Assume a two dimensional lattice. Define $Z(N,i)$ to be the following partition function:

$$Z(N,i) = \sum_{p \in P(0 \to i)} \alpha^{n_0(p)}$$

The sum is over all paths $P(0 \to i)$ starting at the origin and ending at a distance $i$ from the surface; $\alpha = \exp(V/kT)$, and $n_0(p)$ is the number of times a path $p$ touches the surface. The following recursion relation holds:

$$Z(N+1,i) = \begin{cases} 2\alpha Z(N,0) + \alpha Z(N,1) & , \quad i = 0 \\ Z(N,i-1) + 2Z(N,i) + Z(N,i+1) & , \quad i > 0 \end{cases}$$

Hence $p(N,i)$, the propability for the polymer end-point to be at distance $i$ from the surface, obeys:

$$\lambda_{N+1} p(N+1,i) = \begin{cases} 2\alpha p(N,0) + \alpha p(N,1) & , \quad i = 0 \\ p(N,i-1) + 2p(N,i) + p(N,i+1) & , \quad i > 0 \end{cases} , \quad (1)$$

where

$$\lambda_{N+1} = \frac{\sum_{i=0}^{\infty} Z(N+1,i)}{\sum_{i=0}^{\infty} Z(N,i)}$$

Let us assume that $p(N,i)$ approaches a stationary limit, $p(i)$, as $N$ goes to infinity, and find this limit; $\lambda_N$ will also approach a limiting value, which we denote by $\lambda$. Using equation (1)

$$\lambda p(0) = 2\alpha p(0) + \alpha p(1) \quad (2)$$

$$\lambda p(i) = p(i-1) + 2p(i) + p(i+1) \quad , \quad i > 0 \quad (3)$$

The solution of Eq. (3) is

$$p(i) = p_0 a^i \quad (4)$$

where

$$\lambda = \frac{1}{a} + 2 + a \quad (5)$$

Only one of the solutions of Eq. (5) is valid, the one obeying $|a| < 1$. Eq. (4) must hold at all distances from the wall, including $i = 0$. A boundary condition is found from (2)

$$\frac{p(1)}{p(0)} = \frac{\lambda - 2\alpha}{\alpha},$$

This ratio must also be equal to $a$ in light of (4). This equality, together with Eq. (5), yields a quadratic equation for $a$, whose roots are

$$a = -1 \pm \sqrt{\frac{\alpha}{\alpha - 1}}$$

The negative root is not valid because $p(i)$ cannot be negative. At very small temperature ($\alpha \to \infty$) $a$ approaces zero, whereas at large temperature ($\alpha \to 1$) $a$ diverges. We recall now that $a$ must be smaller than one, otherwise the distribution (4) is not normalizable, yielding a requirement that

$$\alpha > \frac{4}{3}$$

and a transition temperature

$$\frac{V}{kT} = \ln\frac{4}{3} \simeq 0.29$$

Above this temperature $p(N, i) \to 0$ as $N \to \infty$.

For a cubic lattice of dimension $d$ a similar derivation yields

$$a = -(d-1) + \sqrt{(d-1)^2 + \frac{1}{\alpha - 1}}$$

and the condition for localization is

$$\alpha > \frac{2d}{2d-1}$$