

# The Diffusion of Innovations: A Methodological Reappraisal

Manuel Trajtenberg

Department of Economics, Tel Aviv University, Tel Aviv 69978, Israel

Shlomo Yitzhaki

Department of Economics, Hebrew University of Jerusalem, Jerusalem 91905, Israel

Studies of diffusion have traditionally relied on specific distributions, primarily the logistic, to characterize and estimate those processes. We argue that such an approach gives rise to serious problems of comparability and interpretation and may result in large biases in the estimates of the parameters of interest. We propose instead the Gini's expected mean difference as a measure of diffusion speed. We discuss its advantages over the traditional approach, present a nonparametric estimation procedure, and tackle with it the problems of truncated processes and of intergroup comparisons. We also elaborate on the use of the hazard rate and suggest various extensions. The diffusion of computed tomography scanners is presented as an illustration.

KEY WORDS: Computed tomography scanners; Gini's expected mean difference; Innovation diffusion; Logistic distribution; Technological change.

## 1. INTRODUCTION

The diffusion of innovations has long been an important research topic in the context of technological change, and lately it has become a prominent theoretical issue in its own right. Given that the actual benefits from innovations are realized only as they spread throughout the economy, economists have naturally been concerned about the features that characterize—and the forces that govern—these rather lengthy time processes.

Empirical research on diffusion started in earnest with Griliches's (1957) pathbreaking work on hybrid corn. In it, he succeeded in bringing diffusion into the realm of the empirically measurable; that is, he pioneered a way to quantify the phenomenon and capture its essentials with the aid of just a few parameters (that could, in turn, be related to optimizing behavior). Yet, and notwithstanding the prolonged success of that approach, some methodological issues have not been satisfactorily resolved, among them those associated with the reliance on specific probability distributions (chiefly the logistic). Thus we intend to reexamine here some key features of diffusion processes, propose new measurement procedures for them, and try to expand the analysis of the phenomenon with the aid of such measures. The focus is on *aggregate* diffusion—that is, the pattern followed by the cumulative percentage of adopters over time—rather than on the underlying behavioral or *micro* aspects of the process. Within that context, we concentrate on diffusion speed, which is the parameter that has commanded the most attention in the literature.

Following a critical review of current methodology in Section 2, we pursue in Section 3 the prime objective of the article, namely, to put forward Gini's expected mean difference (Gini hereafter) as a convenient measure of diffusion speed, elaborate on its meaning and implications, and outline its conceptual and methodological advantages over traditional measures. Section 4 deals with an intricate problem often encountered in diffusion studies, the measurement of diffusion speed of truncated processes. We develop a procedure based on the Gini that allows us to accurately measure the diffusion speed of just the *observed* segment of the process and to compare processes that have been truncated at different levels and/or estimated with different methods. In Section 5, we elaborate on the use of the *hazard rate* as a simple yet incisive tool to probe further into the nature of diffusion processes.

The actual implementation of these ideas is illustrated in Section 6 via selected results from a case study on the diffusion of computed tomography (CT) scanners, a major innovation in medical technology. Finally, Section 7 sketches three possible extensions, linking diffusion with other topics in the economic and statistical literature.

To repeat, our interest throughout the article centers on measurement issues, reflecting not only an obvious concern with measuring things right, but also the belief that good measurement can breed good theory (the converse is usually taken for granted). In that vein, it seems that the replacement of prevailing distribution-specific estimates by a well-defined measure having general applicability can only foster the upgrading of diffusion theory.

## 2. A CRITIQUE OF CURRENT METHODOLOGY

Typically, empirical studies of diffusion of innovations have assumed that these processes follow a logistic pattern—that is, that the cumulative distribution of adopters as a function of time is

$$F(t) = K/[1 + \exp - (\alpha + \beta t)], \quad (1)$$

where  $K$  stands for the effective ceiling (we assume for now that  $K = 1$ ). Those studies proceed then to estimate the logit transform, using weighted or unweighted least squares (LS),

$$\ln[F(t)/(1 - F(t))] = \hat{\alpha} + \hat{\beta}t. \quad (2)$$

In almost all such studies, LS estimates of Equation (2) result in very high  $R^2$ 's (usually better than .90) and, notwithstanding the voicing of some reservations (e.g., Griliches 1957, p. 505; Mansfield 1968, p. 141), that is taken as convincing evidence upholding the logistic specification.

Now if it could be safely assumed that most diffusion processes of interest do correspond to the logistic distribution (or that even if they do not the resulting misspecification biases in the estimates were negligible), then no major objections could be raised against the received methodology. But, as we shall argue, there are not compelling a priori reasons to sustain this particular specification, nor does the available empirical evidence provide strong support to it. The reliance on (2), and on the estimate  $\hat{\beta}$ , is therefore called into question.

Theoretical considerations underlying those empirical studies have to do mostly with some notion of *demonstration* or *contagion* effects; simply put, the idea is that, because of information-spreading mechanisms and/or competitive pressures, the probability of adoption at any time  $t$  would be related to the proportion of individuals that have already adopted by  $t$ , or, more precisely, that the hazard rate, defined as

$$h(t) = f(t)/[1 - F(t)], \quad f(t) = \partial F(t)/\partial t, \quad (3)$$

would be a positive function of  $F(t)$ . Now if one makes the further assumption that  $h(t)$  is *linear* in  $F(t)$ , then the logistic in (1) obtains simply as the solution to the differential equation

$$f(t) = \beta F(t)[1 - F(t)]. \quad (4)$$

Notwithstanding some attempts to provide (4) with a more rigorous base (see, primarily, Mansfield 1968), it is apparent that this amounts to no more than moving the assumption one step back. To be sure, (4) offers some sensible behavioral interpretation (i.e., the contagion effect), whereas (1) by itself has none. The fact remains, however, that it is an ad hoc assumption rather than an analytical result having firm theoretical underpinnings.

Various attempts were made not long ago to model formally the behavioral phenomena underlying diffusion processes (e.g., Jensen 1982, 1983; Reinganum

1981a,b). Not too surprisingly though, what emerges regarding the issue of concern here is essentially a negative result—namely, that no generalizations are possible with respect to the precise functional form of diffusion curves. Theory may provide, at most, a mapping from basic assumptions on individual adoption behavior to broad *families* of diffusion curves (e.g., S-shaped vs. concave, symmetric vs. asymmetric, etc.), but that is a long way from zeroing in on the logistic.

Now to the empirical evidence. To begin, the logistic specification is by no means the only one that has been used in diffusion studies, although it is certainly the most popular. Examples to the contrary include Bain (1964), lognormal; Coleman, Katz, and Menzel (1966), both logistic and exponential; Dixon (1980), Gompertz; and so forth. More important, in the many cases in which the logistic was used, it is, of course, unwarranted to infer from the high  $R^2$ 's obtained that the logistic is in fact the correct specification. The econometric arguments to that effect are well known and will not be repeated here. Moreover, as was forcefully argued by Feller (1940, 1971), the problem regarding the inferential value of goodness-of-fit measures is much more acute when it comes to estimates of the logistic or of any other probability distribution.

Essentially, the trouble stems from the fact that the observed distribution will always fulfill, by necessity, the restrictions that characterize theoretical distributions, namely,  $0 \leq F(t) \leq 1$  and  $F(t') \geq F(t)$  for all  $t' > t$ . Thus, virtually *any* functional form corresponding to commonly used distributions will offer a good fit and result, *inter alia*, in a high  $R^2$ . The crucial point for our purposes here is that, despite the apparent indifference in terms of fit, the choice of specification is by no means inconsequential for the quality of the estimate of diffusion speed (provided, of course, that speed has been clearly defined in advance, independently of any particular distribution).

