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The critical state stability and oscillations in hard superconductors

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Abstract. The limited thermomagnetic instabilities in the critical state have been investigated in hard superconductors under an incomplete penetration of the magnetic flux into the sample and under different external cooling conditions. A flat plate is considered in an external magnetic field parallel to the plate surface. Conditions have been found under which it is possible to observe the electric field and temperature oscillations in a sample; their period and number have been evaluated. The theory is compared with the reported experimental data.

1. Introduction

Magnetic flux jumping is the well-known instability occurring in the critical state in hard and composite superconductors. The fast penetration of the magnetic flux occurring under specific conditions is responsible for this phenomenon (Hancox 1965, Hart 1969). The motion of the fluxoid structure is accompanied by a strong heating and results in a partial or total penetration of the magnetic field into the sample. The corresponding stability criteria have been discussed earlier (Wipf 1967, Swartz and Bean 1968, Mints and Rakhmanov 1977) and are in good agreement with experiment.

However, the conditions under which limited instabilities occur have not yet been researched. The so-called incomplete flux jumps usually lead to partial magnetic flux penetration. Such processes have been experimentally observed in many investigations (see, e.g., Urban 1970, Shimamoto 1974).

The instability criteria and the dynamics of such thermomagnetic instabilities depend largely on the heat transfer conditions, provided that the process goes on slowly compared to the thermal diffusion time (composite superconductors, weak external cooling, . . .). There is no heating in the inner part of the superconductor if the flux jump occurs under an incomplete magnetic field penetration. Therefore, the heat is transferred both to the cooler and to the inner section of the sample. The latter circumstance has so far not been taken into account in the previous analytical treatments of critical state stability. Nevertheless, the experiments have most often been carried out on thick samples.

In the present paper, the critical state stability criteria have been found. The conditions have been determined under which limited flux jumps and the oscillations of electrical field and temperature occur.

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We shall consider a flat plate of thickness $2b$ placed in a magnetic field parallel to the sample surface for the case $b>L$, where L is external magnetic field penetration depth. The theoretical results obtained in this work are compared with the available experimental data (Chicaba 1970, Zebouni *et al* 1964).

2. General equations

The perturbations of temperature θ and electrical field E are described by the heat diffusion and Maxwell equations

$$\left.\begin{aligned} \nu\dot{\theta} &= \kappa\nabla^2\theta + \boldsymbol{jE} \\ \nabla^2\boldsymbol{E} &= \frac{4\pi}{c^2}\frac{\partial \boldsymbol{j}}{\partial t} \end{aligned}\right\} \qquad (1)$$

where ν and κ are the heat capacity and the heat conductivity of the superconductor, respectively, and j is the current density. The expression for j in the linear approximation† is the following:

$$j = j_c(T_0) + \sigma E - \left|\frac{\partial j_c}{\partial T}\right|\theta$$

where $j_c = j_c(T)$ is the critical current density (for simplicity, we are using Bean's (1964) critical state model: $\partial j_c/\partial H = 0$), and σ is the electrical conductivity of the superconductor

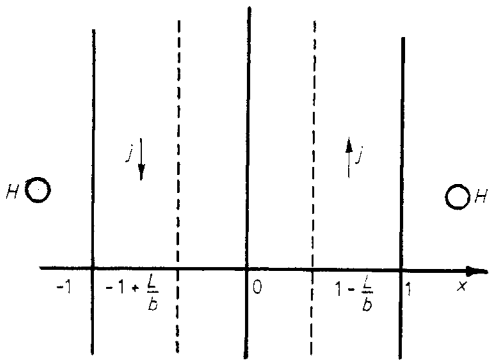

Figure 1. Magnetic field H and current j distribution for the geometry considered.

in the resistive state. Note that magnetic flux jumps are possible only if $(\partial j_c/\partial T)<0$ (de Gennes and Sarma 1966). Let us choose the coordinates in the following manner: the external magnetic field $H\|z$, the (x, y) plane is parallel to the sample surface, and $x=0$ is at the middle of the plate. The magnetic field and current distribution for this geometry is shown in figure 1. The perturbations we are interested in are one-dimensional, i.e.

$$\theta = \theta(x, t), \qquad E = (0, E(x, t), 0).$$

We shall try the solution of the system (1) in the following form:

$$\theta = \theta(x).\exp\,[\lambda(\kappa/\nu L^2)t]$$
$$E = E(x).\exp[\lambda(\kappa/\nu L^2)t]$$

where λ is the eigenvalue to be defined, $L = cH/4\pi j_c$.

