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Flux jumps under time-dependent external conditions

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Abstract. The paper investigates the influence of the time-dependent external conditions on the stability criterion of the critical state in a hard superconductor with respect to flux jumps. It has been shown that the existence of a non-linear region in the current–voltage characteristic can result in a significant delay of the onset of the instability. The methods developed in this paper make it possible to define the dependence of the stability criterion on the rate of change of the external parameters (magnetic field, temperature etc). The theoretical results have been compared with the experimental data.

1. Introduction

The stability criterion of the critical state in hard superconductors is known to have the form (e.g. Swartz and Bean 1968)

$$\Delta H < H_j$$

where ΔH is the magnetic field difference in the sample and H_j is the function of the parameters of the sample and the external conditions. However, some initial perturbation is needed to create the flux jump immediately above the stability threshold (or at $\Delta H = H_j$)†. The disturbance should be (i) strong enough to transfer the superconductor into the flux flow regime, and (ii) should cover a large volume of the superconductor, to prevent heat transfer from the region of the forming flux jump into undisturbed region. Further we shall assume that the second condition is fulfilled.

The nature of the initial perturbation may be different: some uncontrolled disturbances of the temperature, magnetic field, current, etc. or disturbances resulted from some regular influence on the superconductor in the course of the experiment. This paper investigates the dependence of the stability criterion on the rate of variation of the external parameters (the external magnetic field, the temperature, etc.). It is shown that the existence of the nonlinear region in the current–voltage characteristics of the hard superconductor can lead to a sufficient delay in the appearance of instability, i.e. a flux jump occurs at $\Delta H \gg H_j$.

2. The current–voltage characteristics of a hard superconductor and the stability of the critical state

The dependence of the current density j on the electric field E has the form shown schematically in figure 1 (Campbell and Evetts 1972). If j is lower than the critical value

† The disregard of this fact complicates the comparison of the theory with the experiment as well as the comparison of the results of different experiments.

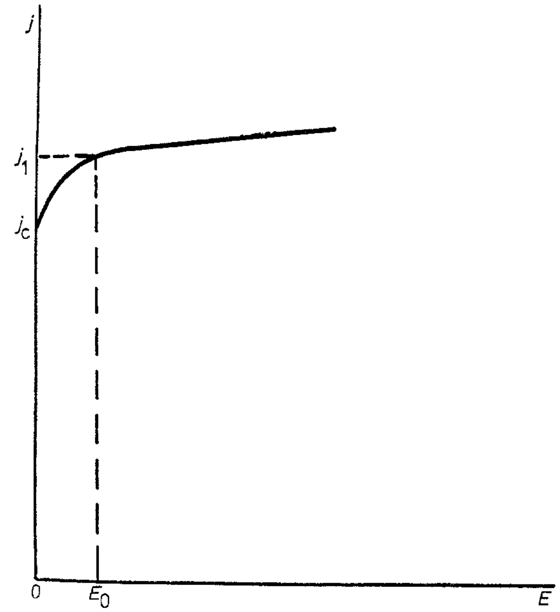

Figure 1. The *I–V* characteristics of the hard superconductor.

$j_c = j_c(T, H)$ (where T is the temperature of the superconductor) then $E=0$. At $j > j_c$, and with electric field E less than a certain value $E_0 = E_0(T, H)$, the function $j = j(E)$ is essentially nonlinear (the regime of the flux creep) and j can be presented in the form:

$$j = j_c(T, H) + \int_0^E \sigma(E)\, dE = j_c + j_N.$$

For $E > E_0$ the dependence $j = j(E)$ is linear: $j = j_1 + \sigma_f E$ (the flux flow regime). As the nonlinear region is usually small ($j_c \gg j_N$), then at $E > E_0$ one can write:

$$j = j_c(T, H) + \sigma_f E. \tag{1}$$

Further on we shall deal with magnetic field $H \gg H_{c_1}$, where one can assume the magnetic field induction being equal to the magnetic field intensity H, and the electric conductivity σ_f may be expressed as follows: $\sigma_f = \sigma_n H_{c_2}/H$ where σ_n is the electric conductivity in the normal state, H_{c_2} is the upper critical field.

Suppose that the sample is placed in the external alternating magnetic field $H_e = H_e(t)$ and consequently the current of the density $j_c = j_c(T, H)$ is excited in it. This current screens the magnetic field and the field H decays to zero at a depth l. Such state is stable only if $\Delta H < H_j$.

The normal current $j_N = \int_0^E \sigma(E)\, dE$, which compensates the drop of j_c at the spontaneous heating and 'retards' the movement of the magnetic flow, is one of the factors stabilising the critical state. Therefore, the value of H_j grows with the increasing of the conductivity. It can be seen from figure 1 that $\sigma(E)$ decreases with increase of E and $\sigma(E)$ achieves the minimum value σ_f at $E > E_0(T, H)$. Hence, the minimum of stability is achieved in the case when the volume of the superconductor, along which the current passes, is in the flux flow regime.

