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LETTER TO THE EDITOR

Crossover behaviour of truncated Eden models

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Abstract. Motivated by recent field-theory approaches to the Eden model of cluster growth, we study a truncated version with a maximal number of three nearest-neighbour particles. Numerical simulations on cubic lattices show that the modification is irrelevant to the asymptotic exponent and affects only the short-distance behaviour. Beyond a crossover scale, which increases with the dimensionality, the total mass of the clusters scales as the volume of a d-dimensional sphere with asymptotic filling factors of ~ 0.7 and ~ 0.5 in $d = 2$ and $d = 3$, respectively.

Renewed interest in a cluster growth model introduced by Eden (1961) was stimulated by the notion that for space dimensionality $d = \infty$ (Parisi and Zhang 1984) and on the Cayley tree (Vannimenus *et al* 1984) it coincides with a physically more relevant diffusion-limited aggregation (DLA) process (Witten and Sander 1981). It was therefore suggested that a field-theory approach to DLA should begin from the description of the Eden mechanism. A possible first step in that direction was proposed by Parisi and Zhang (1985) in the form of a variant of the Reggeon field theory. A systematic Hamiltonian approach has been constructed by Shapir and Zhang (1985). This approach yields a non-Markovian field theory which also has a Markovian representation with more auxiliary fields (Shapir 1985). The picture which emerges is the representation of the Eden cluster as an expanding bubble of the right phase inside the (undercooled) wrong vacuum (Parisi and Zhang 1985). If this picture is adequate then it implies that the growth process is not critical and the systematic analysis of the relevance of the various vertices in the field theory cannot be performed as in the standard critical-point approach. In order to study the effect of changing the bare couplings we may, however, modify the lattice theory to start with.

In the present letter we report first results of Monte Carlo simulations of a drastically modified Eden model in two and three dimensions. We conclude that even a severe restriction of the growth process such as the limitation of the number of nearest neighbours does not change the long range scaling behaviour of the aggregate.

In the standard Eden process a cluster is constructed by sequentially adding particles to a randomly chosen perimeter site. Each perimeter site has the same growth probability. In low dimensions the clusters are almost compact, i.e. their fractal dimension $\bar{d}$ coincides with d of the embedding space (Peters *et al* 1979). While it is usually assumed that $\bar{d} = d$ in all dimensions, some doubts have been raised of whether it is so for large d as well (Parisi and Zhang 1984). Other properties of a different, tree-like, variant of this model in two dimensions were recently investigated (Dhar and Ramaswamy 1985).

In the field-theory approach (Shapir and Zhang 1985) the Lagrangian is formed by a quadratic term, representing the propagation in time and space, and by interaction terms describing the different processes of the birth of new particles. These vertices are derived from the expansion of a local term which schematically has the following simple form:

$$V(\boldsymbol{r}) = \ln\left(1+\sum_{k=1}^{z-1}\Gamma_{1,k}\right) = \sum_{p=1}^{\infty}\frac{(-1)^{p-1}}{p}\left(\sum_{k=1}^{z-1}\Gamma_{1,k}\right)^{p}. \tag{1}$$

In this expression we have defined $\Gamma_{1,k}$, $k=1,\ldots,z-1$ (z is the coordination number of the lattice) as the bare amplitudes for the primary processes in the model. These processes simply describe one incoming particle giving rise to k outgoing particles and are depicted in figure 1 (an integral over time is implicit in that definition).

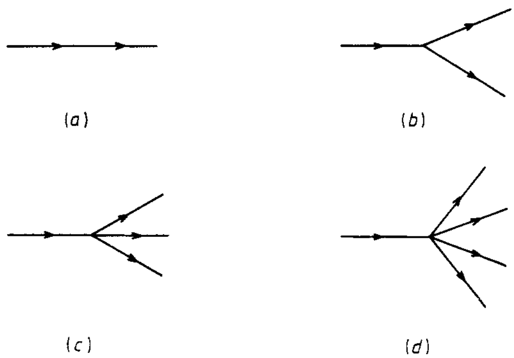

Figure 1. The lowest primary processes in the Eden model. Only processes (a) and (b) are allowed in the truncated version.

It may be instructive to study the effect of the truncation of the bare theory such that only the lowest order primary process $\Gamma_{1,1}$ (figure 1(a)) and $\Gamma_{1,2}$ (figure 1(b)) are allowed. Such a restriction is simply implemented in the lattice model by allowing no more than three nearest neighbours to each particle. Clearly this restriction of the bare theory does not eliminate higher order processes. Firstly, all possible products of these processes are already present in the bare theory due to the expansion of the logarithm (equation (1)) which must be performed, and secondly, renormalised amplitudes $\Gamma_{1,k}^{(\mathrm{R})}$ with $k>2$ will be generated from the consecutive applications of the lowest order primary processes as shown in figure 2. A naive scaling analysis, adequate near criticality, would therefore imply the irrelevance of such a truncation. However, since

Figure 2. The generation of higher primary processes. (a) The process $\Gamma_{1,3}$ (figure 1(c)) is generated by two $\Gamma_{1,2}$ (figure 1(b)) processes. (b) The process $\Gamma_{1,4}$ (figure 1(d)) is generated by three $\Gamma_{1,2}$ (figure 1(b)) processes.

the Eden model may be far from criticality we may wonder what would be the effects of such a modification of the bare process.

