# *SPECTRUM OF MAGNETOSTATIC OSCILLATIONS IN THE PRESENCE OF A DOMAIN STRUCTURE*

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We consider quasistatic oscillations of the magnet with a periodic domain structure. The spectrum of such oscillations has a band character. We obtain the width of the corresponding bands (in terms of frequency) and the form of the spectrum. We present analytic expressions for the spectrum near the characteristic points. We show also that a surface magnetostatic wave exists on the solitary domain wall; we obtain its spectrum and present an explicit expression for it in a number of particular cases.

1. The influence of the domain structure on the oscillation spectrum of a magnet was considered in the literature a number of times. Two limiting cases were investigated, $\lambda \gg L$ and $d \gg \lambda \sim \delta$ ($L$ is the characteristic dimension of the body, $d$ is the transverse dimension of the domain, $\delta$ the thickness of the domain wall, and $\lambda$ the wavelength). In[1,2] it was assumed that $\lambda \gg L$, and thus the precession of the magnetic moments is homogeneous. In this case the connection between the oscillations and the magnetic moments in two groups of flat domains is manifest in the existence of a "demagnetizing field" of the domain boundaries. Oscillations of the domains inside an individual domain boundary ($\lambda \ll d$) were investigated in[3,4].

In this paper we consider the intermediate case $L \gg \lambda \gg \delta$. Since $\lambda \ll L$, the boundary effects and the shape of the magnet are immaterial. Thus, the domain structure will be assumed periodic and the domains will be assumed to be plane-parallel. The condition $\lambda \gg \delta$ makes it possible to neglect the exchange energy and to disregard the structure of the domain boundary, assuming it to be geometrical. The spectrum of the magnetostatic oscillations in the wavelength region in question is determined from the boundary conditions on the domain wall with allowance for the periodicity of the structure. Inside the domain, the connection between the alternating components of the induction and the magnetic field is given by the tensor of the magnetic permeability $\mu_{ik}(\omega)$. It should be noted that the problem of allowance for the periodic domain structure was first formulated in[5,6]. The magnetostatic oscillations were not considered there. In conclusion we note that since the magnetic moments of the neighboring domains are antiparallel, the structure considered by us, relative to long waves, is a "macroscopic antiferromagnet," in which the neighboring domains play the role of atoms. We now proceed to solve the problem.

2. Assume that the magnetization in neighboring domains is $\pm M_S$ ($M_S$—saturation magnetization), and there is no external magnetic field. As is well known, in this case have

$$\mu_{xx} = \mu_{yy} = \mu = 1 + \omega\omega_M / (\omega_a^2 - \omega^2),$$
$$\mu_{xy} = -\mu_{yx} = \pm i\mu' = \mp i\omega\omega_M / (\omega_a^2 - \omega^2),$$
$$\mu_{zz} = 1,$$

the remaining components of the tensor are equal to zero. The z axis is parallel to the anisotropy axis; the y axis is perpendicular to the domain wall; $\omega_2 = \beta\gamma M_S$; $\omega_M = 4\pi\gamma M_S$; $\gamma > 0$ is the gyromagnetic ratio, $\beta > 0$ is the anisotropy constant; $\mu_{xy} = +i\mu'$ for domains in which $M_S \parallel 0z$, $\mu_{xy} = -i\mu'$ for domains with opposite orientation of the magnetization.

The magnetization oscillations are described by the magnetostatic equations $h = -\nabla\varphi$, div $b = 0$ ($b_i = \mu_{ik}h_k$), which reduce to an equation for the magnetic potential $\varphi$

$$\mu\left(\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2}\right) + \frac{\partial^2\varphi}{\partial z^2} = 0. \quad (1)$$