For hard superconductors the representative values of the thermal conductivity κ and the electrical conductivity σ_n are relatively low. As the result, at $E > E_0(T, H)$ the ratio of the coefficients of thermal and magnetic diffusion τ_f

$$\tau_f = \frac{4\pi\sigma_f\kappa}{c^2\nu} \tag{2}$$

is much smaller than 1 (here ν is the specific heat of the superconductor).

Let us assume the surface of the sample being under isothermal conditions (i.e. $T = T_0$ at the boundaries, where T_0 is the bath temperature). Then in the flux flow regime the stability criterion has the form ($\tau_f \ll 1$):

$$\Delta H < H_j = \left(\pi^3 \frac{\nu j_c}{|\partial j_c/\partial T|}\right)^{1/2} \tag{3}$$

(see for example Swartz and Bean 1968). This relation, as well as all those subsequently mentioned, relies on a flat specimen. The corresponding criterion for a sample of other geometry differs from expression (3) only by a constant coefficient.

In order to excite the flux flow in the superconductor the initial disturbance should be strong enough. That is, the electric field E should exceed E_0 in the whole volume of the sample, being in the critical state. If this condition is not fulfilled, then the current density is equal to:

$$j = j_c(T, H) + \int_0^E \sigma(E)\, \mathrm{d}E$$

where $\sigma(E) > \sigma_f$ (and for values of E not too close to E_0 $\sigma(E) \gg \sigma_f$). In this case the parameter $\tau(E) = \tau_f \sigma(E)/\sigma_f > \tau_f$ (or $\tau(E) \gg \tau_f$) and the maximum difference of the magnetic field H_a at which the flux jump occurs is obviously larger than H_j. In the range of $\tau(E) \gg 1$ (weak perturbations), analogous to the case of a superconducting composite with $\tau \gg 1$, the stability criterion can be written in the form (e.g. Hart 1969):

$$\Delta H < H_a = H_j . \bar{\tau}^{1/2} \gg H_j \tag{4}$$

where $\bar{\tau}$ is the averaged value of $\tau(E)$.

The dependence of the field H_a on the rate of variation of the corresponding parameters can be found for the instability developing under time-dependent external conditions.

3. The stability criterion (fast variation of the external parameters)

Let us consider a flat sample in a parallel magnetic field (see figure 2). As it is known, in the range of the magnetic fields $H_{c_1} \ll H \ll H_{c_2}$ the dependence of the critical current density j_c versus H can be chosen in the form:

$$j_c = \frac{\alpha(T)}{H} \tag{5}$$

for many superconducting materials (Saint-James *et al* 1969, Campbell and Evetts 1972).

Taking into consideration the fact that in the equilibrium $j_c \gg j_N$, we find from the Maxwell equation:

$$\frac{\partial H}{\partial x} = \frac{4\pi}{c} j_c(H)$$

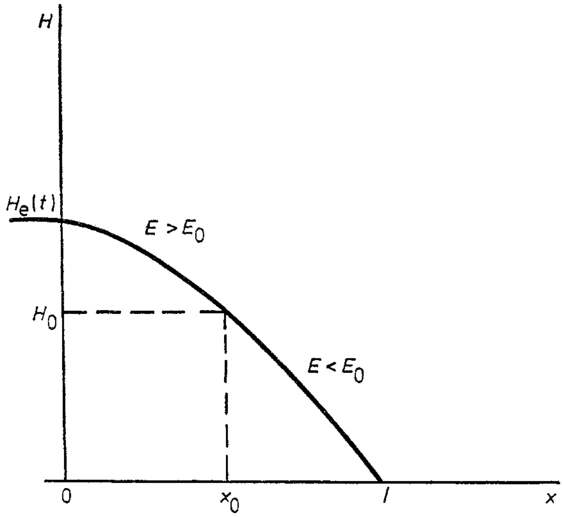

Figure 2. The magnetic field distribution in the sample.

and equation (5) for magnetic field distribution $H = H(x)$:

$$H^2(x) = H_e^2 - \frac{8\pi\alpha(T)}{c} x = H_e^2 (1 - x/l) \tag{6}$$

where $l = cH_e^2/8\pi\alpha(T)$.

To determine the electric field E we have

$$\frac{\partial E}{\partial x} = \frac{1}{c}\frac{\partial H}{\partial t}$$

and the boundary condition $E = 0$ at $x = l$. Then one can find

$$E = -\frac{2\,\dot{H}_e H(x) l}{cH_e}. \tag{7}$$

The expressions (6) and (7) were found by assuming that the heating of the sample by the alternating external field can be neglected. It is easy to estimate that the following conditions are necessary here:

$$\dot{H}_e \ll (8\pi)^3 (T_c - T_0)\,\kappa(\alpha/c)^2/H_e^5$$

$$\dot{H}_e \ll (8\pi)^2 (T_c - T_0)\,W(\alpha/c)/H_e^3$$

where W is the coefficient of the heat transfer into helium. Using the representative values of $T_c - T_0 = 10$ K, $\alpha/c = 10^7$ A Oe cm^{-2}, $\kappa = 10^3$ erg cm^{-1} s^{-1} K^{-1}, $W = 10^6$ erg cm^{-2} s^{-1} K^{-1}, $H_e = 3 \times 10^3$ Oe, we obtain $\dot{H}_e \ll 10^5$ Oe s^{-1}.