To gain some notion of the magnitude of the problem, we ran simulations in which the estimated equations were systematically misspecified, obtaining in all cases very good fits but also large biases in the estimates of diffusion speed (as defined in Sec. 3). In one group of simulations, data were generated by an exponential distribution,  $F(t) = 1 - \exp(-\gamma t)$ , for various values of  $\gamma$  and time lengths, but a logistic (the logit transform) was estimated instead. The  $R^2$ 's ranged from .95 to .99, but the biases in the estimates of diffusion speed were on the order of 50%; the Durbin-Watson statistics were, of course, very low (in the .12-.18 range), reflecting the extent and nature of the misspecification. The following is a typical example: The observations were generated by the distribution  $F(t) = 1 - \exp(-.3t)$  ( $t \in [1, 25]$ ) in intervals of .5. The estimated equation was  $\text{logit} = -.62 + .33t$ ,  $R^2 = .99$ . As will be shown,  $\hat{\beta}$  should be compared to  $2\gamma$ —that is, .33 to .60, indicating a downward bias of almost 50%.

Thus the logistic can by no means be presumed to be

the universal or most accurate representation of diffusion processes and, if used indiscriminately, the estimates might be seriously biased. A possible alternative is to search in each case for the most appropriate specification, but then the question is how to compare between different diffusion processes, bearing in mind that an important goal of these studies is in fact to make consistent and systematic comparisons. This poses a problem because a well-defined concept of speed of diffusion that could readily be computed for any distribution is nowhere to be found. In the case of the logistic,  $\beta$  is usually taken as the appropriate measure, but what are the equivalent parameters in, say, the normal or the lognormal distributions to which  $\beta$  should be compared? On the same note, studies using the logistic have failed to provide  $\beta$  with a clear-cut meaning that will have conceptual and descriptive appeal (calling it the *rate* of diffusion is of no help); they only indicate a way of using  $\beta$  to calculate the time span between two arbitrarily chosen points on the distribution; that is,  $t(F_2) - t(F_1) = (1/\beta)\ln\{[F_2/(1 - F_2)][(1 - F_1)/F_1]\}$  for any  $F_2 > F_1$ . It is rather unsatisfactory that this parameter, which has occupied such a prominent role in diffusion, cannot be grasped on its own terms and depends on an element of arbitrariness for interpretation.

The issue of comparability arises vividly in Dixon (1980). He reexamined Griliches's study of hybrid corn and concluded that the appropriate specification for the majority of states was the Gompertz distribution, defined as

$$F(t) = a^{b^t}, \quad (5)$$

rather than the logistic, as assumed by Griliches. He observed that  $\ln b$  "performs a similar role to the  $b$  [ $\hat{\beta}$  in our notation] coefficient in the logistic function, in that it determines the rate at which  $P_t[F(t)$  in our notation] approaches the ceiling value" (Dixon 1980, p. 1456). That is so, but, more significantly in this context, the parameters  $\hat{\beta}$  and  $\ln b$  are *not exactly equivalent* (see Sec. 3), and therefore what is gained in precision is lost in comparability (it seems that Dixon was aware of this fact, because he ran two separate regressions, one for the states estimated with the Gompertz and a second for those estimated with the logistic).

### 3. THE GINI COEFFICIENT AS A MEASURE OF DIFFUSION SPEED

As already stated, the main purpose of this article is to propose the Gini's expected mean difference as a highly convenient summary parameter of diffusion processes. We contend that this statistic exhibits definite conceptual and methodological advantages over traditional measures, allowing it to overcome the difficulties described previously and to further extend the study of diffusion. The Gini is defined as

$$\Gamma = 1/2 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |x_1 - x_2| f(x_1) f(x_2) dx_1 dx_2, \quad (6)$$

where  $x_1$  and  $x_2$  are two independent, identically distributed random variables (or equivalently, two random realizations of the same variable,  $x$ ). Note that, since (6) is a double integral but  $x_1$  and  $x_2$  correspond in this case to the same variable  $t$ , it has to be divided by 2 so as to avoid "double counting." In the context of diffusion, the variable of interest is obviously time,  $t_1$  and  $t_2$  being the dates of any adoptions. Writing (6) in a slightly different way,

$$\Gamma = \int_{-\infty}^{\infty} \int_{t_1}^{\infty} (t_2 - t_1) f(t_2) f(t_1) dt_2 dt_1, \quad (7)$$

its meaning becomes transparent: *The Gini measures the expected time difference between any two adoptions over the whole diffusion process.* Or, to put it differently, it is the expected *waiting time* of a random potential adopter at a random point in time (within the relevant time period). This constitutes a well-defined notion of speed of diffusion and has general applicability; that is, it is not contingent on specific distributions or any other peculiarities of the processes studied. Notice that speed is commonly defined in terms of distance per unit time, except when the distance is a given, in which case the reciprocal is most often used. In diffusion, the distance is given by the unit interval, and hence it makes more sense to define speed in terms of  $\Gamma$  rather than  $1/\Gamma$ .

Before turning to a more general measurement procedure, it is worth showing what the Gini is for those distributions commonly used in diffusion. This will not only help estimate the Gini in those cases in which the underlying distribution is known, but it will also allow us to bring the results obtained in previous studies to a common denominator.

*The Logistic Distribution.* A straightforward way of proceeding in this case is to use the following representation of the Gini:

$$\Gamma = \int_{-\infty}^{\infty} F(t)[1 - F(t)] dt \quad (8)$$

(see Kendall and Stuart 1977; Lomnicki 1952). Integrating (4),

$$\int_{-\infty}^{\infty} f(t) dt = \beta \int_{-\infty}^{\infty} F(t)[1 - F(t)] dt = \beta\Gamma. \quad (9)$$

But the left side of (9) equals 1; therefore,

$$\Gamma = 1/\beta. \quad (10)$$

*The Exponential Distribution.* Substituting in (8),

$$\Gamma = \int_0^{\infty} e^{-\gamma t}(1 - e^{-\gamma t}) dt \Rightarrow \quad (11)$$

and

$$\Gamma = 1/2\gamma. \quad (12)$$

The Gini for the normal and lognormal distributions

are well known, and that for the Gompertz is derived in the Appendix. Table 1 shows them in a condensed form.

In view of these results, we can now reexamine the problem raised by Dixon (1980); the equivalent parameters there are not  $\beta$  (from the logistic) and  $\ln b$  (from the Gompertz), but  $\beta$  and  $\ln b/.7$  (more precisely, their reciprocals). The diffusion processes of *all* states can be thus compared, regardless of which distribution was fitted. Likewise, there is no longer need to split the states into two groups to assess the effect of profitability variables on interstate differences in diffusion speed, but a single regression having the Gini as dependent variable will do.

