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On the flux jumps in hard superconductors

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Abstract. The paper is devoted to the investigation of critical state stability against flux jumps in hard superconductors accounting the critical current density dependence on the temperature and magnetic field. A flat sample with finite thickness is considered. The stability criteria were found for the Kim–Anderson critical state model as well as for the region close to the upper critical field H_{c2}. It was shown that the stability diminishes near H_{c2}. The stability conditions were found for the total range of external fields below H_{c2}. It was found that under certain conditions the magnetic instabilities may develop in two isolated regions of the external fields.

1. Introduction

The effect of the transport current on the quantized flux lines (Abrikosov vortexes) in hard superconductors (see e.g. de Gennes 1966) is counterbalanced by pinning forces provided the current density does not exceed a certain critical value j_c. At a current density above j_c the fluxoids start moving and an electric field occurs in the specimen. The superconductor passes into a so-called resistive state. According to the critical state concept any applied potential difference causes in hard superconductors the critical current with density j_c. This critical state model provides a satisfactory description, both qualitative and quantitative, for a number of phenomena in hard superconductors.

The critical state may involve instabilities—disruptions of stationary current flow. Flux jumps, occurring at fluctuations of the temperature T or magnetic induction B in a sample, are the typical instabilities of this kind. As $j_c = j_c\ (T, B)$ the current j at growing temperature or magnetic field exceeds j_c and the superconductor goes into the resistive state. This fluctuation, depending on the system parameters and external conditions, either decays or undergoes avalanche-type increase, and the magnitude of the magnetic flux captured in the sample exhibits a jump.

The occurrence of the flux jump and the stability improvement of the superconductive systems has been considered many times. Both qualitative and quantitative theories of magnetic instability have been suggested (Wipf 1967, Swartz and Bean 1968, Wilson *et al* 1970, Kremlev 1973, 1974).

This work investigates the critical state stability with regard to the flux jumps accounting for the dependences of the critical current on temperature and induction inside the sample. A plate of a hard superconductor of finite thickness with an arbitrary transport current within the wide range of external fields is considered. The results are compared with the stability criterion obtained by use of Bean's (1964) model (i.e. $\partial j_c/\partial B = 0$).

2. General equations

We shall consider a flat sample of finite thickness in an external magnetic field (figure 1) with initial temperature T_i when a small perturbation develops. The deviation of the temperature from the initial value of T_i is denoted by $\theta(\theta \ll T_c - T_i$ where T_c is the critical

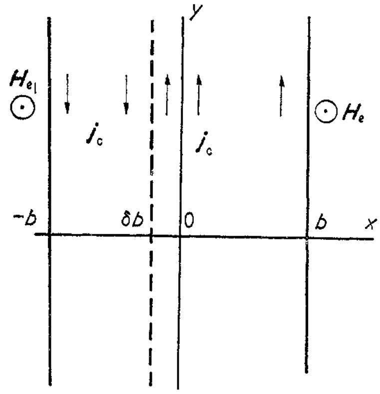

Figure 1. Sample geometry, the distribution of current and external magnetic field.

temperature of superconductor) and the electrical field arising during the motion of a vortex structure as E. We express the relation between current density and E in the form

$$j = j_c(T_i + \theta, H) + \rho_f^{-1} E \tag{1}$$

j_c being the critical current and ρ_f the resistivity of the superconductor in the resistive state. The linear dependence of j on E takes place only for sufficiently strong fluctuations, but this is not significant for the stability research. The nonlinear region of the volt–ampere characteristic is usually small and increasing fluctuations eventually lead to the linear region. Decaying fluctuations are of no interest because only the stability boundary is considered.

The electric field E is taken in the form

$$E = E_0(x/b) \exp [\lambda t (\kappa/\nu b^2)]$$

where ν is the heat capacity and κ is the heat conductivity of superconductor. Using heat conductivity, Maxwell equations and the relation (1), we can easily find the general equation for E, that is the fourth-order differential equation with rather complicated coefficients. This equation has to be furnished with four boundary conditions. The requirement of the existence of a nontrivial solution E determines the dependence of λ on the system parameters. The instability region corresponds to $\lambda > 0$. The stability criterion obtained by this method determines the stability with respect to any small perturbations (of temperature, magnetic field, etc.).