The part of the superconductor $0 < x < x_0$ is in the flux flow regime, where x_0 is defined by the condition $E(x_0) = E_0$:

$$x_0 = l\left(1 - \frac{c^2 E_0^2}{4\dot{H}_e^2 l^2}\right).$$

Let us denote $H(x_0)=H_0$. The magnetic field difference in the region where $E>E_0$ equals

$$H_e-H_0=H_e\left(1-\frac{cE_0}{2\dot{H}_e l}\right). \tag{8}$$

Assuming that $\sigma(E)$ is a rapidly varying function, one may introduce in the zone where $E<E_0$ some average value $\bar{\tau}>\tau_f$. Let $\bar{\tau}\gg\tau_f$ and $\bar{\tau}\gg 1$.

The building-up time of the flux jump in the hard superconductor is small: $t_j\ll t_\kappa=x_0^2\nu/\kappa$ (Mints and Rakhmanov 1977). However, the development of such rapidly growing disturbances is suppressed at $\bar{\tau}\gg 1$ by the normal current j_N. And in the first approximation with respect to $\tau_f\ll 1$, $\bar{\tau}\gg 1$, the existence of the flux creep region in the sample leads to the decrease of the volume where rapidly growing disturbances can appear. Therefore, in the absence of some other initial perturbations, except $\dot{H}_e$, the instability is initiated in the region $0<x<x_0$. And one can conclude that flux jump occurs if

$$H_e-H_0>H_j.$$

For a simple model this statement is proved in the Appendix.

The flux flow is achieved at the current density $j_f=j_1+\sigma_f E_0>j_c$; here $j_f\sim j_c$. Therefore it is convenient to represent E_0 in the form

$$E_0=\frac{j_f-j_1}{\sigma_f}=K\frac{j_c}{\sigma_f}$$

where $K=K(T,H)=(j_f-j_1)/j_c\ll 1$, and substituting j_c from equation (5) and $\sigma_f=\sigma_n H_{c_2}/H$ into the last expression one easily finds:

$$E_0=K(T,H)\frac{\alpha(T)}{\sigma_n H_{c_2}(T)}. \tag{9}$$

The coefficient $K(T,H)$ characterises the initial stage of the movement of the vortex structure and the non-uniformity of the pinning centres. One can assume that in the fields interval $H_{c_1}\ll H\ll H_{c_2}$, the dependence of K versus H is sufficiently weak. Now, using equation (8) and (9) it is easy to find the stability criterion for the critical state in a hard superconductor depending on the rate of variation of the external field:

$$\Delta H<H_a=\frac{H_j}{2}+\left(\frac{H_j^2}{4}+\frac{A(T)}{\dot{H}_e}\right)^{1/2} \tag{10}$$

where

$$A(T)=\frac{4\pi\alpha^2(T)K(T)}{\sigma_n H_{c_2}(T)}.$$

The dependence of H_a on $\dot{H}_e$ is shown qualitatively in figure 3. At $\dot{H}_e\to\infty$, $H_a\to H_j$ (it follows from equation (10) that $H_a\sim H_j$ at $\dot{H}_e\gg A/H_j^2$).

The dependence of H_a on $\dot{H}_e$ is affected by the form of the function $j_c=j_c(H)$. However, the qualitative behaviour of the curves $H_a=H_a(\dot{H}_e)$ does not change. For example, if $\partial j_c/\partial H=0$, $\partial E_0/\partial H=0$, then for H_a we have:

$$H_a=H_j+\frac{4\pi E_0(T)j_c(T)}{\dot{H}_e}. \tag{11}$$

The qualitative appearance of the corresponding curve is just the same as shown in figure 3.

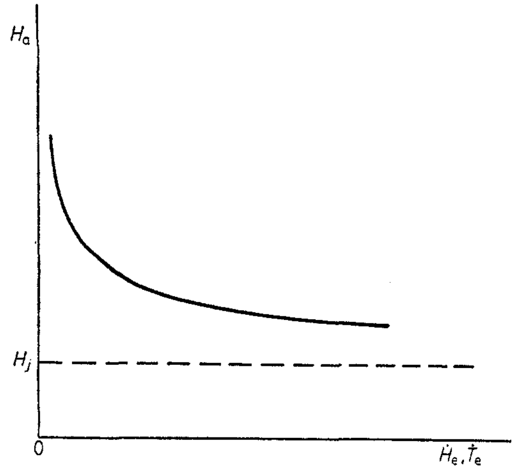

Figure 3. The qualitative dependence of H_a on $\dot{H}_e$ and $\dot{T}_e$ at high $\dot{H}_e$ and $\dot{T}_e$.