In order to answer that question we performed Monte Carlo (MC) simulations of a modified Eden model on simple square and simple cubic lattices. In our version of the model a new particle is allowed to be attached to a perimeter site only if it does not create a situation where one of the cluster particles or the new particle itself has more than three occupied nearest neighbours. We generated several 2D clusters of 7000 particles and a 3D cluster of 800 000 particles. Figure 3 depicts the typical results of the dependence of the average density ρ of the clusters on their radii of gyration R_g (RMS distance of the particles from their centre of mass) as the clusters grow. In this computation we defined the average density in d dimensions as

$$\rho = M / B_d (R_g + A_d)^d, \qquad (2)$$

where M is the mass of the cluster while a numerical factor B_d has been defined to have the value such that ρ will coincide with the actual density in a perfectly homogeneous occupied circle (sphere) which are $B_2 = 2\pi$ and $B_3 = \frac{1}{9} 20\pi (\frac{5}{3})^{1/2}$. The additional factor $A_d \equiv B_d^{1/d}$ appears in the denominator of (2) to provide an adequate interpolation between the discrete and continuum definitions for small R_g. After an initial crossover the densities of the clusters slowly increase and approach asymptotic values of 0.72 ± 0.01 (0.49 ± 0.01) for $d = 2$ ($d = 3$).

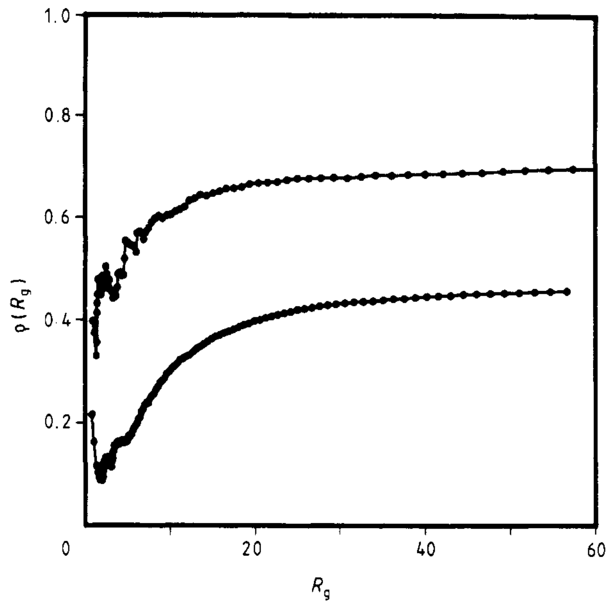

Figure 3. $\rho(R_g)$ against R_g for $d = 2$ (higher curve) and $d = 3$ (lower curve).

It is important to notice that, despite the random generation of these clusters, their densities are fairly close to the maximal possible packing with densities of $\frac{4}{5}$ and $\frac{2}{3}$, in $d = 2$ and $d = 3$, respectively. Note that the interior of the 2D cluster (figure 4) has practically no empty sites which could be occupied without violation of the restriction on the number of allowed nearest neighbours. A similar situation may also be observed in three dimensions. Thus the asymptotic densities of our clusters are smaller than

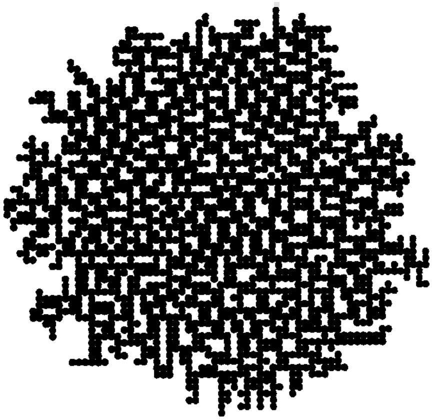

Figure 4. A typical cluster configuration generated in $d=2$.

the maximal possible ones because of the random packing and not due to 'holes' left in the inside of the clusters. Although we cannot rule out the appearance of such 'holes' in higher dimensions this seems, in light of the present results, to be unlikely.

For our low-dimensional clusters it is clear that the effect of the truncation is only short range. The crossover length is approximately 15 and 20 lattice constants in $d=2$ and $d=3$ respectively. For larger distances the dependence of ρ may be fairly well represented by

$$\rho(R_g)=\rho(\infty)-C/R_g. \tag{3}$$

This indicates that we are observing the effect of the fluctuations (and therefore smaller density) on the surface of the growing cluster.

To conclude we have demonstrated the dynamical stability of the Eden process under a rather drastic modification. The field-theory arguments support a similar conclusion in all dimensions but further studies of the potential existence of a critical dimension are certainly worthwhile.

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