As we argued before, however, in most cases there is no good prior regarding the specific form of the underlying distribution, and the search for the correct specification can be cumbersome and often inconclusive. In this respect as well, the Gini proves to be highly advantageous, because it is possible to construct a simple, distribution-free measure of it [recall that the Gini is a  $U$  statistic, so its estimator is unbiased and its distribution converges to the normal for large samples (see, e.g., Hoeffding 1948)]. For that purpose we integrate Equation (8) by parts, defining first  $u = t$ ,  $v = F(t)[1 - F(t)]$ , and  $v' = f(t)[1 - 2F(t)]$ ; then

$$\Gamma = tF(t)[1 - F(t)] \Big|_{-\infty}^{\infty} - \int_{-\infty}^{\infty} tf(t)[1 - 2F(t)] dt. \quad (13)$$

But the first term vanishes and, rearranging the second term,

$$\Gamma = 2 \int_{-\infty}^{\infty} t[F(t) - .5]f(t) dt. \quad (14)$$

Noting that  $dF = f(t) dt$  and changing variables accordingly,

$$\Gamma = 2 \int_0^1 t(F)(F - .5) dF = 2 \text{cov}(t, F), \quad (15)$$

since  $F$  distributes uniformly along the interval  $[0, 1]$ . If detailed information on each adoption is available—that is, if the data consist of the vector  $(t_1, t_2, \dots, t_n)$ , where  $t_i$  is the adoption date of individual  $i$ —then (15) can be computed simply by

$$\hat{\Gamma} = 2 \text{cov}(\bar{F}, t), \quad (16)$$

where  $\bar{F} = R_i/n$  and  $R_i$  is the rank of  $t_i$ . But the data are often aggregated in discrete time periods, in which case the integral in (15) has to be approximated linearly.

$$\Gamma \cong 2 \sum_{i=1}^n \tilde{t}_i (\hat{F}_i - .5) \Delta F_i,$$

$$\tilde{t}_i = (t_i + t_{i+1})/2, \quad \hat{F}_i = (F_i + F_{i+1})/2, \\ \Delta F_i = (F_{i+1} - F_i). \quad (17)$$

Table 1. The Gini for Various Distributions

Distribution <sup>a</sup>	Gini
Logistic: $F(t) = 1/[1 + e^{-(a+bt)}]$	$1/\beta$
Exponential: $F(t) = 1 - e^{-t}$	$1/2\gamma$
Normal: <sup>b</sup> $t \sim N(\mu, \sigma^2)$	$\sigma/\sqrt{\pi}$
Lognormal: <sup>c</sup> $\ln t \sim N(\mu, \sigma^2)$	$e^{\mu+\sigma^2/2} [2\phi(\sigma/\sqrt{\pi}) - 1]$
Gompertz: $F(t) = a^{bt}$	$.7/\ln b$

<sup>a</sup> For convenience, we avoid writing explicitly the cumulative distributions for the normal and lognormal distributions.

<sup>b</sup> See Nair (1936).

<sup>c</sup> See Aitchison and Brown (1963);  $\phi$  represents the standard normal.

It is important to point out that, as with any other nonparametric method, the computation of the Gini in a distribution-free manner entails an efficiency loss. The extent of it, however, and hence the ensuing efficiency-bias trade-off, cannot be actually assessed unless the underlying distribution is known. But if the distribution were indeed known, then we would not go about computing the Gini in a nonparametric fashion; instead we would first estimate the distribution and then compute the Gini from those estimates (as shown in Table 1). The advantage of the Gini in that case is that it allows for consistent comparisons across processes that correspond to different distributions. Still, the loss of efficiency—and the trade-off—entailed by the nonparametric method deserves further examination, but that is beyond the scope of this article.

Finally, a word of caution regarding the reliance on speed, however measured, as a characterization of diffusion processes. Any attempt to subsume a complex phenomenon such as diffusion into a single parameter will result, perforce, only in a partialized picture. Therefore, we do not advocate the use of speed to the exclusion of other measures of diffusion; other features, such as the mean adoption time or the hazard rate, may certainly be of interest as well (in fact, we do resort to them later on). Moreover, speed represents some sort of average over the whole population of adopters and as such might be quite sensitive to outliers, particularly those occurring at the very beginning or at the final stages of the diffusion process. For example, a few "pioneers" (very early adopters) followed by a long waiting period, or a few stubborn holdouts, might distort any measure of speed, as well as any other reasonable characterization of the process as a whole.

#### 4. ESTIMATING THE SPEED OF DIFFUSION OF TRUNCATED PROCESSES

We have assumed that the diffusion process is observed in its entirety—that is, that the data comprise the whole distribution  $0 \leq F(t) \leq 1$ . In many actual cases though,  $F(T) = p < 1$ , where  $T$  is the last date for which data are available. Now, if it could be safely assumed that the process is near completion by  $T$ —that is, that  $F(t)_{t \rightarrow \infty} = K = p + \zeta$ , where  $\zeta$  is a small fraction of  $K$ —then it is possible to estimate  $K$  (usually referred to as the effective ceiling) along with the other diffusion parameters. That necessarily entails the making of as-

assumptions regarding the overall course of the process, most likely on the basis of its observed behavior up to  $T$ —for example, assuming a logistic distribution and estimating it using maximum likelihood or other non-linear methods (see, e.g., Jarvis 1981; Trajtenberg 1983). The quality of the estimates so obtained will obviously depend on the validity of those assumptions (and thus be subjected to the same reservations raised previously) and on how small  $\xi$  is.

It is worth pointing out that what this case implies is that the population of potential adopters was not correctly identified at the outset; the  $(1 - K)$  that did not and presumably will not adopt must have some distinctive characteristics that set them apart in terms of their behavior vis-à-vis the innovation. Thus what is ultimately important is to identify those characteristics and delimit accordingly the “right” population set; a finding that  $K < 1$  does not resolve the issue only indicates that we have insufficient information.

A more serious difficulty arises when there is no indication that the process is near completion by  $T$  (that in turn implying that  $p \ll 1$  and that there is no reason to believe that, if  $K < 1$ ,  $\xi$  is small) and no prior regarding the shape of the whole distribution. Obviously, the only safe, if trivial, solution is to wait until more data become available. But this just evokes a basic tension that arises only too often in empirical research having a claim for “relevance”—the longer the wait and hence the more complete and accurate the data, the more removed the study will turn out to be from current concerns, including those having policy implications. In studies of diffusion, the dilemma is made particularly acute in view of the long time span of those processes (10–30 years).

Clearly, any attempt to estimate the parameters of diffusion in these circumstances will render less than satisfactory results. The main contribution of the Gini in this respect is that it allows us to accurately measure the speed of diffusion of the *observed* segment of the process independent of any assumptions (implicit or explicit) regarding its future trajectory. That is not possible if instead a particular distribution is assumed and estimated on the basis of the truncated distribution. Partitioning the overall time span into two periods, the observed  $(-\infty, T)$  and the unobserved  $(T, +\infty)$ , (8) can be rewritten as

$$\Gamma = \int_{-\infty}^T F(t)[1 - F(t)] dt + \int_T^{\infty} F(t)[1 - F(t)] dt \equiv \Gamma^o + \Gamma^u. \quad (18)$$

Integrating  $\Gamma^o$  by parts as in (13),

$$\Gamma^o = tF(t)[1 - F(t)] \Big|_{-\infty}^T + 2 \int_{-\infty}^T t[F(t) - .5]f(t) dt$$

$$\begin{aligned} &= p(1 - p)T + 2p^2 \int_{-\infty}^T t \left[ \frac{F(t)}{p} - \frac{.5}{p} \right] \frac{f(t)}{p} dt \\ &= p(1 - p)T + 2p^2 \int_0^1 t(F^*)(F^* - .5) dF^* \\ &\quad - p(1 - p) \int_{-\infty}^T tf^*(t) dt, \end{aligned} \quad (19)$$

where an asterisk indicates that the distribution has been normalized (i.e.,  $F^* = F/p$  and  $f^* = f/p$ ) and  $\bar{t} = \int_{-\infty}^T tf^*(t) dt$  is the average time length of the observed period. Thus

$$\Gamma^o = p^2\Gamma^* + p(1 - p)(T - \bar{t}), \quad (20)$$

where  $\Gamma^* = 2 \text{cov}(F^*, t)$  is the Gini of the observed segment, calculated as if it were a complete process in itself (which is the implication of having normalized the distribution); that is, it measures the speed of diffusion among those that adopted up to  $T$ , ignoring the fact that they are only a subset of the population of potential adopters. What  $\Gamma^o$  does is to correct for that fact, thus measuring the contribution of the  $p \cdot N$  initial adopters to the Gini of the entire process,  $N$  being the size of the total population. Equation (20) can be easily generalized to accommodate alternative assumptions regarding the ceiling; that is,  $\Gamma^o(K) = (1/K^2)[p^2\Gamma^* + p(K - p)(T - \bar{t})]$ . Note also that (20) is one of the many forms that the decomposition of the Gini can take (see Yitzhaki 1982a).