Analogous speculations allow us to find the dependence of the field difference H_a on, for example, the rate of the external heating of the sample $\dot{T}_e$. Let $\dot{H}_e=0$ and $\dot{T}_e$ be relatively small, i.e. the characteristic time of the temperature variation $T_0/\dot{T}_e$ is much higher than the time of the thermal diffusion through the scale l:

$$T_0/\dot{T}_e \gg l^2\nu/\kappa$$

(where T_0 is the initial temperature) or

$$\dot{T}_e \ll \kappa T_0/l^2\nu.$$

At the typical values of parameters $T_0=4{\cdot}2$ K, $\kappa=10^3$ erg s^{-1} cm^{-1} K^{-1}, $\nu=10^4$ erg cm^{-3} K^{-1}, $l^2=10^{-2}$–10^{-3} cm we have $\dot{T}_e \ll 10^3$–10^4 K s^{-1}. Under these conditions we can assume that at $0<x<l$ the temperature is uniform. Then one can obtain the expression for the electric field E by means of the magnetic field distribution and the corresponding Maxwell equation ($E=0$ at $x=l$):

$$E=\frac{\dot{H}_e l}{c}\left(\frac{H(x)}{H_e}-\frac{H^3(x)}{3H_e^3}\right)$$

and for H_0 we have the following expressions:

$$E_0=\frac{\dot{H}_e l}{c}\left(\frac{H_0}{H_e}-\frac{H_0^3}{3H_e^3}\right).$$

Using expression (6) we obtain: $\dot{j}=cH_e^2|\alpha^1(T)|\dot{T}_e/8\pi\alpha^2(T)$. Now it is easy to find by means of equation (9) that the stability criterion has the following form:

$$\Delta H < H_a(\dot{T}_e)$$

where $H_a(\dot{T}_e)$ is determined by the equation

$$\frac{2A(T)\alpha(T)}{|\alpha'(T)|}\dot{T}_e^{-1}=(H_a-H_j)H_a^2-\frac{(H_a-H_j)^3}{3}. \qquad (12)$$

The dependence of H_a on $\dot{T}_e$ in the case of small heating ($T-T_0 \ll T_c-T_0$) is shown qualitatively in figure 3. The value of H_a decreases with increase of $\dot{T}_e$ and at $\dot{T}_e \gg A(T)\alpha(T)/|\alpha'(T)|$, $H_a \sim H_j$.

By the same way one can find $H_a = H_a(\dot{T}_e)$ in the case when $\partial j_c/\partial H = 0$, $\partial E_0/\partial H = 0$:

$$H_a = H_j(T)\left(1 + \frac{8\pi j_c^2(T)E_0(T)}{H_j^2(T)|j_c'(T)|\dot{T}_e}\right)^{1/2}. \tag{13}$$

4. The stability criterion (slow variation of external parameters)

It follows from equations (10)–(13) that at $\dot{H}_e$, $\dot{T}_e \to 0$ the field difference $H_a \to \infty$. Actually, H_a is saturated at the low rates of variation of the external parameters, reaching a certain finite value as there always exist uncontrolled disturbances in the system. The corresponding maximum of the magnetic field difference in the sample H_m is determined by the value $\bar{\tau}_0$ at $E \ll E_0$

$$H_m = H_j \, . \, \bar{\tau}_0^{1/2} \tag{14}$$

where $\bar{\tau}_0$ depends on the form of the function $j(E)$ and the spectrum of the uncontrolled fluctuations.

In order to find the dependence of H_a on $\dot{H}_e$, $\dot{T}_e$ in the whole range of variation of $\dot{H}_e$ and $\dot{T}_e$ one has to know the function $j(E)$ at arbitrary E. There are no reliable empirical or theoretical relations for $j(E)$ at $E < E_0$ and, on the other hand, to carry out the necessary calculations with arbitrary function $j(E)$ is too difficult. This is the reason to choose some simple *I–V* curve to find H_a at arbitrary $\dot{H}_e$ and $\dot{T}_e$. The most simple model is the following (see figure 4):

$$j = \begin{cases} j_c; & E = 0 \\ j_c + \sigma_1 E; & 0 < E < E_0 \\ j_c + \sigma_1 E_0 + \sigma_f(E - E_0); & E > E_0. \end{cases}$$

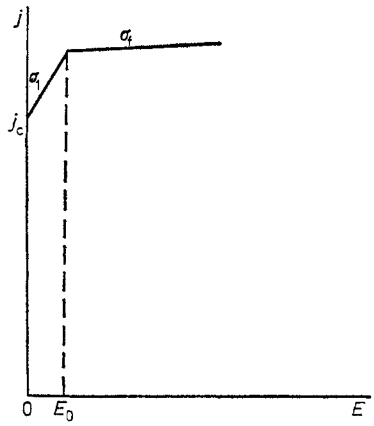

Figure 4. The model for *I–V* characteristics.

This model describes qualitatively well the peculiarities of the actual *I–V* curve and enables us to carry out the necessary calculations. As we are interested here only in the qualitative aspects, then, to make the task easier, we assume that j_c, σ_1, σ_f and E_0 do not depend on the magnetic field.

Under the influence of the changing external variable the part of the superconductor which is in the critical state is divided into two regions: (1) $E > E_0$ at $0 < x < x_0$; (2)

$E<E_0$ at $x_0<x<l$. Now it is easy to find x_0:

$$\left.\begin{aligned} x_0&=l-\frac{cE_0}{\dot{H}_e} \qquad (\dot{H}_e\neq 0,\ \dot{T}_e=0)\\ x_0&=l\left(1-\frac{c^2E_0}{2\pi l^2|j_c'(T)|\dot{T}_e}\right)^{1/2} \qquad (\dot{H}_e=0,\ \dot{T}_e\neq 0).\end{aligned}\right\} \tag{15}$$

As the rate of the variation of the external parameters appears in the stability criterion only through x_0, it is convenient to determine H_a as a function of x_0, and then to calculate the dependence of H_a on H_e and $\dot{T}_e$ by means of equations (15).