Usually it is very difficult to find an analytical solution of the general equation for for j_c versus H dependences of practical interest; therefore only numerical stability research can be carried out. However, in the case of hard superconductors† some simplifications is possible. As was demonstrated by Kremlev (1973) and Mints and Rakhmanov (1975) within the range of hard superconductors, the stability in Bean's

† I.e., for the materials with $D_t/D_m = 4\pi\kappa/c^2\nu\rho_f \ll 1$, where D_t and D_m are thermal and magnetic diffusivities respectively.

model is determined by disturbances with high λ ($\lambda \to \infty$). The heat conductivity of the material becomes insignificant for the research of the most 'dangerous' disturbances (heat transport has no time to take place).

Thus, to consider the flux jump in the case of hard superconductors one can omit the term $\kappa\theta''$ in the heat diffusion equation, and then

$$\nu\dot{\theta} = j_c E. \tag{2}$$

By using (1), (2) and Maxwell equations in the limit of $D_t/D_m \to 0$ we obtain an equation for E_0 as follows:

$$E_0'' + \alpha(x)\, E_0' + \beta(x)\, E_0 = 0 \tag{3}$$

where

$$\alpha(x) = -\frac{4\pi b}{c}\frac{\partial j_c}{\partial H} \qquad \beta(x) = -\frac{4\pi b^2}{c^2}\frac{j_c(x)}{\nu}\frac{\partial j_c}{\partial T}. \tag{4}$$

Equation (3) has to be supplied with electrodynamical boundary conditions only (Mints and Rakhmanov 1975). The current density being equal to 0 at $x/b = \delta$ leads to the natural condition

$$E_0(\delta b) = 0. \tag{5}$$

To obtain the second boundary condition we consider the external magnetic field to remain constant during the fluctuation:

$$E_0'(\pm b) = 0. \tag{6}$$

The stability is violated ($\dot{\theta} > 0$, see (2)) if a nontrivial solution exists for the equation (3) with boundary conditions (5) and (6). In the extreme case under consideration ($\lambda \gg 1$) instability in the regions $-1 < x/b < \delta$ and $\delta < x/b < 1$ develops independently and the total system stability is determined by the least stable zone.

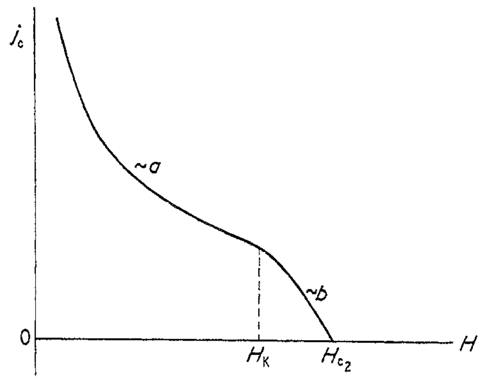

Figure 2. j_c versus H dependence: Kim–Anderson model is applicable on curve (a) section; on curve (b) section (near H_{c_2}) $j_c = j_1(1 - H/H_{c_2})$.

3. The critical state model and stability

In many cases the dependence j_c versus H has the qualitative form presented in figure 2. In a magnetic field not too close to H_{c_2} (section (a)) it follows the empirical relation (Kim *et al* 1964)

$$j_c = j_0 B_0/(H + B_0) \qquad j_0 = j_0(T) \qquad B_0 = B_0(T) \tag{7}$$

(the Kim–Anderson critical state model). Near H_{c_2} (section (*b*)), figure 2 j_c (H) may be taken in the following form:

$$j_c = j_1(1 - H/H_{c_2}) \qquad j_1 = j_1(T) \qquad H_{c_2} = H_{c_2}(T). \tag{8}$$

The initial distribution of magnetic field and current in a sample is easily found for the dependences of j_c on H (equations 7 and 8) by means of corresponding Maxwell equation. Thus for the region $\delta < x/b < 1$ (figure 1) in the Kim–Anderson model:

$$j_c(x) = j_0 B_0[(H_e + B_0)^2 - (8\pi b/c) j_0 B_0(1 - x/b)]^{-1/2}. \tag{9}$$

Similarly near H_{c_2}

$$j_c(x) = j_1(1 - H_e/H_{c_2}) \exp [(4\pi b/c) j_1/(1 - x/b) H_{c_2}] \tag{10}$$

where H_e is the external field.