This is as much as the data can tell without imposing additional structure on it and, as will be shown, there are various ways in which these partial measures can be used for comparative purposes. If there exists additional information that allows formation of priors regarding the unobserved segment of the process (i.e., regarding  $\Gamma^u$ ), however, then the Gini of the entire process,  $\Gamma$ , can be readily obtained by simply adding the prior to  $\Gamma^o$ . The advantage of this procedure over the fitting of a particular distribution to the whole process is that it keeps observed phenomena strictly separate from conjectures (or projections). Thus it is possible to ascertain in a straightforward manner the effect on  $\Gamma$  of different sets of assumptions regarding the remaining diffusion path without these distorting the measure of the observed segment.

An assumption often made in these circumstances is that the process will exhibit in the future the same behavior, on average, as it did up to  $T$  (we call this the *uniformity assumption*), or, more precisely, that

$$\frac{1}{F(T')} \int_{-\infty}^{T'} F(t)[1 - F(t)] dt = \Gamma \quad \text{for all } T' \geq T. \quad (21)$$

It is easy to show that this property holds for the logistic, but for our purposes we need not assume that  $F(t)$  corresponds to that distribution over its entire range, only that (21) holds on average over the period following  $T$ . The estimate of  $\Gamma$  under this assumption

is simply

$$\hat{\Gamma} = \Gamma^o/p = p\Gamma^* + (1-p)(T - \bar{t}). \quad (22)$$

Note that (21) and (22) have important implications for comparing the partial measure  $\Gamma^o$  to the existing body of results from past diffusion studies, which constitutes the only readily available and natural reference group. Given that in most cases the original data used in those studies are not available but only the published estimates of the diffusion parameters (and therefore  $\Gamma^o$  cannot be computed for them) and that in all of them the estimates refer to whole processes (either because the data were indeed complete or because the truncation problem was assumed away), it is imperative to bring these estimates and  $\Gamma^o$  to common ground. This involves using the published estimates to evaluate the integral  $\int_{T-\infty}^T F(t)[1 - F(t)] dt$  [ $T' = T(p)$ ]. As suggested previously, if the estimated distribution was a logistic, then this integral is simply  $p/\beta$  (it is  $p/2\gamma$  in the case of an exponential distribution), but other distributions require that the integral be evaluated numerically. Thus  $\Gamma^o$  can be compared to, say,  $p/\beta$  without this requiring any assumptions regarding  $\Gamma^u$ . This simple result greatly facilitates the required comparisons, more so in view of the fact that most previous studies did use the logistic.

Noting that comparing  $\Gamma^o$  to  $p/\beta$  is formally equivalent to comparing  $\Gamma^o/p = \hat{\Gamma}$  to  $1/\beta$ , it could be argued that there is no way of escaping the uniformity assumption. But this is not so; the comparison of  $\Gamma^o$  to  $p/\beta$  places the uniformity assumption (i.e., the burden of proof) on the other process (the one that  $1/\beta$  corresponds to), and that represents no extra restriction; the assumption, justified or not, was there to begin with, implied in the fitting of a logistic distribution.

A different case arises when original data [i.e., the vectors  $F(t), t$ ] for all of the processes to be compared are actually available. This is the likely situation when the diffusion of a particular innovation is being studied, but the total population of adopters is divided into subsets, each generating its own process, and the objective is to compare between them (e.g., Griliches's study of hybrid corn by geographical areas). Suppose that there are  $m$  such processes, and that they have reached levels  $p_1, p_2, \dots, p_m$  ( $p_i \geq p_j$ ) by  $T$ . As before, if reasonable conjectures can be made on the  $\Gamma^u$ 's, then the processes can be compared in terms of their estimated  $\Gamma_i$ 's ( $\Gamma_i^o + \Gamma_i^u$ ), where the  $\Gamma_i^o$ 's are calculated using all of the data available for each process. Otherwise, the processes have to be truncated at the same cut-off level:  $p_o = \min_i (p_1, p_2, \dots, p_m)$ , and the  $\Gamma_i^o$ 's (in which terms the comparisons are to be made) are calculated using only the  $p_o \cdot n_i$  initial observations of each process. This entails losing information at the upper end of those processes with relatively high  $p_i$ 's, which is the cost to be paid for not resorting to assumptions regarding the unobserved segment of the process. There is no way of avoiding this trade-off and no dominant strategy; the

course of action to be taken will depend on the particulars of each study. Note that truncating the process at a common level,  $p_o$ , usually implies  $T_i \neq T_j$  and  $\bar{t}_i \neq \bar{t}_j$  and that these are parameters of interest in themselves. Now, if  $(T_i - \bar{t}_i) \cong (T_j - \bar{t}_j)$ , then  $\Gamma_i^o = p_o^2 \Gamma_i^* + p_o(1 - p_o)(T - \bar{t})$ , and hence there is no need to compute  $\Gamma_i^o$ , but  $\Gamma_i^*$  (which enjoys some advantage in interpretation) suffices for comparative purposes.

It is important to stress again the partial and hence tentative nature of all these comparisons; as more data become available, the measures ought to be revised and the comparisons redone.

Finally, it should be clear that the procedures described here apply to any case in which any part of the distribution is missing, not just its upper end. In Russell (1977), for example, the data on the initial stages of two of the innovations are severely lacking (one starts at 19% and the other at 48%), which is not an uncommon situation; data on diffusion are often gathered only after the innovation has become important enough and hence widely spread.

## 5. THE ADOPTION RATE

So far we have been concerned exclusively with the measurement of the speed of diffusion as a one-parameter representation of the diffusion process. The next problem is whether it is possible to learn more about the process itself, still within the same restrictive framework—that is, having data only on adoption dates (or percentages of adopters in discrete time periods)—and without imposing additional structure on it. The answer is a qualified “yes”; there are obviously numerous properties of the observed diffusion process that could be sought and at least as many statistical tests that could be applied to them. But apart from purely descriptive purposes, it is worth investigating a characteristic only insofar as it enhances the understanding of diffusion as a socioeconomic phenomenon, or if it gives some indication for further research. Of course, it is theory's customary role to provide guidance in that respect, but, as stated previously, the results are found lacking. Therefore, any further step taken in this direction will be necessarily tentative, and no general conclusions can be expected.

We want to suggest the behavior over time of the hazard rate (which could be appropriately relabeled in this context as the *adoption rate*) as a likely candidate for investigation [see Barlow and Proschan (1975) for a description of the properties of the hazard rate and Chandra and Singpurwalla (1981) for the relationship between the hazard rate and the Gini]. To recall, the adoption rate is formally defined as  $h(t) = f(t)/[1 - F(t)]$ . By analogy to its meaning in reliability theory,  $h(t)$  can be interpreted here as the conditional probability of adoption at  $t$  for what has been a holdout until then. Now, if  $h(t) = \gamma$ —that is, if the conditional probability is constant over time—then the underlying distribution is necessarily exponential, suggesting that the

contagion—or demonstration—effect (i.e., the direct influence of previous adopters on would-be adopters) is *not* the predominant force driving the diffusion process. On the other hand, if  $h'(t) > 0$ , the corresponding distribution is *likely* to be S-shaped, and the contagion effect *may* be at work, but no solid inferences can be drawn without further information [the case of  $h'(t) < 0$  does not seem to be relevant in diffusion].