Figure 5 shows the results of the numerical calculations of the dependence of the dimensionless value H_a/H_j versus x_0 at various σ_1/σ_f. The calculations have been

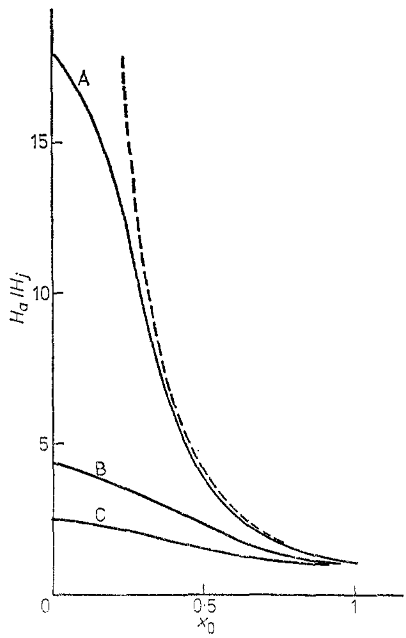

Figure 5. The curves $H_a/H_j=H_a(X_0)/H_j$. Solid curves, numerical calculations; curve A corresponds to $\tau_f=10^{-2}$, $\tau_1=10$; curve B, $\tau_f=10^{-1}$, $\tau_1=1$; curve C, $\tau_f=10^{-2}$, $\tau_1=1$. Dashed curve, the result given by equation (11).

carried out using the methods applied for defining H_j (see for example Mints and Rakhmanov 1977). All basic relations are given in the appendix for readers' convenience. It should be noted that to determine the values of H_j and H_m more accurate relations (3) and (4) can be substituted by the expressions (Kremlev *et al* 1977)

$$H_j=\left(\frac{\pi^3\nu j_c(T)}{|j_c'(T)|}\right)^{1/2}(1+1{\cdot}1\ \tau_f^{1/3}) \qquad (\tau_f\ll 1)$$

$$H_m=\left(\frac{\pi^3\nu j_c(T)}{|j_c'(T)|}\right)^{1/2}(\tau_1+1{\cdot}1\ \tau_1^{2/3}) \qquad (\tau_1\gg 1)$$

where

$$\tau_1=\frac{4\pi\kappa\sigma_1}{c^2\nu}.$$

The curves $H_a=H_a(x_0)$ found by means of equations (15) and (11) or (13) are shown by the dashed line in figure 5. At $x_0=1$ (high $\dot{H}_e$, $\dot{T}_e$) these curves agree well with the numeric calculations. As may be expected when the region of x_0, where the curves are close to each other, is wider, the ratio σ_1/σ_f is higher.

5. Comparison of theory and experiment

Experiments to study magnetic instabilities are performed as a rule with a specimen placed in a varying external magnetic field. One must initiate the flux jump in the course of the experiment, e.g. by mechanical shock (Evetts *et al* 1964), in order to determine the true instability threshold H_j. This fact has not been taken into account in the vast majority of the studies and the flux jumps have been initiated by varying the external field or by random factors, and this fact is probably responsible for the contradictions in the results of experiments investigating the dependence of the field at which the flux jump occurs (H_a in our terms) on $\dot{H}_e$. So for example, Watson (1967) and Wipf and Lubell (1965), and some others, mention a regular decrease of H_a with increase of $\dot{H}_e$. On the contrary, the experiments of Neuringer and Shapira (1966), Levy *et al* (1970), Harrison *et al* (1975) did not register such dependence. In particular, the authors of the latter paper had noted that the dependence of the external field at which the first flux jump occurred on $\dot{H}_e$ was of accidental nature and the actual stability boundary does not depend on it. Such an accidental nature of the instability is proved also by the 'disruptions' at the regular experimental curves.†

To compare the theory developed in this paper with the experimental results we shall use the relations obtained in §3, although the dependences found in §4 would certainly give more accurate coincidence between the theory and the experiment. But to take into account the saturation of the value H_a at $\dot{H}_e \to 0$ one must introduce an additional adjusting parameter σ_1/σ_f, which could not be found from experiment.

The experimental data obtained by Watson (1967) at various T_0 and $\dot{H}_e$ are shown in figure 6. Figure 7 contains the points of H_a versus $\dot{H}_e$ at $T_0=2{\cdot}5$ and $4{\cdot}2$ K; these data were presented in the paper by Wipf and Lubell (1965). The theoretical curves $H_a = H_a(\dot{H}_e)$ obtained by means of equation (10) are also shown there. The value $A(T)$ at every fixed T_0 was chosen so that the theoretical curves lay above the corresponding experimental points.

