Quantization of Electron Energy near a Domain Wall

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As well known, a ferromagnet in the non-magnetized state is stratified into domains [1]. The saturation magnetic moment inside each domain equals $M_0 = M_0(T) \sim 10^2 - 10^3$ Oe. In the transition region between the domains, the magnetic-moment vector is rotated through an angle, corresponding to the domain structure. The width of the transition region (the domain wall) is $\delta \sim 10^{-5} - 10^{6}$ cm. The induction $\vec{B} = 4\pi\vec{M}$ is thus homogeneous inside the domain and inhomogeneous in the domain wall. In a ferromagnet, the induction field $\vec{B}$ plays a role of an external magnetic field relative to the conduction electrons. The characteristic size of the orbit in the homogeneous induction field $\vec{B}_0 = 4\pi\vec{M}_0$ is $R \sim 10^{-2} - 10^{-4}$ cm, i.e., $R \gg \delta$. This makes it possible to identify the conduction electrons by the character of their motion, in the following manner. One group of electrons moves without crossing the domain wall, i.e., in a homogeneous induction field. The other group (near the domain wall) crosses the region of the inhomogeneous induction field; these electrons "feel" the field $\vec{B}_1$ and the field $\vec{B}_2$ ($\vec{B}_1$ and $\vec{B}_2$ are the induction vectors in the neighboring domains).

We consider in this paper the quantization of the conduction-electron energy near the domain wall. We recall that the quantization of electron energy inside the domains is well known (Landau quantization) [2]. Of course, it is necessary here to satisfy the condition $\Omega\tau \gg 1$, where $\Omega = eB_0/mc$ is a cyclotron frequency and τ is the electron free-path time.

It is clear from the foregoing that the motion of an electrons in ferromagnet is determined by the domain structure, i.e., by the relative orientations of $\vec{B}_1, \vec{B}_2$, and the domain wall. The domain structure can be one of two types: 1) The projections B_{1y} and B_{2y}, of the induction vectors $\vec{B}_1$ and $\vec{B}_2$ on the direction perpendicular to the domain wall (the y-axis) are not equal to zero (then $B_{1y} = B_{2y} = B_y$ by virtue of $\operatorname{div}\vec{B} = 0$). 2) The projections of the induction vectors $\vec{B}_1$ and $\vec{B}_2$ on the y-axis are $B_{1y} = B_{2y} = 0$.

In the domain structure of the first kind, the motion of the electrons is in general infinite and aperiodic, owing to the presence of B_y components. As is well known, such a motion is not quantized.

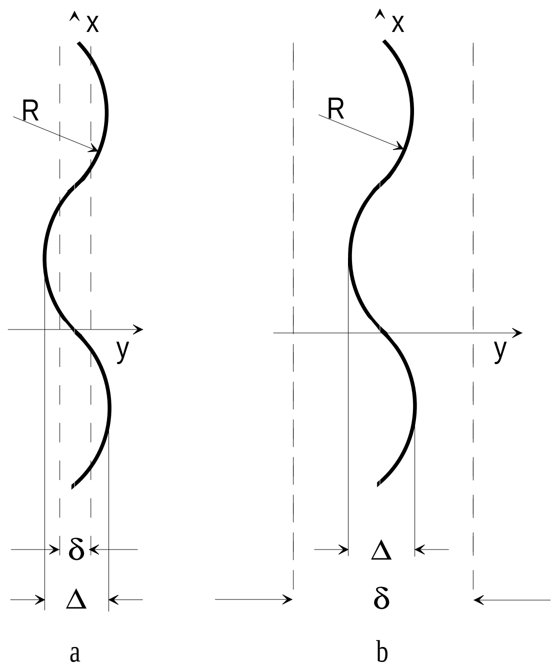

Fig. 1. Trajectory of electron motion near the domain wall: a) $\Delta \gg \delta$, b) $\Delta \ll \delta$.