A good example is provided in the study of Coleman et al. (1966) on the diffusion of the use of a new drug among physicians. They used the adoption rate (without referring to it as such) to distinguish between what they called “snowball” versus “individual” diffusion processes. The most telling finding is that the adoption rate of socially integrated doctors increases sharply over time, whereas that of isolated doctors oscillates without displaying a trend. Thus they concluded that the contagion effect [associated with  $h'(t) > 0$ ] is at work in the former case but not in the latter, for which  $h'(t) = 0$ . Following the same reasoning, they characterized the diffusion process among socially integrated doctors as logistic and that of isolated doctors as exponential. Note that what allows the authors to draw these conclusions is not just the sign of  $h'(t)$  but the fact that adopters were separated into subgroups according to variables reflecting the extent of their social integration, which has a direct bearing on the plausibility of the demonstration effect.

The simplest procedure will thus be to regress  $h(t)$  on  $t$  [and/or on  $F(t)$ ] and test for the significance of the slope coefficients. But this will be meaningful only if the diffusion process displays a uniform behavior over time. In some cases, however, the observed distribution results from the concatenation of different sequential processes, each initiated in response to discrete changes in the exogenous variables governing the adoption decision (e.g., major technological improvements, jumps in prices, changes in government policies, etc.). This sort of phenomenon cannot be captured in the simple correlation between  $h(t)$  and  $t$ , but will most likely show up in a plot of  $h(t)$  on  $t$ . As will become apparent in the following empirical illustration, the visual inspection of such plots can be highly informative and can provide the researcher with much-needed guidance for analyzing the diffusion process.

## 6. AN EMPIRICAL ILLUSTRATION: THE DIFFUSION OF CT SCANNERS

The approach laid out in the preceding sections can be best appraised by applying it to a concrete case. For this purpose, we have chosen the diffusion of CT scanners in U.S. hospitals. CT is a revolutionary diagnostic technology that produces highly detailed and accurate pictures of thin “slices” of any section of the human body using a sophisticated configuration of X rays, detectors, and computers. Developed in the late 1960s, the first operational prototype was built by the British firm EMI in 1971, and the first commercial installation

in the United States took place in June 1973. It has commanded a great deal of public attention ever since, partly because of its scientific merits, but also because of growing concerns that this kind of expensive advance in medical technologies may have been fueling the rapid rise in health-care costs. Acting on this belief, the government enacted in the mid-1970s a series of regulations designed to slow down the diffusion of CT scanners, but it relaxed them a few years later. It is still a matter of controversy whether diffusion was indeed too fast and whether those regulations have had a noticeable effect on it. Thus the interest in the case is not only academic, but it has policy implications as well.

The data used in this study consist of the adoption dates (month, year) of all first scanners installed in community hospitals during the period 6/73–12/81 [these data are taken from Trajtenberg (1983); see Table 2], and supplementary information from the American Hospital Association (AHA) Annual Surveys of Hospitals. Table 3 shows the distribution of hospitals and adopters categorized by number of beds. Given that only 1.2% of hospitals with fewer than 100 beds had CT scanners, we decided to exclude them from the study. Nonetheless, the diffusion process for community hospitals with more than 100 beds (hereafter re-

Table 2. Data on the Adoption of CT Scanners

Month <sup>a</sup>	Adopters <sup>b</sup>	Month	Adopters	Month	Adopters
7	1	44	12	77	10
8	1	45	14	78	13
10	3	46	23	79	17
11	1	47	12	80	7
14	4	48	10	81	17
15	1	49	20	82	14
17	1	50	23	83	13
18	1	51	24	84	8
19	2	52	19	85	10
20	2	53	23	86	19
21	2	54	35	87	9
22	1	55	35	88	17
23	4	56	34	89	8
24	5	57	34	90	12
25	4	58	26	91	8
26	5	59	21	92	12
27	7	60	16	93	7
28	1	61	15	94	13
29	5	62	20	95	8
30	12	63	13	96	10
31	12	64	13	97	7
32	12	65	19	98	6
33	14	66	15	99	8
34	8	67	14	100	12
35	21	68	16	101	13
36	10	69	15	102	13
37	9	70	12	103	8
38	17	71	11	104	8
39	15	72	14	105	12
40	8	73	13	106	16
41	14	74	6	107	13
42	18	75	17	108	11
43	17	76	10	109	1

<sup>a</sup> Number of months elapsed from November 1972 (the date of announcement of CT).

<sup>b</sup> Number of community hospitals with more than 100 beds that adopted their first CT scanner during that month.

Source: Data collected by M. Trajtenberg.

Table 3. Distribution of Community Hospitals and Adopters of CT Scanners by Number of Beds

Number of beds	Total number of hospitals	Hospitals with CT scanners	$p$ : (2) + (1)
Up to 99	2,848	34	.012
100-199	1,417	187	.132
200-299	719	285	.396
300-399	384	254	.662
400-499	244	201	.824
500+	314	275	.876
Total	5,926	1,236	.209
100+	3,078	1,202	.390

NOTE: The definition of community hospitals used by the AHA is "Non-federal, short-term general and other special hospitals, excluding hospital units of institutions." This excludes approximately 1,200 long-term and/or federal hospitals, of which only 37 had CT scanners.

ferred to just as *hospitals*) was far from complete by the end of 1981 ( $p = .39$ ), and, therefore, the methods developed in Section 4 for truncated distributions are called into action.

The previous statement implies having an idea of the value of the ceiling  $K$ , and indeed, although it is difficult to venture a point estimate, there are clear indications of what the bounds might be. Trajtenberg (1983) studied the effect of technological change on the diffusion process of CT scanners and found that the ceiling was highly responsive to successive improvements in the technology. Actual estimates of these effects show that, considering the innovations that took place up to 1982, the ceiling stood then at almost 50%. Assuming that technological advance will continue for another decade at the same pace as in 1982 (probably a conservative assumption), the ceiling will rise as a consequence by five points. In addition, regulatory measures restraining the adoption of CT scanners have been eased in the early 1980's and, according to preliminary evidence, this change will bring in at least 5% additional adopters. Thus a lower bound for the final ceiling will be 60%. As to the upper bound, it is safe to assume that the ceiling for CT scanners will not surpass the .84 mark reached by diagnostic radioisotopes, a previous innovation in imaging technology that has already completed—*asymptotically*—its diffusion process (the .84 figure is taken from the 1977 survey of the AHA). Therefore, we shall consider in the computations  $K = .60$ ,  $K = .84$ , and, as a midpoint,  $K = .70$ . As we shall see, none of the qualitative results depend on the specific value of  $K$  in that range.