† The applicability of such an approach has been discussed in detail in the earlier papers of the present series (Mints 1978, Maksimov and Mints 1979).

By using system (1) and the current density expression, it is now easy to obtain an equation for $\theta = \theta(x)$ (Kremlev 1973, Mints and Rakhmanov 1975):

$$\theta^{\mathrm{IV}} - \lambda(1+\tau)\theta^{\mathrm{II}} - \lambda(\beta - \lambda\tau)\theta = 0. \tag{2}$$

Here

$$\beta = \frac{4\pi}{c^2}\left|\frac{\partial j_c}{\partial T}\right| \frac{j_c L^2}{\nu} \sim H^2$$

$$\tau = \frac{4\pi}{c^2}\frac{\sigma\kappa}{\nu} = \frac{D_t}{D_m}.$$

D_t and D_m are the thermal and magnetic diffusion coefficients, respectively. The differentiation is carried out with respect to the dimensionless variable $x(-b/L \leqslant x \leqslant b/L)$. The relationship between the electric field and θ is

$$E = \begin{cases} 0, & 0 \leqslant |x| \leqslant (b-L)/L \\ (\kappa/j_c L^2)(\lambda\theta - \theta^{\mathrm{II}}), & (b-L)/L < |x| \leqslant b/L. \end{cases} \tag{3}$$

To determine $\theta(x)$, $E(x)$ and the eigenvalue spectrum $\lambda = \lambda(\beta, \tau \ldots)$, equation (2) has to be supplied with four boundary conditions. Both the temperature and electric field are continuous at $x = x_0 = (b-L)/L$. So we obtain

$$\theta^{\mathrm{I}} - \tilde{W}\theta|_{x=x_0} = 0 \tag{4}$$

$$\lambda\theta - \theta^{\mathrm{II}}|_{x=x_0} = 0 \tag{5}$$

where $\tilde{W} = \lambda^{1/2} \tanh [\lambda^{1/2}(b-L)/L]$. Since the solutions we look for are obviously symmetrical relative to the x axis, the other two conditions are the surface cooling and the electrodynamical boundary condition $\dot{H}(b/L) = 0$

$$\theta^{\mathrm{I}} + W_1\theta|_{x=b/L} = 0 \tag{6}$$

$$\lambda\theta^{\mathrm{I}} - \theta^{\mathrm{III}}|_{x=b/L} = 0 \tag{7}$$

where $W_1 = W_0 . L/\kappa$, and W_0 is the coefficient of heat transfer from the superconductor to the cooler. Note that the intensity of the heat transfer $\tilde{W}$ inside the sample strongly depends on λ and on the ratio $(b-L)/L$.

3. The qualitative theory

In the case of an incomplete magnetic field penetration into the sample $(L < b)$, the heat is generated in part of the superconductor volume. Accordingly, the heat is removed not only to the cooler but also to the internal zone of the sample. This circumstance is the main physical feature of the present situation. Evidently, the heat transfer to the inside of the sample can be effective only if the perturbation develops slowly compared to heat diffusion (in a layer of thickness L), which is equivalent to the condition $|\lambda| \ll 1$. However, from the expression for $\tilde{W}$ we have $\tilde{W} \leqslant \lambda^{1/2} \ll 1$ when $|\lambda| \ll 1$. Thus, the critical state stability criterion can differ appreciably from the case $b = L$ only if the heat transfer to the cooler is small, $W_1 \lesssim \tilde{W} \ll 1$. When $W_1 \gtrsim 1$, the presence of an internal zone, especially in the case $|\lambda| \ll 1$, has an influence merely on certain features of the temperature and electric field distribution. As shown by Maksimov and Mints (1979) it can change only the critical state dynamics considerably.