Let us consider the other type domain structure. The motion of the electrons in the fields $\vec{B}_1$ and $\vec{B}_2$ in the y direction is finite. As a result, the motion in the direction transverse to the domain wall is finite and periodic. Such a motion is quantized, and energy levels are produced spaced a distance $\Delta\epsilon_n \sim \hbar\omega$ apart (ω is the frequency of the classical motion). Let us now estimate the order of magnitude of ω for the electrons near the domain wall in the simplest case. Let the electrons have a quadratic dispersion law, $\vec{B}_1 = -\vec{B}_2 = \vec{B}_0$ and $\Delta \gg \delta$, where Δ is the characteristic dimension of the transverse motion of the electrons. It is clear that $\omega = \Omega\pi/2\phi$ (see Fig. 1); if $\Delta \ll R$, then $\phi \sim \sqrt{\Delta/R} \ll 1$ and $\omega \sim \Omega\sqrt{R/\Delta} \gg \Omega$. Thus $\Delta\epsilon_n \sim \hbar\Omega\sqrt{R/\Delta} \gg \hbar\Omega$. In a field $B_0 \sim 10^2 - 10^3$ Oe we have $\Delta\epsilon_n \sim 1 - 10$ K. It is easily seen that the level system which we consider as an example and which arises in the domain structure with $B_1 = -B_2$ is similar to the system of magnetic structure levels. The exact correspondence is established by the formula $\epsilon_{\rm dom}(n) = \epsilon_{\rm sur}(n/2)$.

It should be noted that, owing to the spin, near the domain wall the conduction electron interacts via exchange with the spin system of the ferromagnet, which is inhomogeneous in the wall. The energy of this interaction is of the order of $\epsilon_0\,(a/\delta)^2$, where $\epsilon_0 \sim 10^4$ K is the Fermi energy and a is the interatomic distance. Thus, $\Delta\epsilon >> \epsilon_0\,(a/\delta)^2$ and the exchange interaction with wall spins can be neglected.

We consider now the solution of the classical problem of electron motion in a domain structure of the second kind, assuming for simplicity a quadratic dispersion law. The motion of the particle in the magnetic field is described by the Hamiltonian equation

$$\dot{\vec{p}} = -\frac{\partial H}{\partial \vec{r}}, \quad \dot{\vec{r}} = -\frac{\partial H}{\partial \vec{p}}, \qquad (1)$$

where $H = \left(\vec{P} - e\vec{A}/c\right)^2/2m$. As already mentioned, $\vec{B} = \vec{B}(y) = \mathrm{curl}\,\vec{A}$. By virtue of the fact that $|B_1| = |B_2| = 4\pi M_0$, the z axis can be chosen such that $B_z(0) = 0$. Then $B_z(+\infty) = -B_z(-\infty) = B_z$,

$B_x(-\infty) = B_x(+\infty) = B_x$, and $B_x^2 + B_z^2 = B_0^2$. We take the vector potential $\vec{A}$ in the form

$$A_x(y) = -\int^y B_z\, dy', \quad A_z = \int^y B_x\, dy', \quad A_y = 0.$$

Since $P_y = p_y$, we obtain for the function H the expression

$$H = \frac{p_y^2}{2m} + U(y),$$

where

$$U\,(y) = \frac{1}{2m}\left\{\left(P_x - \frac{e}{c}A_x\right)^2 + \left(P_z - \frac{e}{c}A_z\right)^2\right\}.$$

From (1) we obtain

$$P_x = p_x + \frac{e}{c}A_x\,(y) = const, \quad P_z = p_z + \frac{e}{c}A_z\,(y) = const,$$

$$\frac{p_y^2}{2m} + U\,(y) = E. \qquad (2)$$

Thus, the motion of the electron along the y axis has been reduced to uniform motion of a particle in a field $U(y)$ with energy E at specified P_x and P_z. Substituting the solution of this problem in (2), we can readily obtain the electron motion along the x and z axis. We consider now in greater detail the case $\Delta \gg \delta$. Then $A_x = -B_z|y|$, $A_z = B_x y$, and we obtain for the field $U(y)$ the expression

$$U\,(y) = \frac{P_x^2 + P_z^2}{2m} - \frac{m\Omega^2 y_0^2}{2} + \frac{m\Omega^2\,(y - y_0)^2}{2},$$

where

$$\Omega = \frac{eB_0}{mc}, \quad y_0 = \frac{c}{eB_0}\,\frac{B_x P_z - B_z P_x \operatorname{sign} y}{B_0} = \frac{B_x R_z - B_z R_x \operatorname{sign} y}{B_0}.$$

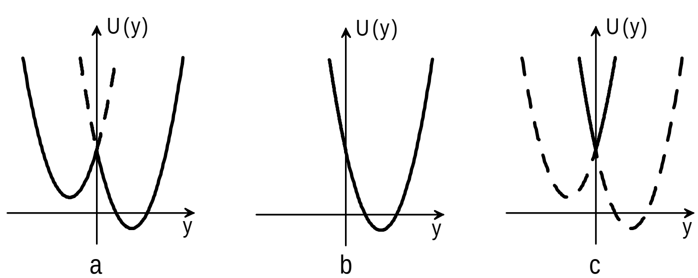

Fig. 2. The field $U(y)$ is represented by the solid line. Depending on the sign of d, it consists of different parts of two identical parabolic potential wells: a) $d < 0$, b) $d = 0$, c) $d > 0$.