As a first step, we proceed to estimate the diffusion speed of CT scanners to compare it with other innovations. To recall, if we want to avoid making assumptions about the future course of the process, the comparisons are to be based on the partial measure  $\Gamma^\circ$  [see Eq. (20)], which, allowing for different  $K$ 's, can be written as

$$\Gamma^\circ(p/K) = (p/K)[(p/K)\Gamma^* + (1 - (p/K))(T - \bar{t})]. \quad (23)$$

The availability of very detailed data allowed us to compute  $\Gamma^*$  using (16) rather than (17) (i.e., using ranks), rendering a value of  $\Gamma^* = 12.89$  (recall that the time unit is months); that is, the average time difference between any two adoptions for the set of hospitals that adopted CT scanners up to  $T = 12/81$  was of slightly more than a year. Plugging it in (23),  $\Gamma^\circ$  was computed for the various  $K$ 's, and the results are shown in Tables 4 and 5. In Table 6 we compare these with estimates of the diffusion speed of 20 innovations reported in the literature. All of them were estimated using the logistic specification and, as shown in Section 4, the transformation of the reported  $\hat{\beta}$ 's that allows comparability with our figures is simply  $\Gamma^\circ(p/K) = (p/K)/\hat{\beta}$ . Figure 1 summarizes the results: The diffusion of CT scanners was indeed quite fast; that is, it belongs to the fastest third of the innovations studied (this is true for the three alternative ceilings considered). Moreover, it was more than three times faster than the diffusion of the other two reported innovations in diagnostic technologies (electroencephalograph and diagnostic radioisotopes). Needless to say, this finding does not—and could not—resolve the question of whether or not the diffusion of CT scanners was *too* fast. Nevertheless, it is informative and relevant for that issue, because it provides a factual perspective on the standing of CT scanners relative to other innovations. It is important to note that such perspective would have been seriously distorted had we used the logistic to estimate speed instead of the Gini; comparing the estimates in Table 5 to those obtained with the Gini in Table 4, the logistic overstates the speed of diffusion by 50%–80%. According to these later results, CT scanners would have been the second fastest in the set of 20 innovations shown in Table 6, the first being tin containers in Mansfield's study (which is a puzzling outlier). This underlines our contention that the two methods can lead to very different conclusions in a comparative context.

We examine now the behavior of the adoption rate, which, as argued in Section 5, may provide some insight into the nature of the diffusion process itself. Regressing it on time and on  $F(t)$  (using  $K = .84$ ),

$$h = .003 + .000058t, \quad r^2 = .20, \quad (24)$$

(3.5)            (4.9)

and

$$h = .004 + .01F(t), \quad r^2 = .19. \quad (25)$$

(7)            (4.7)

Table 4. Estimates of Diffusion Speed of CT Scanners, Using the Gini

$K$	$\Gamma^\circ(K)$ (in months)	$\Gamma^\circ(K)$ (in years)
.84	14.06	1.17
.70	15.20	1.27
.60	15.77	1.31

Table 5. Estimates of Diffusion Speed of CT Scanners, Logistic Estimates

$K$	$\hat{\beta}$	$r^2$	Implied Gini: $\Gamma^o = (p/K)/\hat{\beta}$ (in years)
.84	.059	.85	.657
.70	.061	.86	.760
.60	.063	.88	.860

These results are informative only in a negative sense; (25) makes it highly unlikely that the process as a whole corresponds to the logistic because of (a) the very low  $r^2$ , which indicates that  $h(t)$  was not smoothly increasing with  $F(t)$ , as it should have been if the underlying distribution was a logistic, and (b) the very small coefficient of  $F(t)$ , which, appropriately transformed, renders an estimate of diffusion speed *three times* larger (i.e., three times slower) than that obtained previously with

the Gini (for  $K = .84$ ). Likewise, (24) rules out the possibility that the process conforms to an exponential-type distribution because the time coefficient, although very small, is nevertheless significant. Thus, a more complex pattern is suggested. The plot of  $h(t)$  on time (see Fig. 2) clarifies the issue; there was a sharp discontinuity midway along the diffusion path (in the third quarter of 1977), the process behaving very differently before and after. In the first period  $h'(t) \geq 0$ , suggesting an S-shaped distribution (and the underlying demonstration effect), whereas afterward there is a drop in the level of  $h(t)$ , and  $h'(t) \approx 0$ , implying a slowdown in the process and an exponential-like pattern thereafter. Following this lead, we computed separate  $\Gamma^o$ 's for each period and found that the first was faster than the second by a factor of 2. Furthermore, the logistic fits well the first period, rendering an estimate of diffusion speed consistent with the partial Gini. A discus-

Table 6.  $\hat{\beta}$  and  $\Gamma^o(K)$  for 20 Innovations

Innovation	(1) Reported $\hat{\beta}$	(2) $\Gamma^o(K = .84)$	(3) $\Gamma^o(K = .7)$	(4) $\Gamma^o(K = .6)$
Griliches (1957) <sup>a</sup> Hybrid corn	.54	.86	1.03	1.20
Jarvis (1981) Improved pastures in Uruguay <sup>b</sup>	.55	.84	1.01	1.18
Mansfield (1968) Bituminous coal mining:				
Shuttle car	.32	1.45	1.74	2.03
Trackless mobile loader	.32	1.45	1.74	2.03
Continuous mining loader	.49	.95	1.14	1.33
Iron and steel:				
By-product coke oven	.17	2.73	3.28	3.82
Continuous wide-strip mill	.34	1.37	1.64	1.91
Continuous annealing line for tin plate	.17	2.73	3.28	3.82
Brewing:				
Pallet loading machine	.55	.84	1.01	1.18
Tin container	2.40	.19	.23	.27
High-speed bottle filler	.36	1.29	1.55	1.81
Railroads:				
Diesel locomotive	.20	2.32	2.79	3.25
Centralized traffic control	.19	2.44	2.93	3.42
Car retarders	.11	4.22	5.06	5.91
Romeo (1975) <sup>a</sup> Numerically controlled machine-tools	.35	1.33	1.59	1.86
Russell (1977) <sup>a</sup> Postoperative recovery room	.31	1.50	1.80	2.10
Intensive care unit	.30	1.55	1.86	2.17
Electroencephalograph	.12	3.87	4.64	5.42
Diagnostic radioisotopes	.13	3.57	4.29	5.00
Respiratory therapy	.41	1.13	1.36	1.59

NOTE: The  $\hat{\beta}$ 's are the logistic coefficients reported in the studies listed here (the time unit is years). In columns 2-4 the  $\hat{\beta}$ 's are transformed into the partial measure  $\Gamma^o(p/K)$  for different  $K$ 's to enable comparisons with CT scanners. If  $F$  is logistic then  $\Gamma^o(p/K) = \int_0^1 F(1-F) dt = p/K\hat{\beta}[T' = T(p/K)]$ . According to Table 3,  $p = .39$ ; hence  $\Gamma^o = .39/K\hat{\beta}$ .

<sup>a</sup> We have taken the average of the group coefficient; states in Griliches, industries in Romeo, and different classes of hospitals in Russell (the estimates for hospitals with less than 100 beds were excluded). This is not equivalent to the coefficient of the aggregate process, but it is a good enough approximation for our purposes.

<sup>b</sup> Jarvis tried several specifications, but the coefficients did not vary much; the range was .51-.59 with an average of .55, which is the figure reported here.