All relations for the region $-1 < x/b < \delta$ have the identical form.

For the model relations in the form (7) and (8) the stability region can be found exactly. It is possible to perform in general terms the qualitative research of the influence of the critical current dependence on magnetic field. We try a solution of equation (3) in the following form:

$$E_0 = \exp(-\mathrm{i}K(x)\, x/b). \tag{11}$$

We shall assume that $K'(x)$ and $K''(x)$ are small, which is true, as will be shown, at least for sufficiently high magnetic fields. For $K(x)$ from (3) we get the following:

$$K(x) = (i\alpha(x)/2) \pm [\beta(x) - (\alpha^2(x)/4)]^{1/2}. \tag{12}$$

Employing boundary conditions (5) and (6) we find that a nontrivial solution of equation (3) exists if

$$\cos [(\beta - \alpha^2/4)^{1/2}(1 - \delta) + \psi] = 0$$

where

$$\psi = \tan^{-1} \{\alpha/[2(\beta - \alpha^2/4)^{1/2}]\}.$$

The values α and β are taken in a certain intermediate point x.

Thus the system is stable at

$$(\beta - \alpha^2/4)^{1/2}(1 - \delta) + \psi < \pi/2. \tag{13}$$

The relative magnitude of terms α^2 and β in (13) is

$$\alpha^2/\beta = 4\pi\nu \left(\frac{\partial j_c}{\partial H}\right)^2 \Big/ j_c \left|\frac{\partial j_c}{\partial T}\right|.$$

The following denotations are introduced:

$$\frac{\partial j_c}{\partial T} = -\frac{j_c}{T_0} \qquad \frac{\partial j_c}{\partial H} = -\frac{j_c}{H_1} \qquad T_0 = T_0(H, T) \qquad H_1 = H_1(H, T).$$

Hence

$$\alpha^2/\beta = \frac{4\pi\nu T_0}{H_1^2}.$$

In the fields much less than H_{c_2} the value of T_0 is evidently $\sim T_c$, and $H_1 \sim H_e$. Thus in sufficiently strong fields it can be assumed that

$$\alpha^2/\beta \ll 1$$

(for hard superconductors $\alpha^2/\beta \ll 1$ even at H_1 about several kG). In this case from (13) we come to the conclusion that the system is stable when

$$\frac{4\pi b^2(1-\delta)^2}{c^2}\frac{j_c^2}{\nu T_0}<\frac{\pi^2}{4} \tag{14}$$

which coincides with the stability criterion found in the preceding papers for the Bean model (Swarts and Bean 1968, Kremlev 1973, Mints and Rakhmanov 1975).

Near H_{c_2} one easily finds that

$$H_1 \sim H_{c_2} - H \qquad T_0 \sim T_c(1-H/H_{c_2}).$$

As a result it follows that

$$\frac{\alpha^2}{\beta} \sim \frac{4\pi\nu T_c}{H_{c_2}{}^2}\frac{1}{(1-H/H_{c_2})}.$$

Thus the presence of $\alpha(x)$ term in the equations is of any importance only in the immediate vicinity of H_{c_2} (for $1-H/H_{c_2} \lesssim 10^{-3}$). Out of this region $\alpha(x)$ is still much less than $\sqrt{\beta}$ and the stability criterion takes the following form:

$$\frac{4\pi b^2(1-\delta)^2}{c^2}\frac{j_c^2}{\nu T_c(1-H/H_{c_2})}<\frac{\pi^2}{4}. \tag{15}$$

Since the density of critical current decreases with growing external field, it follows from (14) that system stability increases at high fields. However, near H_{c_2} stability drops again because of the small value of $(I-H/H_{c_2})$ in (15).