Depending on the value of $d = y_0(y < 0) - y_0(y > 0)$, the field $U(y)$ has a different character (see Fig. 2). It is easily to find that $\omega(d > 0) > \omega(d = 0) = \Omega > \omega(d < 0)$, where ω is the frequency of the oscillations in the field $U(y)$. In the case $\Delta \ll R(d > 0)$ we have $\omega \sim \Omega\sqrt{R/\Delta}$. For a domain structure with $B_x = 0$ ($B_1 = B_2$) we get $U(y) = U(-y)$.

Let us consider now the case $\Delta \ll \delta$. Near the origin, in the zeroth approximation in Δ/δ, we have $B_x(y) = B_x'$ and $B_z(y) = B_z'(y/\delta)$, where $B_x' \sim B_z' \sim B_0$. From this we obtain for the vector potential A and the field $U(y)$ the expression

$$A_x = -B_z'\,\frac{y^2}{2\delta}, \quad A_z = B_x' y,$$

$$U(y)=\frac{P_x^2+P_z^2}{2m}+\frac{B_z'eP_x}{2mc\delta}\left(y-\frac{B_x'P_z\delta}{B_z'P_x}\right)^2=\frac{P_x^2+P_z^2}{2m}+\frac{m\Omega'^2}{2}\frac{R_x'}{\delta}(y-a)^2.$$

If $P_x>0$ and $B_x'P_z/B_z'P_x\sim P_z/P_x\ll 1$, then the motion along the axis takes place in a potential well and the condition $\Delta\ll\delta$ is fulfilled. It is clear that the motion in the field $U(y)$ is a harmonic oscillation with frequency $\omega=\Omega'\sqrt{R_x'/\delta}\gg\Omega'\sim\Omega_0$.

The foregoing solution of the classical problem can be generalized in a natural manner to include the quantum case by making the substitution $p_y\to\hat{p}_y=-i\hbar(\partial/\partial y)$ and solving the Schrodinger equation in the field $U(y)$. The resultant quantization of the energy levels is of interest only in the quasiclassical region, and is obtained from the formula $\oint p_y\,dy=2\pi n\hbar$. In the case $\Delta\gg\delta$ we have

$$\epsilon_n=\frac{P_x^2+P_z^2}{2m}+\left(\frac{3\pi}{4}\right)^{2/3}(n\hbar\Omega)^{2/3}\left(\frac{P_x^2}{2m}\right)^{2/3}\left[1-\frac{P_zB_x}{P_xB_z}\right]^{2/3}.$$

If $B_1=-B_2$, we obtain from this

$$\epsilon_n=\frac{P_x^2+P_z^2}{2m}+\left(\frac{3\pi}{4}\right)^{2/3}(n\hbar\Omega)^{2/3}\left(\frac{P_x^2}{2m}\right)^{1/3}.$$

For the case $\Delta\ll\delta$ we get

$$\epsilon_n=\frac{P_x^2+P_z^2}{2m}+n\hbar\Omega\sqrt{\frac{R_x'}{\delta}}=\frac{P_x^2+P_z^2}{2m}+n\hbar\sqrt{\frac{eB_z'P_x}{m^2c\delta}}.$$

Thus, the distance between the levels arising near the domain walls is $\Delta\epsilon_n\approx\sqrt{\hbar\Omega R/\Delta}$, and for an electron moving inside the wall we have $\Delta=\delta$; on the other hand, when $\Delta\gg\delta$ the character of the spectrum (the dependence on n, P_x, P_z) differs from that of the case $\Delta\ll\delta$.

The existence of the considered energy levels leads, for example, to resonant absorption of ultrasound ($\omega\sim 10^{10}-10^{11}\ \mathrm{sec}^{-1}$). A detailed discussion of this effect will be presented in a later paper.

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English translation:

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