The experimental data given by Watson (1967) (figure 6) are the most complete for the comparison of theory and experiment. Following equation (10), we have obtained the dependence $A(T)$ for the superconductor used in the experiment by Watson (1967) (sample with $T_c=3{\cdot}2$ K). The corresponding curve is given in figure 8. In the vicinity of T_c the upper critical field $H_{c_2} \sim (1-T/T_c)^{1/2}$ (De Gennes 1966), and in the same region $\alpha(T) \sim 1-T/T_c$. If $K(T)$ has no singularity at $T \to T_c$, then $A(T) \to 0$ at $T \to T_c$. Actually, $A(T)$ decreases sharply at $T_0 \sim 2{\cdot}5$ K (see figure 8). Besides $A(T)$, the expression (10) includes $H_j(T)$. This value has been evaluated by Watson (1967) using equation (3) and the experimental data for the specific heat and T_c (see solid curve in figure 6).

As can be seen from figure 6, the experimental points at $\dot{H}_e < 2$ kOe min^{-1} lie noticeably lower than the theoretical curves. H_a is apparently close to saturation in this region. Quantitative agreement of the theory and the experimental data is worse in the range of high $\dot{H}_e$, as in this range the form of $H_a(T)$ greatly depends on $H_j = H_j(T)$, which is not known with sufficient accuracy (Watson 1967).

† In some experiments the growth in $H_a = H_a(\dot{H}_e)$ has been registered in the region of high $\dot{H}_e$, which is apparently explained by sufficient heating of the sample (Rothwarf *et al* 1968, Chikaba *et al* 1968). We shall not discuss it.

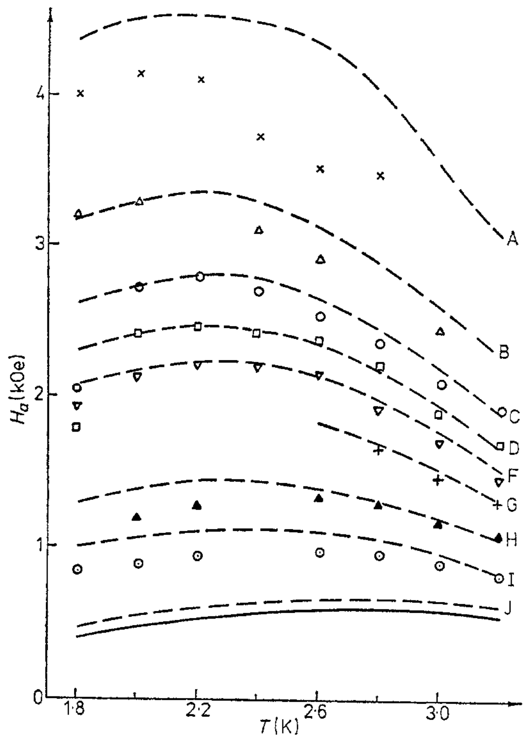

Figure 6. The dependence of H_a on T_0 at different $\dot{H}_e$. Experimental points by Watson (1967) and our theory (dashed curves). Solid line, $H_j = H_j(T)$ found by Watson (1967). Line A and points denoted by × correspond to $\dot{H}_e = 2$ kOe min^{-1}; B and △, 3 kOe min^{-1}; C and ○, 4 kOe min^{-1}; D and □, 6 kOe min^{-1}; F and ▽, 10 kOe min^{-1}; G and +, 15 kOe min^{-1}; H and ▲, 30 kOe min^{-1}; I and ⊙, 58·5 kOe min^{-1}; J, 10^3 kOe min^{-1}.

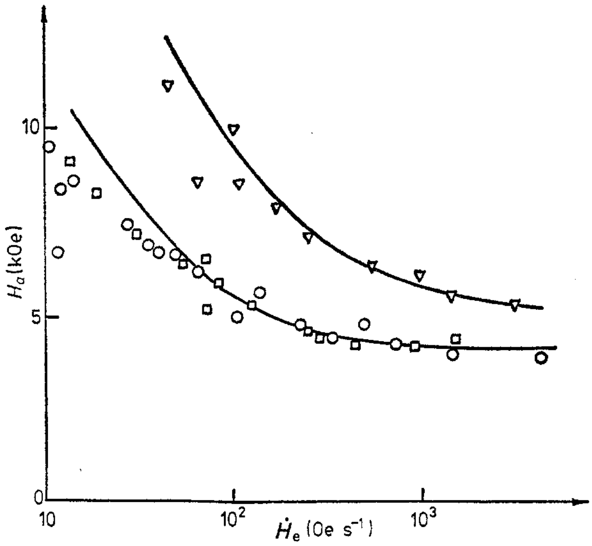

Figure 7. The experimental points by Wipf and Lubell (1965) and theory (solid lines); points denoted by ▽ correspond to $T_0 = 4·2$ K; □, ○, two different samples at $T_0 = 2·5$ K.