Qualitative theory shows the dual effect of j_c versus H dependence on the stability. In the region of the parameters, where the condition $\alpha^2 \ll \beta$ is not satisfied, the derivative $\partial j_c/\partial H$ is directly included into stability criterion; and more essentially, near the upper critical field H_{c_2} the derivative $\partial j_c/\partial T$ may grow drastically and the stability undergoes a considerable reduction (compare (14) and (15)). The growth of $\partial j/\partial T$ occurs due to the fact that $j_c(H_{c_2})=0$, and H_{c_2} is in its turn a function of temperature.

4. Quantitative results

In this section we present the stability criterion obtained by means of the exact solution of equation (3) with boundary conditions (5) and (6) in the Kim–Anderson model (7) and in the field region near H_{c_2}.

4.1. Stability criterion in Kim–Anderson model

Let us find the expressions for the coefficients $\alpha(x)$ and $\beta(x)$ in equation (3). It is easy to obtain from (4), (7) and (9):

$$\alpha(x)=\alpha_0/[1-2\alpha_0(1-x/b)]$$

where the following designation is introduced:

$$\alpha_0=\alpha(b)=(4\pi b/c)\,[j_0B_0/(H_e+B_0)^2]$$

and for $\beta(x)$:

$$\beta(x)=\beta_0\left(1-\frac{\gamma}{[1-2\alpha_0(1-x/b)]^{1/2}}\right)\left(\frac{1}{1-2\alpha_0(1-x/b)}\right)$$

where

$$\beta_0 = \frac{4\pi b^2}{c^2} \frac{j_c^2(H_e)}{\nu T_0}$$

$$\gamma = \frac{B_0}{H_e + B_0} \frac{1}{T_2/T_1 + 1}.$$

Here $T_0(T) \sim T_1(T) \sim T_2(T) \sim T_c$ and those are found from the conditions

$$\frac{\mathrm{d}j_0}{\mathrm{d}T} = -\frac{j_0}{T_1} \qquad \frac{\mathrm{d}B_0}{\mathrm{d}T} = -\frac{B_0}{T_2} \qquad T_0^{-1} = T_1^{-1} + T_2^{-1}.$$

Let us introduce the following denotation and a new variable y:

$$\rho = \beta_0^{1/2} \frac{\gamma}{\alpha_0}$$

$$y = \frac{\beta_0^{1/2}}{\alpha_0} [1 - 2\alpha_0(1 - x/b)]^{1/2}. \tag{16}$$

Then from (3)

$$yE_0'' + (y - \rho) E_0 = 0. \tag{17}$$

The solution of (17) is a linear combination of Whittaker functions:

$$E_0 = c_1 W_{\mathrm{i}\rho/2,\, 1/2}(2\mathrm{i}y) + c_2 W_{-\mathrm{i}\rho/2,\, 1/2}(-2\mathrm{i}y). \tag{18}$$

Usually $B_0 \lesssim 1$ kG, and in the most interesting case $H_e \gg B_0$. Then it follows from (16) that $y \gg \rho$ at any x. Besides, it is clear from (16) that $\rho \ll 1$ for hard superconductors. Under these conditions the term with ρ in (17) can be assumed to be zero to the first approximation and from (17) we obtain

$$E_0 = c_1 \exp(\mathrm{i}y) + c_2 \exp(-\mathrm{i}y). \tag{19}$$

The boundary conditions (5) and (6) require, for the existence of a nontrivial solution (19) of equation (17),

$$\cos[y(1) - y(\delta)] = 0$$

and the stability criterion takes the form

$$|y(1) - y(\delta)| < \pi/2. \tag{20}$$

Let us consider now two cases of magnetic field distribution inside the sample as presented in figure 3. In the first case (figure 3*a*) the magnetic field penetrated the entire volume of the sample. Here δ is a free parameter of the problem determined by the relation of external magnetic fields to the left side (H_{e1}) and right side of the sample (H_e) (at $H_e = H_{e1}$ $\delta = 0$).