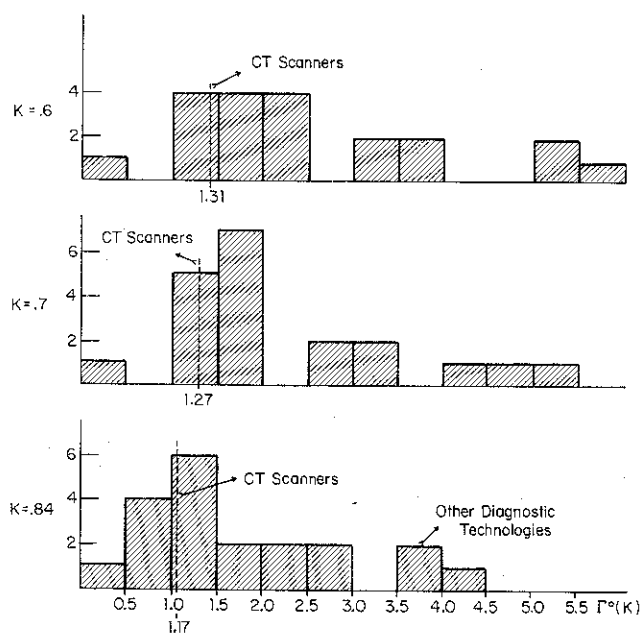


Figure 1. The Frequency Distribution of  $\Gamma^*(K)$  for 20 Innovations. Source: Tables 4 and 5 and Table 6, columns (2)-(4).

sion of the causes underlying this rather dramatic change is beyond the scope of this article; suffice it to say that it had to do with the implementation of government regulations (and prior expectations in that regard) and with a large drop in the pace of technical advance. The important point is that, had we proceeded according to the received methodology, we would have probably overlooked this crucial feature of the diffusion process [as can be seen in Fig. 3, the plot of  $F(t)$ , often shown in diffusion studies, does not reveal it either] and assuming that a particular distribution would have resulted in biased estimates.

Now to the effect of size on adoption—as shown in Table 3, the percentage of hospitals that adopted CT scanners by 12/81 varies a great deal across the number-

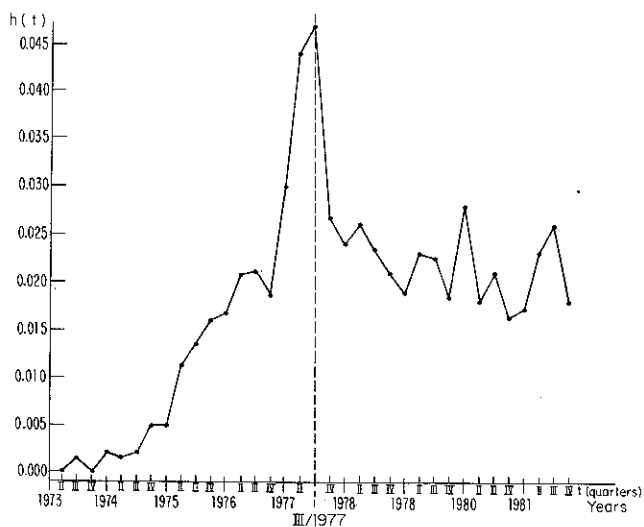


Figure 2. The Adoption Rate of CT Scanners, Assuming  $K = .84$ .

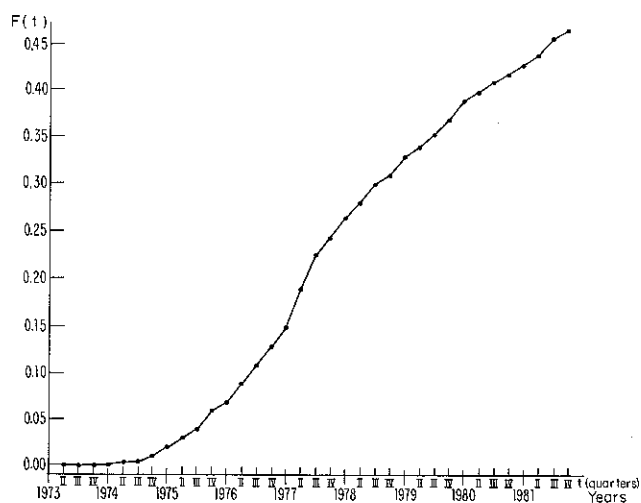


Figure 3. The Diffusion Path of CT Scanners:  $F(t)$ , Assuming  $K = .84$ .

of-beds groups, ranging from .13 for hospitals with 100–199 beds to .88 for the largest hospitals. The question is how to compare them in terms of diffusion speed in view of these disparities in the proportion of the process that is observed. In Section 4, we argue that there are two alternative ways of proceeding: (a) truncate the processes at the same cut-off level (minimum  $p_i$ ), or (b) make assumptions regarding  $\Gamma_i^*$ . In this case alternative (a) has to be discarded, because it will imply doing away with most of the available data. On the other hand, we do not have specific priors regarding the unobserved segment of each group's process, and hence the only alternative left is to resort to the uniformity assumption—that is, to estimate Equation (22) for each group. This is certainly troublesome in the case of small hospitals in view of their very low  $p_i$ ; hence the estimates for them should be viewed as tentative.

Table 7 presents both the estimates  $\Gamma_i$  and their components to provide a better idea of the nature of these calculations. Except for hospitals with 200–299 beds versus those with 100–199 beds, diffusion speed increases substantially with number of beds, a result consistent with previous studies (e.g., Russell 1979). But a closer look at the table indicates that the intragroup behavior of adopters was fairly similar across groups (as evidenced by the small variation in  $\Gamma^*$ ) and that the observed differences in speed were caused mostly by differences in  $p/K$  and in the mean adoption time. To illustrate the point, suppose that the total number of hospitals with 100–199 beds was much smaller so that  $p/K$  was equal to that of the largest hospitals; in that case, diffusion in small hospitals would have been faster than in the 500+ group because of the high  $\bar{t}$  of the former—that is, because the process was “crammed” in the later period. This underscores the fact that diffusion speed as defined here and elsewhere is only a measure of dispersion and by no means the only relevant aspect of diffusion; the location in time of the process

Table 7. Estimates of Diffusion Speed Under the Uniformity Assumption by Number of Beds

Number of beds	$p/K^*$	$\Gamma$	$(T - \bar{t})$	$\hat{\Gamma}$ (months)	$\hat{\Gamma}$ (years)
100-199	.189	11.089	30.872	27.14	2.26
200-299	.430	12.346	39.839	28.01	2.33
300-399	.676	11.982	42.732	21.96	1.83
400-499	.841	11.866	46.891	17.44	1.45
500+	.894	11.476	57.527	16.36	1.36

\* The  $p$ 's are taken from Table 3, and the  $K$ 's correspond to the ceilings of diagnostic radioisotopes: .70 for hospitals with 100-199 beds, .92 for those with 200-299 beds, and .98 for hospitals with 300 beds or more.

is at least as important, for which the mean adoption time,  $\bar{t}$ , is an appropriate measure [the "origin" of Griliches (1957), defined as  $(-2.2 - \alpha)/\beta$ , is precisely the mean adoption time as estimated by the logistic  $(-\alpha/\beta)$ , plus an arbitrary constant].

## 7. EXTENSIONS

One of the main methodological advantages of studying diffusion with the aid of tools that are well known and widely applied (rather than issue-specific) is that the analysis of the phenomenon can be readily extended by drawing from the literature in which these and related tools play a key role. The Gini is certainly a tool of that nature, and it opens up numerous possibilities for further research. We would like to suggest three such extensions; the first links diffusion with stochastic dominance, the second with rank-order tests of linear hypotheses, and the third with the capital asset pricing model (CAPM).

Although the treatment of diffusion has been carried out almost exclusively in positive terms, there is certainly a normative aspect to it—namely, the assessment of the relative desirability of alternative diffusion processes, in view of the costs and benefits of delaying adoption, and society's time preferences. The following highly simplified formulation of this issue suggests a line of inquiry that may prove fruitful.