In the case of complete penetration of the magnetic field ($b=L$), 'slow' perturbations dominate either at $\tau \gg 1$ or at $W_1 \ll 1$ in the parameter range preceding the stability threshold. In the latter case, $|\lambda| \ll 1$, provided that the ratio of the times of magnetic (t_m) and thermal (t_κ) diffusion $t_m/t_\kappa=\tau$ exceeds a certain critical value $\tau>\tau_c$ (Mints and Rakhmanov 1977). The incomplete magnetic field penetration ($L<b$) is characterised by 'slow' perturbations under the same conditions. Note that now $t_\kappa=b^2\nu/\kappa$ and $t_m=4\pi\sigma L^2/c^2$, and, consequently, $t_m/t_\kappa=(L/b)^2\tau$. As a result, in the case of a weak external cooling $|\lambda| \ll 1$, provided that $\tau_c(L/b)<\tau$, where $\tau_c \sim (b/L)^2$ for $L/b \ll 1$.

Let us consider now the simplest situation: $W_1=\infty$, $\tau \gg 1$, and $b/L \gg 1$ (correspondingly, $\tilde{W}=\lambda^{1/2}$). The 'slow' perturbations occur in the immediate vicinity of the stability threshold. In the first approximation with respect to $\lambda\tau \gg 1$, the process proceeds under a fixed magnetic flux (Mints and Rakhmanov 1977). With the same accuracy it follows from the Maxwell equation

$$E=\frac{1}{\sigma}\left|\frac{\partial j_c}{\partial T}\right|\theta. \tag{8}$$

Substituting the latter into the heat diffusion equation we get for θ

$$\theta^{\mathrm{II}}+(\beta/\tau)\theta=0. \tag{9}$$

From equation (9) and the boundary conditions we come to the conclusion that in the first approximation ($\lambda=0$) the 'slow' perturbations appear at $\beta/\tau=\pi^2/4$ (the length L corresponds to one-quarter of a wavelength).

To determine λ it is necessary to take into account the heat transfer to the inside of the sample, i.e. the boundary condition (4). At $x=x_0$, $E=0$, the relation between the electric field and the temperature perturbations (equation (8)) is valid, therefore, only when $(x_0+\delta l) \leqslant |x|$. Here $\delta l \sim 1/(\lambda\tau)^{1/2}$ is the dimensionless magnetic diffusion length.

With the required accuracy the boundary condition (4) becomes the following:

$$\theta^{\mathrm{I}}/\theta\,|_{x=x_0+\delta l}=\lambda^{1/2}. \tag{10}$$

Using equations (9) and (10) it is easy to obtain a relationship for $\lambda=\lambda(\beta, \tau\ldots)$

$$\frac{\beta}{\tau}=\frac{\pi^2}{4}+2\lambda^{1/2}+\frac{\pi^2}{2(\lambda\tau)^{1/2}}. \tag{11}$$

From equation (11) it follows that in the range of

$$\frac{\pi^2}{4}\tau\left(1+\frac{4\sqrt{2}}{\pi\tau^{1/4}}\right)=\beta_0<\beta<\beta_c=\frac{\pi^2}{4}\tau\left(1+\frac{8}{\pi\tau^{1/4}}\right)$$

λ appears to be a complex value, $\lambda=\lambda_0+i\lambda_1$, where $\lambda_0>0$. For $\beta=\beta_0$

$$\lambda=i\lambda_c=i\,\pi^2/4\tau^{1/2}$$

where $\lambda_c=\lambda(\beta_c)$. At $\beta=\beta_c$ a real positive value $\lambda=\lambda_c$ appears, and consequently, $\beta=\beta_c$ represents the critical state stability criterion. The function $\lambda=\lambda(\beta)$ is shown in figure 2.

As is seen from equation (11), the value λ_c undergoes the greatest variation compared to the similar situation with $b=L$ (see Maksimov and Mints 1979). Comparison of the corresponding results shows that the characteristic perturbation development time has increased (when $b=L$, $\lambda_c=(\pi^4/16\tau)^{1/3}$).