It is easily found from (9) that such a situation takes place when external field $H_e > H_p$, where H_p is

$$H_p^2 = (8\pi b/c)(1 - \delta) j_0 B_0. \tag{21}$$

From (20), (21), (16) at $H_e > H_p$ we can obtain the stability criterion in the form

$$\frac{H_e^2}{4\pi} \frac{1}{\nu T_0} \left[1 - \left(1 - \frac{H_p^2}{H_e^2}\right)^{1/2}\right]^2 < \frac{\pi^2}{4}. \tag{22}$$

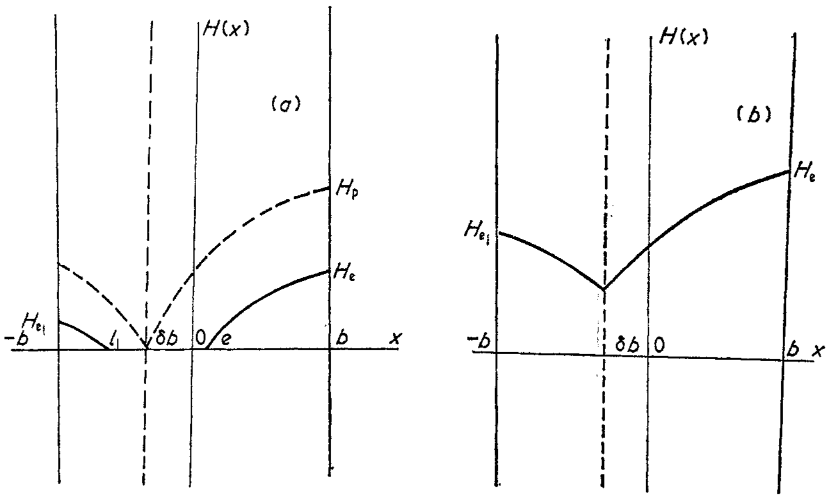

Figure 3. The distribution of the magnetic field $H(x)$ in the sample: (*a*) magnetic field penetrates all the volume of superconductor; (*b*) no field and transport current are present in the inner regions in the sample (similar to the case of semi-infinite sample). At $H_e = H_p$ surface current layers join, and the magnetic flux penetrates all the superconductor volume.

If $H_e^2 \gg H_p^2$ then it follows from (22)

$$\frac{4\pi b^2 (1-\delta)^2 j_0^2}{c^2} \frac{B_0^2}{H_e^2} \frac{1}{\nu T_0} < \frac{\pi^2}{4}. \tag{23}$$

Criterion (23) coincides evidently with (14) obtained from the Bean model or by use of qualitative theory if j_c, in the left part of inequality (14), is assumed equal to $j_c(H_e)$.

For $H_e < H_p$ in the inner layer of a superconductor the current density is zero (figure 3*b*) which is apparently equivalent to the case of a semi-infinite sample. The thickness of current carrying layer l is determined by an external magnetic field H_e. The boundary condition (5) should be rewritten as

$$E_0(l) = 0. \tag{5'}$$

In the determination of l from $H(l) = 0$ one can easily find the stability criterion by means of solution (19) with boundary conditions (5) and (6):

$$\frac{H_e^2}{4\pi} \frac{1}{\nu T_0} < \frac{\pi^2}{4}. \tag{24}$$

At $H_e = H_p$ (24) and (23) naturally coincide.

It follows from (24) that with growing external field the stability decreases ($H_e < H_p$).

The phenomenon is readily explained by the growing depth of the current-carrying layer l. At $H_e > H_p$ stability increases with growing external field ((22) and (23)). It follows from (22) and (23) that the system is least stable at $H_e = H_p$. Thus, if (24) is satisfied at $H_e = H_p$, the stability is preserved within all ranges of external field not too close to H_{c_2} ($H_e < H_K$ see figure 2). Substituting (21) in (24) we obtain a condition of the stability for all range of fields $H_e < H_k$

$$\frac{2\pi b\,(1-\delta)}{c} \frac{j_0 B_0}{\nu T_0} < \frac{\pi^2}{4}. \tag{25}$$

Stability criterion (24) for a semi-infinite sample in a Kim–Anderson critical state model was found by Wipf (1967). Note that for a semi-infinite sample in the Bean model the stability criterion is exactly identical to (24).