As the value of H_j had not been evaluated in the paper (Wipf and Lubell 1965) we have to choose two adjusting parameters H_j and $A(T)$ to compare the experimental data presented in this paper with theory (see figure 7). The values of A and H_j used to plot the theoretical curves were: at $T_0=2{\cdot}5$ K, $A=0{\cdot}85$ kOe^3 s^{-1}, $H_j=4$ kOe; and at $T_0=4{\cdot}2$ K, $A=4$ kOe^3 s^{-1}, $H_j=5$ kOe. Expressions (9) and (10) make it possible to estimate

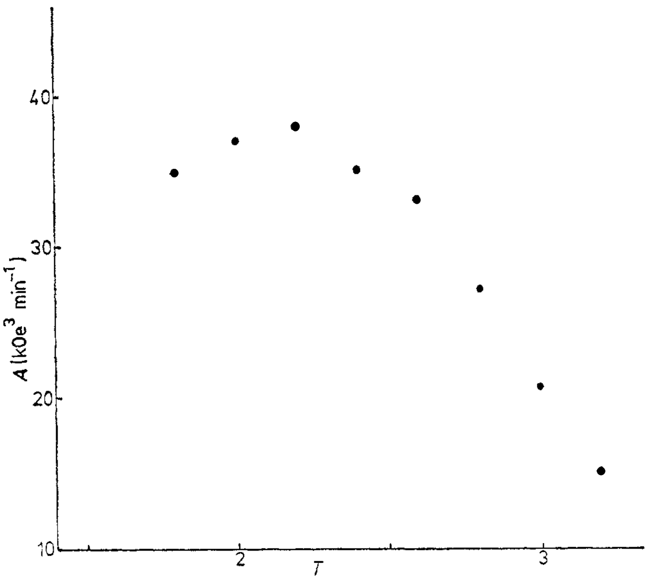

Figure 8. The dependence of A on T_0 for the curves shown in figure 6.

$E_0=A(T)/4\pi\alpha(T)$ using the value of $A(T)$ found from the experimental data. For example, let $A=4$ kOe^3 s^{-1}, $\alpha(T_0)=\alpha(0)(1-T_0/T_c)$, $\alpha(0)=3\times10^8$ A Oe cm^{-2}, $T_0=4$ K, $T_c=10$ K then $E_0\simeq10^{-7}$ V cm^{-1}. Using the values of A obtained by means of the dependence $H_a=H_a(\dot{H}_e)$, and the values for $H_j=5$ kOe, $\alpha(0)=3\times10^8$ aOe cm^{-2}, by equation (12), it is easy to find out that the magnitude of $\dot{T}_e$ at which a considerable influence of temperature variations on the H_a should be expected lies within the range $\dot{T}_e=10^{-2}$–10^{-1} K s^{-1} for the samples used by Wipf and Lubell (1965). The magnitude of H_a achieves the value of the order of H_j at $\dot{T}_e\sim1$ K s^{-1}.

6. Conclusions

(i) The dependence of the maximum difference of the magnetic field in hard superconductor at which the critical state is stable has been found as a function of the the rate of variation of the external magnetic field $\dot{H}_e$ and the rate of external heating $\dot{T}_e$ at sufficiently high $\dot{H}_e$ and $\dot{T}_e$. Two different models of the critical state have been considered (§2).

(ii) A simple approximate current–voltage characteristic has been chosen for a hard superconductor. For this model the stability criterion of the critical state has been found at any value of $\dot{H}_e$ and $\dot{T}_e$ (§ 3).

(iii) A comparison of the theory and the experiment has been made (§4).

Appendix

To define the field difference H_a for a hard superconductor with the model of *I–V* characteristics shown in figure 4, we shall consider for example a flat sample of thickness $2b$ placed in the external magnetic field $H_e > H_p = 4\pi b j_c/c$ (see figure 9).

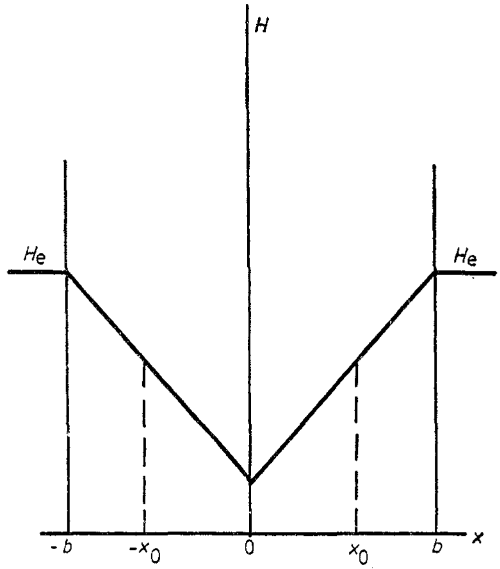

Figure 9. The magnetic field distribution in a flat sample at $H_e > H_p$, $\partial j_c/\partial H = 0$.