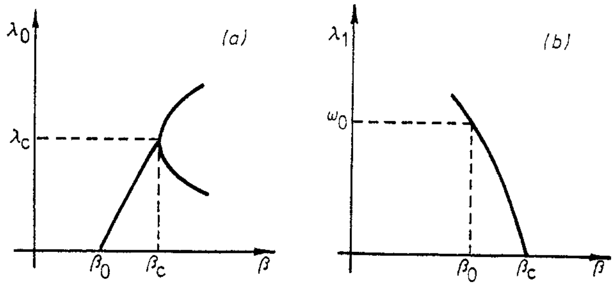

Figure 2. The function $\lambda=\lambda(\beta)$. (*a*) Real and (*b*) imaginary part of $\lambda(\beta)$.

Thus, electric field and temperature oscillations may precede the magnetic flux jump. Their nature strongly depends on the electric field intensity in the sample (Mints 1978). When the critical state stability is investigated in an external magnetic field varying at a rate of $\dot{H}$, an electric field of the order $E_i \sim (L/c)\dot{H}$ arises inside the sample. When $E_i > E_0(T)$, where $E_0(T)$ is the linear section boundary of the curve $j=j(E, T)$, the above relationships are valid. Thus, the oscillations with an amplitude up to $L\dot{H}/c$ can really occur, if $E_i > E_0$. The typical magnetic field variation rate is, as a rule, small compared to the magnetic field variation rate due to a magnetic flux jump. This circumstance enables one to estimate the number of oscillations N observed under a given $\dot{H}$. The eigenvalue spectrum region $\lambda_0 > 0$, $\lambda_1 \neq 0$ apparently corresponds to the interval

$$\Delta H = \frac{\Delta\beta}{(\partial\beta/\partial H)} = \frac{\beta_c - \beta_0}{2\beta_c} H_j$$

where H_j is determined from the equality $\beta(H_j)=\beta_c$. The variable magnetic field is in the interval ΔH during the time $\Delta t = H/\dot{H}$. Hence the number of oscillations N may be evaluated as the ratio of Δt to the oscillation period

$$\sim \frac{t_\kappa}{\lambda_1} \frac{L^2}{b^2}.$$

The result is that

$$N \sim \frac{\Delta H}{\dot{H} t_\kappa} \lambda_1 \frac{b^2}{L^2}.$$

It is seen that $N \gtrsim 1$ if the external magnetic field variation rate is within the range

$$\frac{cE_0}{L} < \dot{H} < \frac{\Delta H}{t_\kappa} \lambda_1 \frac{b^2}{L^2}.$$

4. The limited instabilities region

It has been shown that if the complex eigenvalue spectrum $\lambda=\lambda(\beta, \tau, W_1)$ exists, the consecutive series of limited flux jumps, i.e., electric field and temperature oscillations, are observed (Maksimov and Mints 1979). The range of parameters β, τ, W_1 in which the limited instabilities exist are given by the relationships

$$\lambda_0(\beta, \tau, W_1) \geqslant 0 \qquad \lambda_1(\beta, \tau, W_1) \neq 0.$$

These inequalities determine the following range of parameter β

$$\beta_0(\tau,\ W_1) \leqslant \beta < \beta_c(\tau,\ W_1).$$

The maximum possible number of oscillations can be evaluated as

$$N \sim \frac{H_j}{\dot{H} t_\kappa} \cdot \frac{\beta_c - \beta_0}{\beta_c} \cdot \frac{b^2}{L^2}\, \omega_0$$

where $\omega_0 = \lambda_1(\beta_0)$. The requirement of the existence of a nontrivial solution of equation (2) with the corresponding boundary conditions determines the relationship $\lambda = \lambda(\beta,\ \tau,\ W_1)$ In the general case, the resultant equation may be solved only numerically. Here, we shall consider the most interesting extreme case $|\lambda| \ll 1$, which permits an analytical solution. We shall obtain simple algebraic equations for the determination of $\lambda = \lambda(\beta,\ \tau,\ W_1)$ and find β_0, β_c, λ_c, ω_0, as well as evaluate N.