Assume that the net social loss caused by postponing the adoption of the innovation until time  $t$  is  $v(t)$ , exhibiting the properties  $v(0) = 0$ ,  $v' > 0$ , and  $v'' < 0$ , and that the objective is to minimize the expected value of the loss,  $E[v(t)]$ , for all  $t \in (0, \infty)$ . This is analogous to the problem dealt with in the literature on stochastic dominance; as shown in Yitzhaki (1982b), given two diffusion processes,  $F_1(t)$  and  $F_2(t)$ , the necessary conditions for second-order stochastic dominance are  $\mu_1 \leq \mu_2$  and  $\mu_1 - \Gamma_1 \leq \mu_2 - \Gamma_2$ , where  $\mu_i$  is the expected adoption time and  $\Gamma_i$  is the Gini of  $F_i$  ( $i = 1, 2$ ). Sufficient conditions can also be derived for distributions that intersect once at most with the aid of the extended Gini. If the factors affecting the value of these parameters were known (e.g., the effect of tax incentives, regulation, market structure, etc.), then it could be possible to design optimal or second-best diffusion policies.

The second extension has to do with an issue of prime concern in diffusion studies, the identification of the

variables that affect the decision to adopt and the timing of adoption. This has been approached in various ways in the literature—linear probability models (Russell 1979), discrete choice models (Sommers 1980), simple regressions with the estimated  $\beta_i$ 's as the dependent variable (Griliches 1957; Mansfield 1968), and so forth. Consider now the linear model

$$t_i = \sum_{j=1}^J \alpha_j x_{ij} + \varepsilon_i, \quad i = 1, \dots, N, \quad (26)$$

where  $t_i$  is the time of the  $i$ th adoption,  $x_j$  are the variables presumed to affect the adoption decision (e.g., the characteristics of adopters, time-dependent attributes of the innovation and of the environment, etc.), and  $\varepsilon_i$  is the error term assumed to be iid but not necessarily normally distributed. Substituting (26) for  $t$  in (15),

$$\begin{aligned} \Gamma &= 2 \operatorname{cov}[t, F(t)] \\ &= 2 \sum_{j=1}^J \alpha_j \operatorname{cov}[x_j, F(t)] + 2 \operatorname{cov}[\varepsilon, F(t)], \quad (27) \end{aligned}$$

where  $2\alpha_j \operatorname{cov}[x_j, F(t)]$  is the contribution of the  $j$ th variable to the Gini and  $2 \operatorname{cov}[\varepsilon, F(t)]$  is its unexplained portion. If instead of estimating (26) we substitute  $\bar{F}_i = R_i/n$  for  $t_i$  (where  $R_i$  is the rank of  $t_i$ ), we obtain in fact Bennett's (1968) model for nonparametric tests of linear hypotheses (see also Shirley 1981); that is, this specification allows us to perform  $\chi^2$  tests of the null hypothesis  $H_0: \alpha_1 = \dots = \alpha_j = \dots = \alpha_L = 0$  ( $L < J$ ), which meaning can be best understood in the context of (27).

Although the relative merits of this approach vis-à-vis those mentioned previously are yet to be examined, it is worth noting some of the features that make it attractive: It provides a *direct* way of assessing the effect of exogenous variables on the diffusion process [rather than indirect or two-stage procedures as in Griliches (1957) and Mansfield (1968)], it does not require restrictive assumptions regarding the distribution of the error term, and it enhances the coherence and scope of the methodology presented here for the study of diffusion.

Now to the third possible extension. As mentioned in Section 4, it is often interesting to divide the population of adopters into subgroups and do a comparative

study of their diffusion processes. A related idea is investigating the impact of each group on the aggregate process—that is, the extent to which diffusion within each group accelerates or slows down overall diffusion. Formally, this involves decomposing the overall Gini into group-specific components that capture the relative size of each group and the similarity of its process to (or its correlation with) the aggregate process.

Let  $F_i(t)$  be the diffusion process and  $n_i$  be the size of the  $i$ th group,  $i = 1, \dots, m$ . The aggregate process will then be

$$F_o(t) = \frac{1}{N} \sum_{i=1}^m n_i F_i(t), \quad N = \sum_{i=1}^m n_i, \quad (28)$$

and the overall Gini will be

$$\Gamma_o = \int_{-\infty}^{\infty} [1 - F_o(t)] F_o(t) dt. \quad (29)$$

Substituting (28) for  $F_o$  in (29),

$$\Gamma_o = \sum_{i=1}^m \Gamma_{o,i}, \quad (30)$$

where

$$\Gamma_{o,i} = \frac{n_i}{N} \int_{-\infty}^{\infty} [1 - F_o(t)] F_i(t) dt, \quad i = 1, \dots, m. \quad (31)$$

The  $\Gamma_{o,i}$ 's are the magnitudes of interest; the larger the correlation between the diffusion process in group  $i$  and the aggregate process [i.e., the larger is the integral in (31)], the larger is  $\Gamma_{o,i}$ , and hence the more group  $i$  will slow down aggregate diffusion (the same holds, mutatis mutandis, for the relative size of group  $i$ ,  $n_i/N$ ). For comparative purposes, though, it is more convenient to use the shares

$$w_i = \Gamma_{o,i}/\Gamma_o, \quad \sum_{i=1}^m w_i = 1, \quad (32)$$

the meaning of which is immediate— $w_i$  is simply the fraction of the overall Gini accounted for by group  $i$ , or, in other words, it is the percentage of contribution of the diffusion process in group  $i$  to the average waiting time between adoptions in the total population.

The actual computation of (32) is done as follows: Let  $t_o$  be the vector of adoption times of the aggregate process and  $t_i$  be the analogous vector for group  $i$ . Define a new vector,  $\tilde{t}_i$ , for each  $i$  so that its  $j$ th element is

$$\begin{aligned} \tilde{t}_{ij} &= t_{oj} \quad \text{if } t_{oj} \in t_i \\ &= 0 \quad \text{if } t_{oj} \notin t_i; \end{aligned}$$

thus  $t_o = \sum_{i=1}^m \tilde{t}_i$ ,  $\Gamma_o = 2 \text{cov}(F_o, t_o) = 2 \sum_{i=1}^m \text{cov}(F_o, \tilde{t}_i)$ , and

$$w_i = \text{cov}(F_o, \tilde{t}_i) / \text{cov}(F_o, t_o). \quad (33)$$

Finally, it is worth noting that the decomposition here is formally similar to the one performed in the context of the familiar CAPM; the groups in diffusion can be thought of as different stocks, and aggregate diffusion as the market portfolio. As shown in Shalit and Yitzhaki (1984), the  $\Gamma_{o,i}$ 's—properly normalized—are analogous to the  $\beta$ 's in CAPM, a fact that facilitates the interpretation of these measures and may prove useful in further exploring the links between diffusions by groups and aggregate diffusion.

**ACKNOWLEDGMENTS**

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**APPENDIX: THE GINI OF THE GOMPERTZ DISTRIBUTION**

Let  $t$  be a random variable having a Gompertz distribution  $F(t) = a^{b^t}$ . Inverting it,  $t(F) = (1/\ln b) \ln(\ln F/\ln a)$ . Applying Equation (15),

$$\begin{aligned} \Gamma &= \frac{2}{\ln b} \int_0^1 \ln(\ln F/\ln a) (F - .5) dF \\ &= \frac{2}{\ln b} \int_0^1 (\ln|\ln F| - \ln|\ln a|) (F - .5) dF; \quad (A.1) \end{aligned}$$

hence

$$\Gamma = \frac{2}{\ln b} \int_0^1 \ln|\ln F| (F - .5) dF. \quad (A.2)$$

Evaluating numerically the integral in (A.2),

$$\int_0^1 \ln|\ln F| (F - .5) dF \cong .35. \quad (A.3)$$

Thus  $\Gamma \cong .7/\ln b$ .

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