On the domain boundary we have continuous $\varphi$ and $b_y = -\mu\partial\varphi/\partial y - \mu_{yx}\partial\varphi/\partial x$. The boundary conditions should be supplemented by the condition of translational invariance $\varphi(y + 2d) = e^{2i\kappa d}\varphi(y)$ ($\kappa$ is the transverse wave number and $2d$ is the period of the magnetic structure). Since $\omega$ is a periodic function of $\kappa$ with a period $2d$, it can be assumed that $0 \le \kappa \le \pi/d^{-1}$). We seek the solution of (1) in the form

$$\varphi_n = (A_n \exp\{ik_y y\} + B_n \exp\{-ik_y y\}) \exp\{ik_\parallel \rho\},$$
$$\mathbf{k}_\parallel = \{k_x, 0, k_z\} = \{k\sin\theta, 0, k\cos\theta\},$$

$k$ is the longitudinal wave number, $\theta$ is the angle between $k_\parallel$ and $0z$, and $n = 0, \pm 1, \pm 2, \ldots$ are the numbers of the domains. Substituting this expression in (1) gives an equation for $k_y$:

$$\mu(k_x^2 + k_y^2) + k_z^2 = 0. \quad (2)$$

Using the boundary conditions for $y = 0$, $d$ and the condition for translational invariance, we obtain after simple calculations a dispersion equation which, together with (2), determines the spectrum of the oscillations of the magnet with a periodic domain structure:

$$k_y^2\mu^2 \sin^2\varkappa d = (\mu'^2 k_x^2 + \mu^2 k_y^2)\sin^2 k_y d. \quad (3)$$

The solutions of (1) are divided into two classes. Solutions with $k_y^2 > 0$ can naturally be called volume solutions. The spectrum of the volume oscillations is

1)We note that the conditions of translational invariance transform the "levels" $\omega_n(k_x, k_z)$ which occur in the analogous single-domain problems [7], into the bands $\omega_n(k_x, k_z, \kappa)$. The wave number $\kappa$ plays here the role of the corresponding quasimomentum.

determined from (3) and lies in the frequency interval

$$\omega_2^2 = \omega_a(\omega_a + \omega_M \sin^2\theta) < \omega^2 < \omega_a(\omega_a + \omega_M) = \omega_3^2.$$

Solutions with $k_y^2 = -q^2$ are also possible, and constitute surface magnetostatic oscillations. The boundaries of the spectrum of the surface oscillations, with respect to frequency, are determined by the condition $q^2 > 0$ and by the relation $(\mu'^2 k_x^2 - \mu^2 q^2) > 0$, which follows from (3); this yields

$$\omega_1^2 = \omega_a^2 < \omega^2 < \omega_a(\omega_a + \omega_M \sin^2\theta),$$
$$\omega_a(\omega_a + \omega_M) < \omega^2 < (\omega_a + \omega_M)(\omega_a + \omega_M \sin^2\theta) = \omega_4^2. \quad (4)$$

Thus, there exist two branches of surface oscillations.

Let us investigate first the spectrum of the volume oscillations. For the frequencies of the volume oscillations, we have the equation

$$\sin^2\left\{kd\sqrt{\frac{\omega^2-\omega_2^2}{\omega_3^2-\omega^2}}\right\} = \sin^2\varkappa d \frac{(\omega^2-\omega_2^2)(\omega_3^2-\omega^2)}{(\omega^2-\omega_1^2)(\omega_4^2-\omega^2)}. \quad (5)$$

The left side of (5) as a function of $\omega$ at any fixed kd experiences an infinite number of oscillations in the interval $(\omega_2, \omega_3)$. The right-hand side vanishes when $\omega = \omega_2$, $\omega = \omega_3$, and reaches a maximum at point $\omega = \sqrt{\omega_2\omega_3} = \sqrt{\omega_1\omega_4}$. The maximum value is smaller than unity. Thus, at fixed k, $\kappa$, and $\theta$ there exists an infinite set of solutions (5)—"levels"—and the spectrum of the volume oscillations consists of an infinite set of overlapping bands, as expected. We consider now several particular cases.

