Problem 6

The system to be stabilized is a third-order disturbed integrator,

$$\dddot{\sigma} = h_0(t,\vec{\sigma}_2)\,\|\vec{\sigma}_2\|_h^{1+3q} + g(t,\vec{\sigma}_2)u, \quad \sigma, u \in \mathbb{R},$$
$$\vec{\sigma}_2 = (\sigma, \dot{\sigma}, \ddot{\sigma}), \ \ |h_0(t,\vec{\sigma}_2)| \le C, \ 0 < K_m \le g(t,\vec{\sigma}_2) \le K_M, \tag{1}$$

where $\|\vec{\sigma}_2\|_h$ is a homogeneous norm corresponding to the weights

$$\deg\sigma = 1, \ \deg\dot{\sigma} = 1+q, \ \deg\ddot{\sigma} = 1+2q, \ q \ge -1/3,$$

and $h_0, g : \mathbb{R}^4 \to \mathbb{R}$ are some Lebesgue-measurable functions, $t \ge 0$. The functions $h_0, g, \|\cdot\|_h$ are concretized further.

The task is as follows:

1. To develop a universal homogeneous control template $u = \alpha U_3(\vec{\sigma}_2, \beta, q)$, $\beta \in \mathbb{R}^k, \alpha \in \mathbb{R}$, globally asymptotically stabilizing system for each q for some properly chosen parameters α, β and the whole state $\vec{\sigma}_2(t)$ available. The β dimension k is chosen by the student.
2. To find appropriate values of α, β for q = -1/3 by simulation. Two-three initial values of the form $\vec{\sigma}_2(0) = (\pm 2, \pm 2, \pm 2)$ are to be taken.
3. To apply a homogeneous output-feedback control including a filtering differentiator of the differentiation order $n_d = 2$ and the filtering order $n_f \ge 5$. To show the exact finite-time stabilization for only $\sigma(t)$ available by simulation, zero differentiator initial values and **some** $\vec{\sigma}_2(0) = (\pm 2, \pm 2, \pm 2)$.
4. To show the robustness of the system with respect to some large noise in sampled $\sigma(t)$ (random noise of the magnitude ~ 0.1, high-frequency noise $A\cos(\omega t)$ for $\omega \sim 10^5$ and $A \sim 10^3$ or even larger).
5. The integration is performed by the Euler method with the sampling/integration step $\tau \le 10^{-4}$. In the presence of sampling noises, if the noise contains a random component, $\tau \le 10^{-6}$ is to be taken for better results. Harmonic components $A\cos(\omega t)$, $A, \omega \gg 1$, usually require $\tau \le 10^{-5}$.

The report should contain the template development, parametric values, essential graphs and the maximal values of $|\sigma|, |\dot{\sigma}|, |\ddot{\sigma}|$ over the last 1/3 of the simulation interval for $\tau = 10^{-4}, 10^{-5}$.

The functions from (1) are as follows:

$$h_0 = \frac{\sigma^2 \operatorname{sign}\dot{\sigma} - 2\dot{\sigma}\operatorname{sign}\ddot{\sigma} + \ddot{\sigma}^2 \operatorname{sign}\sigma}{\sigma^2 + 2|\dot{\sigma}| + \ddot{\sigma}^2}\sin(88t), \ \ g = 2 + \frac{(\sigma\operatorname{sign}\dot{\sigma} - \dot{\sigma} + \ddot{\sigma}\operatorname{sign}\sigma)\cos(50t)}{|\sigma| + |\dot{\sigma}| + |\ddot{\sigma}|}, \ \ \|\vec{\sigma}_2\|_h = [|\sigma|^{\frac{1}{3}} + |\dot{\sigma}|^{\frac{1}{3(1+q)}} + |\ddot{\sigma}|^{\frac{1}{3(1+2q)}}]^3$$

Essential information appears in the document

https://www.tau.ac.il/~levant/Levant-SMC2019_for_students.pdf

Any questions are welcome.

בהצלחה!