4.1. Adiabatic boundary conditions ($W_1 = 0$)

The eigenvalue spectrum $\lambda = \lambda(\beta,\ \tau)$ is determined in this case from the equation

$$k_2(k_1{}^2 - \lambda) \tan k_2 - k_1(k_2{}^2 + \lambda) \tanh k_1 - \lambda^{1/2}(k_1{}^2 + k_2{}^2) \tanh\,[\lambda^{1/2}(b-L)/L] = 0 \qquad (12)$$

where

$$k_{1,2}^2 = \pm \frac{\lambda(1+\tau)}{2} + \left[\lambda(\beta - \lambda\tau) + \frac{\lambda^2}{4}(1+\tau)^2\right]^{1/2}.$$

It is easy to find that λ equals zero when $\beta = \beta_1 = 3\ b/L$. By investigating the relation $\lambda = \lambda(\beta,\ \tau)$ in the vicinity of the point $\beta = \beta_1$ one may obtain an expression for the critical value $\tau = \tau_c = \tau_c(L/b)$:

$$\tau_c = \tfrac{5}{6}(b^2/L^2) - \tfrac{9}{7}(b/L) + \tfrac{1}{2}. \qquad (13)$$

When $\tau_c < \tau$, $\lambda_c = 0$ and consequently, the limited instabilities and oscillations cannot occur (Maksimov and Mints 1979). The value $\beta_c = \beta_1$ here increases with b/L. When $\tau < \tau_c$, $\lambda_c \neq 0$, and consequently, there exist limited instabilities. The evolution of the $\lambda = \lambda(\beta,\ \tau)$ curve with the increase of $\tau_c(L/b)$ is shown in figure 3 for $\tau \gg 1$.

In the range $1 \ll \tau \lesssim \tau_c(L/b)$ one easily finds

$$\beta = \tau^{1/2} \frac{(\lambda\tau)^{1/2} \tanh\,[\lambda^{1/2}(b-L)/L]}{[1 - \tanh\,(\lambda\tau)^{1/2}/(\lambda\tau)^{1/2}]}. \qquad (14)$$

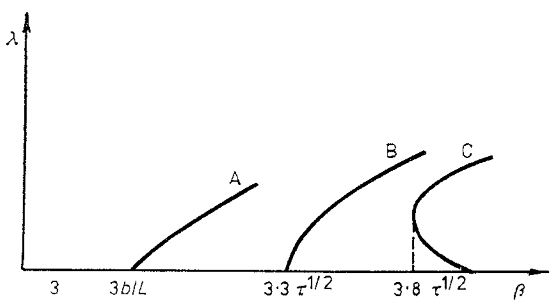

Figure 3. The evolution of the $\lambda = \lambda(\beta,\ \tau)$ curve with increasing $\tau_c(L/b)$. Curve A, $\tau_c < \tau$; B, $\tau_c = \tau$; C, $\tau_c > \tau$.

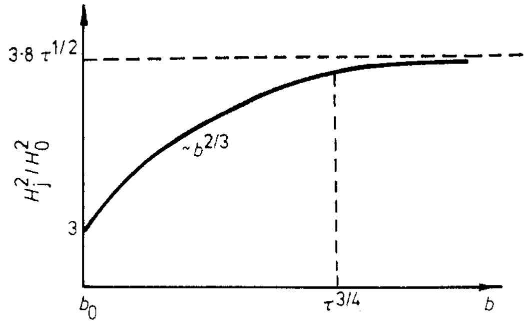

Figure 4. Graph of H_j versus b for $\tau \gg 1$.

If $1 \ll \tau \ll b^2/L^2$ one may set tanh $[\lambda^{1/2}(b-L)/L]=1$ and easily compute from equation (14):

$$\lambda_c = 2{\cdot}5/\tau \qquad \beta_c = 3{\cdot}8\ \tau^{1/2}$$

$$\omega_0 \simeq 2{\cdot}5/\tau \qquad \beta_0 = 2{\cdot}4\ \tau^{1/2}$$

$$N \sim \frac{1}{\tau}\frac{H_j}{\dot{H}t_\kappa}\frac{b^2}{L^2}.$$

It is seen that under the adiabatic insulation, the heat transfer to the inside of the sample may appreciably increase the magnitude of magnetic flux jump field H_j. The value H_j versus b is shown in figure 4 for

$$\tau \gg 1 (H_0{}^2 = 4\pi\nu j_c/|\partial j_c/\partial T|,\ b_0 = \sqrt{3}\,.cH_0/4\pi j_c).$$

