastic strain energy, there apparently occurs the release of additional heat equal to:

$$Q_1 + Q_2 = \int j_c E \, dt + \sigma \delta\varepsilon .$$

Here, j_c is the critical current density, E is the electric field, $\delta\varepsilon$ is the variation of the value of plastic strain ε , and $\sigma\delta\varepsilon$ is the plastic strain energy. If no magnetic flux jumps and plastic strain jerks occur, while a new position of equilibrium is set up in the sample at a temperature exceeding the starting temperature by a value of θ , use can be made of the law of conservation of energy for finding the value of θ :

$$Q = \nu\theta = Q_0 + Q_1 + Q_2 = \nu\theta_0 + Q_1 + Q_2 \quad (1)$$

The value of Q_1 can be readily estimated with the aid of Maxwell equations[8]. As a result, we find:

$$Q_1 = \frac{1}{\gamma^2}\,\frac{4\pi b^2}{c^2}\, j_c \left|\frac{dj_c}{dT}\right|\theta , \quad (2)$$

where γ^2 is a factor depending on the sample geometry and the value of transport current; for simplicity, we shall assume $\partial j_c/\partial H = \partial j_c/\partial\varepsilon = 0$. An exact calculation[6] shows that $\gamma^2 = \frac{3}{i^3}$ where $i = I/I_c$, $I_c = 2bj_c$.

The value of Q_2 can be estimated on the basis of the fact that, in the case of slow processes ($t_\varepsilon \gg t_{\ae}$), the rate of plastic strain $\dot\varepsilon$ can be regarded as constant[5], then:

$$\delta\varepsilon = \frac{\frac{\partial\dot\varepsilon}{\partial T}}{\left|\frac{\partial\dot\varepsilon}{\partial\varepsilon}\right|}\,\theta$$

As a result,

$$Q_2 = \sigma\frac{\frac{\partial\dot\varepsilon}{\partial T}}{\left|\frac{\partial\dot\varepsilon}{\partial\varepsilon}\right|}\,\theta \quad (3)$$

By substituting Eqs.(2) and (3) in Eq.(1), we find the expression for θ :

$$\theta = \frac{\theta_0}{1 - \left(\frac{\beta}{\beta_1} + \frac{\alpha}{\alpha_1}\right)} \quad (4)$$

where

$$t_{\ae} = \frac{b^2\nu}{\ae}, \quad \beta = \frac{4\pi}{c^2}\,\frac{j_c b^2}{\nu}\left|\frac{\partial j_c}{\partial T}\right| ,$$

$$\alpha = \frac{\sigma b^2}{\ae}\,\frac{\partial\dot\varepsilon}{\partial T}, \quad \beta_1 = \gamma^2 = \frac{3}{i^3}, \quad \alpha_1 = \left|\frac{\partial\dot\varepsilon}{\partial\varepsilon}\right| t_{\ae} .$$

It follows from Eq.(4) that, at $\beta/\beta_1 + \alpha/\alpha_1 \to 1$, an unrestricted temperature growth is observed at any value of initial fluctuation and, consequently, the condition

$$\beta/\beta_1 + \alpha/\alpha_1 < 1 \quad (5)$$

serves the criterion of stability of critical state and plastic yield of the material relative to thermomagnetomechanical perturbations.

In hard superconductors with $\tau \ll 1$ the critical state stability may be affected by rapid perturbations ($t_j \ll t_{\ae}$) as well, and the corresponding stability criterion has the form[8]:

$$\beta < \beta_2 \quad (6)$$

where $$\beta_2 = \frac{\pi^2}{4i^2}\left(1 + 2\tau^{1/2}\right).$$

On comparing Eqs.(5) and (6), we find the region of stability of critical state and plastic yield on the plane of parameters α , β as:

$$\begin{cases} \dfrac{\beta}{\beta_2} < 1 , & \alpha < \alpha_0 \\ \dfrac{\beta}{\beta_1} + \dfrac{\alpha}{\alpha_1} < 1 , & \alpha_0 < \alpha \end{cases} \quad (7)$$

where

$$\alpha_0 = \left(1 - \frac{\pi^2 i}{12}\right)\alpha_1$$

The dependence (7) is shown in Fig.2a.