4.2. *Stability criterion in fields close to H_{c_2}*

Let us determine the stability criterion for j_c versus H dependence (8). Assuming the sample to be thin enough and $H > H_K$ in all the volume, it is easy to find from (8):

$$\alpha(x) = \alpha_c = \frac{4\pi b}{c}\frac{j_1}{H_{c_2}}$$

and for $\beta(x)$:

$$\beta(x) = \beta_c \exp\,[\alpha_c(1 - x/b)]\,\{\gamma_c + \exp\,[\alpha_c(1 - x/b)]\}$$

where

$$\beta_c = \frac{4\pi b^2}{c^2}\frac{j_c^2\,(H_e)}{\nu T_0'} \qquad \gamma_c = \frac{1}{(1 - h_e)\,(T_2'/T_1' - 1)}.$$

Here T_1', T_2' and T_0' can be defined from the conditions

$$\frac{\mathrm{d}j_1}{\mathrm{d}T} = -\frac{j_1}{T_1'} \qquad \frac{\mathrm{d}H_{c_2}}{\mathrm{d}T} = -\frac{H_{c_2}}{T_2'} \qquad \frac{1}{T_0'} = \frac{1}{T_1'} - \frac{1}{T_2'} \qquad h_e = \frac{H}{H_{c_2}}.$$

Let us introduce the following denotation and a new variable Z:

$$\mu = \sqrt{\beta_c}\,\frac{\gamma_c}{\alpha_c}$$
$$Z = \frac{\sqrt{\beta_c}}{\alpha_c}\exp\,[\alpha_c(1 - x/b)]. \tag{26}$$

Then we get from (3) a new equation analogous to (17)

$$ZE_0'' + (Z + \mu)\,E_0 = 0. \tag{27}$$

It is solved by a linear combination of Whittaker functions similar to (18):

$$E_0 = c_1 W_{-\mathrm{i}\mu/2,\,1/2}(2\mathrm{i}Z) + c_2 W_{\mathrm{i}\mu/2,\,1/2}(-2\mathrm{i}Z). \tag{28}$$

It is evident from (26) that for hard superconductors $\mu \gg 1$. Then provided

$$|\gamma_c| \gg e^{\alpha_c} \tag{29}$$

and employing the properties of Whittaker functions one can put down solution (28) in the form

$$E_0 = Z^{1/2}\{c_1 J_1\,[2(\mu Z)^{1/2}] + c_2 N_1\,[2(\mu Z)^{1/2}\} \tag{30}$$

where J_1 and N_1 are Bessel functions of the first and second kind. The nontrivial solution of equation (27) with boundary conditions (5) and (6) exists if the following relation takes place:

$$J_0(\sigma)\,N_1(\eta\sigma) - J_1(\eta\sigma)\,N_0(\sigma) = 0 \tag{31}$$

where

$$\eta = \exp\,[\alpha_c\,(1 - \delta)/2] \qquad \sigma = H_{c_2}\,(1 - h_e/\pi\nu T_2')^{1/2}.$$

Since $\sigma \gg 1$ up to the fields very close to H_{c_2}, Bessel functions in (31) can be substituted with their asymptotic values. Then from (31) for the case

$$\sigma^2\alpha_c \gg 1$$

we obtain

$$\cos\left[\sigma(\eta-1)\right]=0.$$

Hence the stability criterion is

$$\frac{H_{c_2}{}^2(1-h_e)}{\pi\nu T_2'}\left[1-\exp\left(\frac{2\pi b(1-\delta)}{c}\frac{j_1}{H_{c_2}}\right)\right]^2<\frac{\pi^2}{4}. \tag{32}$$

At $\sigma \gg 1$ and arbitrary $\alpha_c \ll 1$ we get from (32) stability criterion in the form

$$\frac{4\pi b^2(1-\delta)^2}{c^2}\frac{j_c{}^2(H_e)}{\nu T_2'(1-h_e)}<\frac{\pi^2}{4}. \tag{33}$$

Since $T' \sim T_{c_2}$ criterion (33) coincides with (15), $j_c = j_c(H_e)$ and $H = H_e$ in the left part of (15). The stability decrease near H_{c_2} found on the basis of qualitative speculations is confirmed by inequalities (32) and (33). In this case the condition of applicability of the qualitative theory (K', $K'' \ll K$) coincides with the requirement $\alpha_c \ll 1$.