Let us consider the close vicinity of $\tau = \tau_c (0 \leqslant (\tau_c - \tau)/\tau_c \ll 1)$. From equation (12) a simple algebraic equation for the determination of $\lambda = \lambda(\beta, \tau)$ immediately follows:

$$\left(\frac{b}{L}\right)^6 \lambda^3 - \frac{1}{2}\left(\frac{b}{L}\right)^4 \lambda^2 + 8\left(\frac{b}{L}\right)^2 \lambda\left(\frac{\tau_c - \tau}{\tau_c} - \frac{\beta - \beta_1}{\beta_1}\right) - 24\,\frac{\beta_1 - \beta}{\beta_1} = 0. \qquad (15)$$

From equation (15) it follows that

$$\lambda_c = 8\,\frac{\tau_c - \tau}{\tau_c}\frac{L^2}{b^2} \qquad \beta_c = 3\ b/L$$

$$\omega_0 = 3\left(\frac{\tau_c - \tau}{\tau_c}\right)^{1/2}\frac{L^2}{b^2} \qquad \beta_0 = 3\,\frac{b}{L}\left(1 - 0{\cdot}2\,\frac{\tau_c - \tau}{\tau_c}\right)$$

$$N \sim \left(\frac{\tau_c - \tau}{\tau_c}\right)^{3/2}\frac{H_j}{\dot{H}t_\kappa}.$$

The period of the oscillations and their number N are relatively small

$$(\lambda_c/\omega_0 \sim [(\tau_c - \tau)/\tau_c]^{1/2} \ll 1)$$

just as it is in the analogous case for $b = L$.

4.2. The isothermal boundary conditions ($W_1 = \infty$)

As mentioned by Maksimov and Mints (1979), in the presence of external cooling there always exists a range of parameters where the limited instabilities and oscillations occur.

In the case of $\tau^{-1} \ll |\lambda| \ll 1$ the value $\lambda = \lambda(\beta, \tau)$ is determined by the equation

$$\lambda\tau\{2 \tanh [\lambda^{1/2}(b-L)/L] + \lambda^{1/2}\} + [(\pi^2/4)\tau - \beta]\lambda^{1/2} + \pi^2\tau^{1/2}/2 = 0. \tag{16}$$

If $1 \ll \lambda^{1/2}\, b/L$, then equation (16) is identical with equation (11) that we have derived on the basis of qualitative considerations. Therefore, all the results obtained from equation (11) are valid if $1 \ll \tau \ll 0{\cdot}25(b/L)^4$. The estimation of N is

$$N \sim \frac{H_\mathrm{j}}{\dot{H}t_\kappa} \cdot \frac{1}{\tau^{3/4}} \frac{b^2}{L^2}.$$

Equation (16) permits an analytical solution in the contrary extreme case $\lambda^{1/2}(b-L)/L \ll 1$, as well. From equation (16) it follows that

$$\lambda^{3/2}\tau[2(b/L) - 1] + [(\pi^2/4)\tau - \beta]\lambda^{1/2} + \pi^2\tau^{1/2}/2 = 0. \tag{17}$$

Equation (17) enables us to determine the parameters

$$\lambda_\mathrm{c} = \left(\frac{\pi}{2}\right)^{4/3} \left(\frac{1}{\tau[2(b/L) - 1]^2}\right)^{1/3} \qquad \beta_\mathrm{c} = \frac{\pi^2}{4}\, \tau \left(1 + 2{\cdot}2\, \frac{[2(b/L) - 1]^{1/3}}{\tau^{1/3}}\right)$$

$$\omega_0 = 1{\cdot}26\, \lambda_\mathrm{c} \qquad \beta_0 = \frac{\pi^2}{4}\, \tau \left(1 + 0{\cdot}9\, \frac{[2(b/L) - 1]^{1/3}}{\tau^{1/3}}\right)$$

$$N \sim \frac{H_\mathrm{j}}{\dot{H}t_\kappa} \frac{b^2}{L^2} \left[\left(2\,\frac{b}{L} - 1\right)\tau^2\right]^{-1/3}.$$

It follows from the expression for λ_c that equation (17) is valid if

$$\frac{[(b/L) - 1]^6}{[2(b/L) - 1]^2} \ll \tau.$$

Comparison of these results with the corresponding ones for the case of $b = L$ shows that the appearance of the heat transfer to the inside of the sample can significantly enhance the critical state stability. The range in which it is possible to observe oscillations expands, and their period relatively increases.