For arbitrary k and $\kappa$ and for $\theta = 0$ we get

$$\omega_\pm^{(m)} = \left[\frac{\omega_2^2 + \omega_3^2(m\pi/kd \pm \varkappa/k)^2}{1+(m\pi/kd \pm \varkappa/k)^2}\right]^{1/2}_{m=0,1,2\ldots}. \quad (6)$$

When $\theta \neq 0$ the form of the spectrum is similar, but the bands do not coalesce at $\kappa = \pi/d$ (see Figs. 1 and 2). We note only that at $\theta \neq 0$ the branch $\omega^{(0)}$ vanishes if

$$kd < k_0 d = \frac{\omega_1 \cos^2\theta \sin\varkappa d}{\omega_2 |\sin\theta|}.$$

We present also for illustration the form of the branch $\omega^{(0)}$ at $\theta \neq 0$ and $k \ll k_0$:

$$\omega^{(0)} \approx \omega_3 - \frac{kd\omega_M |\sin\theta|}{2\sin\varkappa d}. \quad (7)$$

Let us consider further the spectrum of the surface oscillations. The dispersion equation has in this case the form

$$\mathrm{sh}^2\left\{kd\sqrt{\frac{\omega^2-\omega_2^2}{\omega^2-\omega_3^2}}\right\} = \sin^2\varkappa d \frac{(\omega^2-\omega_2^2)(\omega^2-\omega_3^2)}{(\omega^2-\omega_1^2)(\omega_4^2-\omega^2)} \quad (8)$$

As already mentioned, there are two branches of surface waves[2)]. The high-frequency branch $\omega_I$ (see Fig. 3, upper curve) exists for any k and $\kappa$. In the limiting case of long waves ($kd \ll 1$) we easily obtain for it

$$\omega_I \approx \omega_3 + \frac{\omega_M kd |\sin\theta|}{2\sin\varkappa d}. \quad (9)$$

The character of the spectrum in the long-wave approximation can be understood from intuitive physical considerations. The "activation" frequency $\omega_3 = \sqrt{\omega_a(\omega_a + \omega_M)}$ is the frequency of the homogeneous precession of the magnetic moments inside the domains in the field $H_a = \beta_M$, with allowance for the demagnetizing field of the domain walls.

[2)]The solution $\omega = \omega_2$ ($q = 0$) corresponds, as can be easily verified, to the trivial solution $\varphi \equiv 0$.

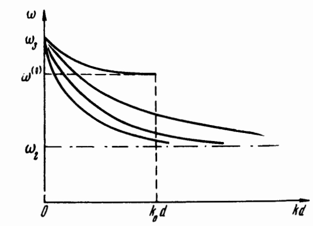


FIG. 1. Dependence of $\omega$ on k at fixed values of $\kappa$ and $\theta$ for volume oscillations.

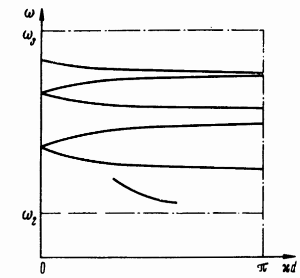


FIG. 2. Dependence of $\omega$ on $\kappa$ at fixed values of k and $\theta$ for volume oscillations.

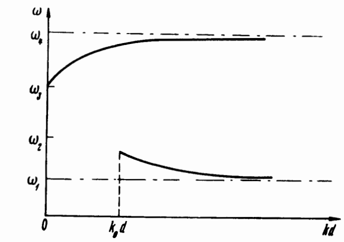


FIG. 3. Dependence of $\omega$ on k for surface waves.

In the case of short waves ($kd \gg 1$), $\omega_I(k)$ approaches asymptotically $\omega = \omega_4$.

This means that on a "solitary" domain boundary there exists a surface magnetostatic wave with frequency $\omega_4$.

The low-frequency surface branch $\omega_{II}$ appears at $k > k_0$ (Fig. 3, lower curve). $\omega_{II} \to \omega_1$ at $kd \gg 1$. This wave is a surface oscillation of the magnetic moment (it is easy to verify that $|h| \ll |m|$). A wave of this type is missing in an individual domain wall, and for its formation it is necessary to have an increasing exponential in the solution of (1).

In conclusion, we note that the complicated spectrum of the oscillations of a magnet with a periodic domain structure has a strong influence on the nonlinear and magnetoelastic effects.

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