5. Discussion

In the preceding section for j_c versus H dependence shown in figure 2 the stability criteria were defined. If $H_e{}^2 < H_p{}^2 = 8\pi b(1-\delta)j_0B_0/c$ (i.e. the magnetic field has not penetrated the entire volume of the sample (figure 3*b*)) stability decreases with growing external field. The stability criterion is determined by inequality (24):

$$\frac{H_e{}^2}{4\pi}\frac{1}{\nu T_0}<\frac{\pi^2}{4} \tag{24}$$

which shows that the region of sufficiently low fields is always stable.

If the magnetic field $H_e > H_p$ (figure 3*a*) the stability grows with increase of external field, and the corresponding criterion takes the form (22)

$$\frac{H_e{}^2}{4\pi}\frac{1}{\nu T_0}\left[1-\left(1-\frac{H_p{}^2}{H_e{}^2}\right)^{1/2}\right]^2<\frac{\pi^2}{4} \tag{22}$$

at $H_e{}^2 \gg H_p{}^2$. (22) transforms into (23):

$$\frac{4\pi b^2(1-\delta)^2 j_c{}^2(H_e)}{c^2}\frac{}{\nu T_0}<\frac{\pi^2}{4}. \tag{23}$$

It is clear from (22) and (24) that the critical state is least stable in external field $H_e = H_p$ i.e. when both current-carrying layers overlap (figure 3*b*). Thus the system is stable throughout the whole range of fields $H_e < H_K$ if the condition (22) is true for $H_e = H_p$ (see (25)):

$$\frac{2b(1-\delta)}{c\nu T_0}j_0B_0<\frac{\pi^2}{4}. \tag{25}$$

At $H_e = H_K$ j_c versus H dependence undergoes a qualitative change (figure 2). Within the scope of fields $H_e > H_K$, derivatives $\partial j_c/\partial T$ and $\partial j_c/\partial H$ grow and the stability decreases considerably, the corresponding criterion being determined from inequality (32) which at $\alpha_c = (4\pi b/c)(j_1/H_{c_2}) \ll 1$ transforms into (33). These inequalities demonstrate the stability growing with increasing external field. Hence, the critical state is stable for the total range of fields $H_K < H_e < H_{c_2}$ if (32) is true at $H_e = H_K$:

$$\frac{H_{c_2}^2 (1-h_K)}{\pi \nu T_2'} \left[1 - \exp\left(\alpha_c \frac{1-\delta}{2}\right)\right]^2 < \frac{\pi^2}{4} \qquad h_K = \frac{H_K}{H_{c_2}} \tag{34}$$

or for $\alpha_c \ll 1$ from (33):

$$\frac{4\pi b^2 (1-\delta)^2}{c^2} \frac{j_c^2 (H_K)}{\nu T_2'(1-h_K)} < \frac{\pi^2}{4}. \tag{35}$$

The presented results enable us to find the stability criterion for the total range of external fields $B_0 \ll H_e < H_{c_2}$. First of all, let us find out which of the conditions is more restrictive—(25) or (34) (at $\alpha_c \ll 1$: (35)). To compare (25), (34) and (35) parameters j_0 and B_0 from (25) shall be related with H_K. The necessary relation will be found by equating expressions (7) and (8) for j_c at $H = H_K$:

$$j_0 B_0 = j_1 H_K (1 - h_K).$$

Substituting this into (25) we obtain

$$\frac{2b(1-\delta)}{c\nu T_0} j_c(H_K)\, H_K < \frac{\pi^2}{4}. \tag{36}$$

The case of $\alpha_c \ll 1$ shall be considered first. From inequalities (36) and (35) the former is more restrictive as $T_2' \sim T_0 \sim T_c$, $H_K \sim H_{c_2}$, $|\delta| \lesssim 1$ and the ratio of the left parts of (36) and (35) is about $\alpha_c^{-1} \gg 1$. Hence in the case of $\alpha_c \ll 1$ the system is stable for the total range of external fields $B_0 \ll H_e < H_{c_2}$ if inequality (36) satisfied.

Let us assume the inequality (36) to be violated. If the condition (35) remains true, flux jumps may occur in the part of the external fields close to H_p where inequalities (22) and (24) are not satisfied.

If inequality (35) is also violated, then two alternatives arise depending on the parameters of the sample.