4.3. Arbitrary thermal boundary conditions

The presence of finite external cooling does not qualitatively change the relationship $\lambda(\beta)$, as compared to the case of isothermal boundary conditions. In the general case, the solution of equations for $\lambda = \lambda(\beta, \tau, W_1)$ can be obtained only numerically. Therefore, we shall consider a situation where $W_1 \ll 1$ and $|\lambda| \ll 1$. An analytical solution of this extreme case can be found.

The equation for $\lambda = \lambda(\beta, \tau, W_1)$ in the most interesting range of parameters $\tau > \tau_\mathrm{c}$ is

$$1{\cdot}2(b/L)(\tau - \tau_\mathrm{c})\lambda^2 + (\beta_1 - \beta)\lambda + 3W_1 = 0. \tag{18}$$

From equation (18) it follows that

$$\lambda_\mathrm{c} = 1{\cdot}6\left(\frac{L}{b} \frac{W_1}{(\tau - \tau_\mathrm{c})}\right)^{1/2} \qquad \beta_\mathrm{c} = 3\,\frac{b}{L} + 3{\cdot}8\left(\frac{b}{L}\, W_1(\tau - \tau_\mathrm{c})\right)^{1/2}$$

$$\omega_0 = \lambda_\mathrm{c} \qquad \beta_0 = \beta_1 = 3\, b/L$$

$$N \sim \frac{b}{L}\, W_1\, \frac{H_\mathrm{j}}{\dot{H}t_\kappa}.$$

Note that equation (18) is valid for an arbitrary relation between L and b. Both the oscillation period and the magnetic flux jump time are relatively large (to the measure of $1/\lambda_c$), as in the case of $b=L$.

5. Results, discussions and conclusions

Electric field and temperature oscillations in hard superconductors have been observed experimentally in an external magnetic field which varied at a constant rate (Chicaba 1970, Zebouni *et al* 1964). We shall now compare the reported experimental data with the theory developed by Maksimov and Mints (1979) and that of the present paper.

The oscillations under investigation can be observed immediately before the magnetic flux jump, provided that $b>L$. In the increasing external magnetic field, the latter series of oscillations corresponds to the case $b=L$ due to the dependence of j_c on H, and may not end in a magnetic flux jump. This situation was observed by Chicaba (1970).

From estimates of N, it appears that the number of oscillations increases as the rate of external magnetic field variation decreases. This fact is in agreement with the corresponding experimental results (Chicaba 1970).

The period and number of oscillations as a function of the sample temperature has been investigated by Zebouni *et al* (1964). The relatively long oscillation period and the limited contact between the sample and the helium bath suggest that these experiments were carried out under weak external cooling ($W_1 \ll 1$). Thus it follows from the results of the present work that the number of oscillations increases and their period decreases as the temperature decreases (basically, due to the heat capacity dependence on temperature). In fact, this situation was realised in the experiment (Zebouni *et al* 1964). It is not possible to compare the quantitative results, since Zebouni *et al* give no data concerning the materials employed and the nature of the external heat transfer. Note that cylindrical samples were used in these experiments. Naturally, the comparison between the present theory and the experiment can be only qualitative.

6. Summary

The critical state stability has been studied under an incomplete magnetic field penetration into the sample. The conditions have been determined under which the limited instabilities and oscillations occur. The instability dynamics depending on the external cooling parameter and the ratio of the thermal and magnetic flux diffusion coefficients have been determined. The oscillation period has been calculated and the maximum possible number of oscillations has been estimated. It is shown that the theory and experiments are in agreement.

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