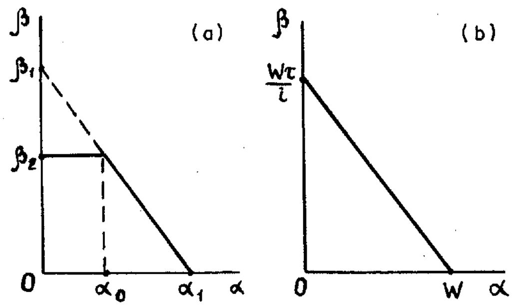

Fig.2

2.2. Weak external cooling ($\tau \gg 1$)

Let us now find an analogous criterion of stability of critical state and plastic yield of the material under weak external cooling for the case of composite superconductors with $\tau \gg 1$.

In the course of instability development, the heating takes place under conditions of frozen-in magnetic flux (to an accuracy of $1/\tau \ll 1$)[8]. As a result, the relationship between E and θ has the form[8]: $E = \rho \left| dj_c/dT \right| \theta$. The power of heat release, q, dissipated per unit length of the sample is accordingly equal to:

$$q = S(ij_c E + \sigma \delta \dot{\varepsilon}) = \\ = S\left(i\rho j_c \left|\frac{dj_c}{dT}\right| + \sigma \frac{\partial \dot{\varepsilon}}{\partial T}\right)\theta, \tag{8}$$

where S is the cross-sectional area.

The critical state and plastic yield of the material are apparently stable if the value of q does not exceed the power q_1 transferred to the cooler via side surface:

$$q_1 = P W_o \theta \tag{9}$$

where P is the cross-sectional perimeter and W_o is the coefficient of heat transfer to the cooler. Using the expressions (8) and (9), we finally derive the sought-for stability criterion as:

$$i \frac{\beta}{W\tau} + \frac{\alpha}{W} < 1 \tag{10}$$

where $W = \frac{W_o b}{\varkappa}$. The dependence (10) is shown in Fig.2b. Note that the criterion (10) was first obtained by the authors of Ref.[4] from analogous qualitative considerations; the region of applicability of Eq.(10) is restricted by the inequality $1/\tau < W < 1$.

3. Training in superconductors

The inequalities (7) and (10) define, in particular, the region of values of external parameters (current, mechanical stress, intensity of external heat transfer) at which the critical state and plastic yield of the material are stable. A violation of the condition (7) or (10) causes an instability accompanied by intensive heating and plastic strain increase stimulated by thermal softening of the material (an increase of $\dot{\varepsilon}$ with temperature). This process terminates in strain hardening of the superconductor (an increase of $\left|\frac{\partial \dot{\varepsilon}}{\partial \varepsilon}\right|$ and decrease of $\frac{\partial \dot{\varepsilon}}{\partial T}$ at the same mechanical stress). According to the obtained criteria, this should lead to an increase of the quenching current i_q in the subsequent (n + 1 -th) test record, i.e., $(i_q)_{n+1} > (i_q)_n$; an analogous conclusion can also be made with respect to the value of quenching stress, σ_q. Therefore, the training phenomenon in superconductors can be interpreted as a successive process of strain hardening of the material, stimulated by its thermal softening.

Note in conclusion that the obtained criteria (7), (10) exhibit a qualitatively good correlation with the results of experimental studies [1-3] while a quantitative comparison of theory and experiment appears impossible at present because the authors of Refs.[1-3] provided no requisite values of physical characteristics included in the expressions (7), (10).

References

1. O.P. Anashkin, V.E.Keilin, V.V.Lykov. Appl.Supercond.Conf. Alushta, U.S.S.R., 1975 (BNL-TR-624).
2. C.Schmidt. Appl.Phys.Lett., 28, 463 (1976).
3. C.Schmidt, G.Pasztor. IEEE Trans.Magnet. MAG-13, 116 (1977).
4. R.G.Mints. J.Phys. D13, 847 (1980).
5. R.G.Mints, B.V.Petukhov. Sov.Sol.State Phys. 22, 1085 (1980).
6. I.L.Maksimov, R.G.Mints. J.Phys. D to be published.
7. B.V.Petukhov, Yu.Z.Estrin. Sov.Sol.State Phys. 17, 2041 (1975).
8. R.G.Mints, A.L.Rakhmanov. Sov.Usp.Fiz. Nauk 121, 499 (1977).
9. I.L.Maksimov, R.G.Mints. J.Phys. D13 (1980).