In the first case at $H_e = H_K$ the condition (22) is true and flux jumps occur in two isolated regions of external fields—near H_K and near H_p. The intermediate fields region is stable. As $H_p^2/H_K^2 \sim \alpha_c(1-h_K)$, then at $\alpha_c \ll 1$, $H_p^2 \ll H_K^2$. Using (36) and (23) we come to the conclusion that the system is stable in the range of the fields

$$\frac{16b^2 (1-\delta)^2 h_K^2 j_c^2 (H_K)}{c^2 \pi \nu T_0} < h_e^2 < h_K^2 \tag{37}$$

provided the condition is fulfilled that

$$1 - \frac{c(\pi \nu T_0)^{1/2}}{4b(1-\delta) j_1} < h_K < 1 - \frac{c^2 \pi \nu T_2'}{16b^2 (1-\delta)^2 j_1^2}. \tag{38}$$

In the second case, inequality (22) is not true at $H_e = H_K$ and both instability regions join together. Flux jumps occur throughout the whole range of external fields except in the immediate vicinity of H_{c_2} where criterion (33) is satisfied and at very low fields where condition (24) is satisfied for certain.

Consider the case of $\alpha_c \gtrsim 1$. The comparison of inequalities (34) and (36) shows the ratio of their left parts to be of the order of

$$\frac{2\{\exp[\alpha_c/2(1-\delta)] - 1\}^2}{\alpha_c(1-\delta)}.$$

Hence at $\alpha_c \gtrsim 1$ the condition (34) is more restrictive and the most 'dangerous' region of external fields is now found close to H_{c_2}. Inequality (34) is the stability criterion for the whole range of external fields. It is to be mentioned that the criterion (34) cannot be used at arbitrarily high value of α_c as it was obtained from the assumption (29):

$$|(1-h_e)(T_2'/T_1'-1)|^{-1} \gg e^{\alpha_c}.$$

With condition (34) violated but inequality (36) remaining true flux jumps occur in a zone of external fields H_e near H_K (but with $H_e > H_K$).

If the condition (36) is violated, two alternatives are possible depending on the parameters of the system (as in the case of $\alpha_c \ll 1$). In the first case at $H_e = H_K$ inequality (22) is satisfied. Then, the magnetic instabilities occur in two isolated regions of external fields close to H_p and H_K.

In the second case the condition (22) is not satisfied at $H_e = H_K$. Instability areas join together. Flux jumps occur in the whole range of external fields except for the immediate vicinity of H_{c_2}, where criterion (32) is satisfied and at very low fields where the condition (24) is satisfied for certain.

6. Conclusions

In the present paper the critical state stability in a flat sample of a hard superconductor against the flux jumps accounting for the critical current dependence on temperature and magnetic field is determined. An equation was found describing the development of a small disturbance in a superconductor.

The thermal diffusion accounts just for small corrections in the stability criterion. An equation was found describing the development of the fast-growing disturbances in a hard superconductor. The corresponding stability criterion does not depend on the cooling conditions on the surface.

A qualitative theory (§3) was elaborated, enabling us to conduct the investigation of the critical state stability within the scope of general assumptions. The j_c versus H dependence was shown to produce the essential effect on the stability of the critical state. At $H_e \sim H_{c_2}$ this effect is connected with the change of T_0 character determining the derivative $\partial j_c/\partial T$, and in massive samples $\partial j_c/\partial H$ is directly included into stability criterion.

For the j_c versus H dependence presented in figure 2 the corresponding stability criteria were also found (§4). It was demonstrated that at $H_e < H_p$ (the magnetic field did not penetrate the whole volume of a superconductor (figure 3*b*)) stability decreases with growing external field. At $H_e > H_p$ (magnetic flux fills up all the volume of the sample (figure 3*a*)) stability grows with growing H_e. At $H_e > H_K$ the dependence of j_c on H changes (figure 2) and the stability of critical state undergoes substantial reduction.

A condition was found determining the stability throughout the total range of the external magnetic fields (§5). At $\alpha_c \ll 1$ it is inequality (36) or the equivalent (25) and at $\alpha_c \gtrsim 1$ inequality (34). Under certain conditions instabilities are shown to occur in two isolated regions of external fields—near H_p and H_K.

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