Let us write down the equation for a small temperature perturbation δT. The value δT may be presented in the form

$$\delta T = \theta(x/b) \exp(\lambda t\kappa/\nu b^2).$$

Then using Maxwell's equation and thermal diffusion equation it is easy to find

$$\theta^{IV} - \lambda(1+\tau_f)\theta^{II} - \lambda(\beta - \lambda\tau_f)\theta = 0 \qquad |x| > x_0$$

$$\theta^{IV} - \lambda(1+\tau_1)\theta^{II} - \lambda(\beta - \lambda\tau_1)\theta = 0 \qquad |x| < x_0$$

where

$$\beta = \frac{4\pi b^2 j_c |j_c'(T)|}{c^2\nu} = (\Delta H)^2 \frac{|j_c'|}{4\pi\nu j_c}; \qquad \Delta H = H_e - H(x=0).$$

As the solution is symmetrical with respect to the axis $x=0$ it has the form $(x>0)$:

$$\theta = c_1 \sinh k_1 x + c_2 \cosh k_1 + c_3 \sin k_2 x + c_4 \cos k_2 x \qquad x > x_0$$

$$\theta = c_5 \sinh q_1 x + c_6 \cosh q_1 x + c_7 \sin q_2 x + c_8 \cos q_2 x \qquad x < x_0 \qquad (16)$$

$$\theta^I = 0,\ E = \frac{\kappa}{j_c b^2}(\lambda\theta - \theta^{II}) = 0 \qquad \text{at } x=0$$

$$k_{1,2} = \left[\left(\frac{\lambda^2(1-\tau_f)^2}{4} + \lambda\beta\right)^{1/2} \pm \frac{\lambda(1+\tau_f)}{2}\right]^{1/2}$$

$$q_{1,2} = \left[\left(\frac{\lambda^2(1-\tau_1)^2}{4} + \lambda\beta\right) \pm \frac{\lambda(1+\tau_1)}{2}\right]^{1/2}.$$

To determine c_i one has to use the boundary conditions and also the conditions of continuity at $|x|=x_0$.

Under the isothermal boundary cooling we have:

$$\theta=0, \qquad x=\pm b. \tag{17}$$

It follows from the continuity of the magnetic field at $x=0$ that

$$\lambda\theta^{\mathrm{I}}-\theta^{\mathrm{III}}=\frac{j_c b^3}{\kappa c}\dot{H}_e. \tag{18}$$

The temperature, heat flow, and electrical and magnetic fields are continuous at $|x|=x_0$. The set of these conditions leads to the continuity of θ, θ^{I}, θ^{II} and θ^{III} at $|x|=x_0$.

Finally, we obtain the system of the non-uniform linear equations to define c_i

$$\sum_j a_{ij}c_j=c_i \tag{19}$$

where $a_{ij}=a_{ij}(\beta, \tau_t, \tau_1, x_0)$, all c_i except one equal to zero, c_i different from zero equals: $j_c b^3 \dot{H}_e/\kappa c$.

The existence of the growing eigen solutions (i.e., the solutions with $\lambda>0$) of the system of the equations for c_i corresponds to the instability. And therefore the stability criterion may be determined from the equation: det $\|a_{ij}\|=0$ at $\lambda>0$, where det$\|a_{ij}\|$ is the determinant of the system (19). (This question is discussed in more detail in the papers by Kremlev *et al* (1977) and Mints and Rakhmanov (1977).) Let the minimum value of β, at which the eigen solution with $\lambda>0$ appears first, equals β_c, and $\lambda=\lambda_c$ respectively. Then the stability criterion has the form

$$\Delta H<H_a=[4\pi\beta_c\nu j_c/|j_c'(T)|]^{1/2}$$

and the increment of the instability equals $t_j^{-1}=\lambda_c\kappa/\nu b^2$. The results of the numerical solution of this problem are given in figure 5.

Figure 10 shows schematically the spectrum of the eigen values $\lambda=\lambda(\beta)$ at various x_0.

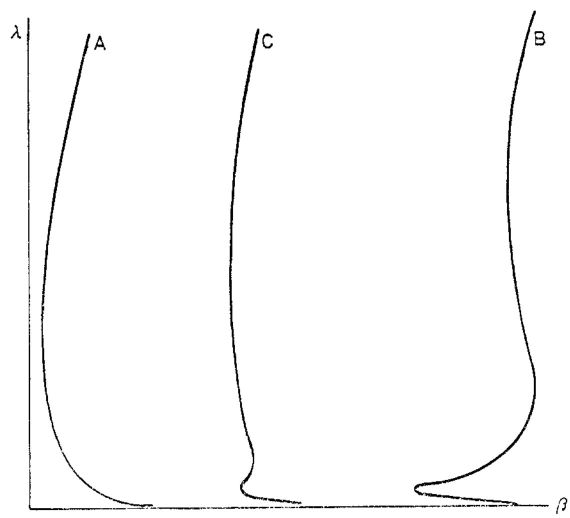

Figure 10. The functions $\lambda=\lambda(\beta)$ at various x_0. Curve A: x_0 is small, the critical state is destroyed by the 'fast' perturbations; curve B, $x_0\sim 1$, the critical state is destroyed by the 'slow' perturbations; curve C, intermediate value of x_0.

When $x_0 \sim 0$ the stability is broken by 'fast' ($\lambda_c \gg 1$) disturbances. At $\tau_1 \gg 1$ such disturbances exist in the region $|x| > x_0$ and are practically suppressed in the creep zone. Vice versa, at $x_0 \to 1$ the stability is broken by 'slow' ($\lambda_c \ll 1$) disturbances developing in the whole volume of the sample. These results for the spectrum $\lambda = \lambda(\beta)$, in particular, confirm the assumptions made in §2 about the influence of the creep zone on H_a at high $\dot{H}_e$ (or $\dot{T}_e$).

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