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LOCALIZATION IN A ONE-DIMENSIONAL DISORDERED MODEL

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Numerical investigation of a random, one dimensional Kronig-Penny-like model is performed using long chains and large ensembles. Dependence of the inverse localization length α on randomness, irreproducibility of resistance measurements and the dependence of the standard deviation of α on α and the length of the chain were studied. For energies, $E=k^2$ close to the zone boundary $k=\pi$, we have found $\alpha\sim(\pi-k)$.

Recently the electrical properties of one-dimensional disordered chains of scatterers have been studied extensively. Both analytical[1-4] and numerical[4-6] calculations have received attention. We investigated variety of properties of one such system. In our model an electron moves in a Kronig-Penny like potential, which consists of L equally spaced δ-functions with random strengths. The state of an electron at the Fermi level, whose energy is k^2, thus satisfies the following Schrodinger equation:

$$[-\frac{d^2}{dx^2}+\sum_{n=1}^{L} U_n\delta(x-n)]\psi = k^2\psi \qquad (1)$$

where $U_n=U(1-cy_n)$. The randomness of the system is determined by the parameter c and the binary distributed random variable y_n, which can take the values 1 or 0 with the probabilities p and 1-p, respectively. If p is small enough we can still talk about the original, undisturbed band structure. The location of a state in the band will be determined by U and k^2. Most of the calculations, unless specified otherwise, refer to the middle of the first allowed band: $U=\pi$, $k=3\pi/4$. Long random chains (up to L=16000 in most cases) were generated by computer, and large ensembles (up to N=400 systems) were used to get reliable results. The dimensionless resistance R of each chain was evaluated from Landauer's formula[7] using the transfer matrix method[2,3,5]. The inverse localization length $\alpha\equiv \ln(1+R)/L$ was calculated for each chain.

The statistical distributions of α for different values of c and p, i.e. different values of $\langle\alpha\rangle$ (the brackets denote averaging over the ensemble), and different L were calculated. We found that α is really the well behaved variable of the problem in agreement with previous investigations[2,5]. The typical results are depicted by the histograms in Fig.1. For $\langle\alpha\rangle\cdot L\gtrsim 10$ the distribution of α appears to be approaching a Gausian distribution. For smaller values of $\langle\alpha\rangle\cdot L$ the standard deviation σ_α becomes of the order of $\langle\alpha\rangle$, and the distribution is highly assymetric.

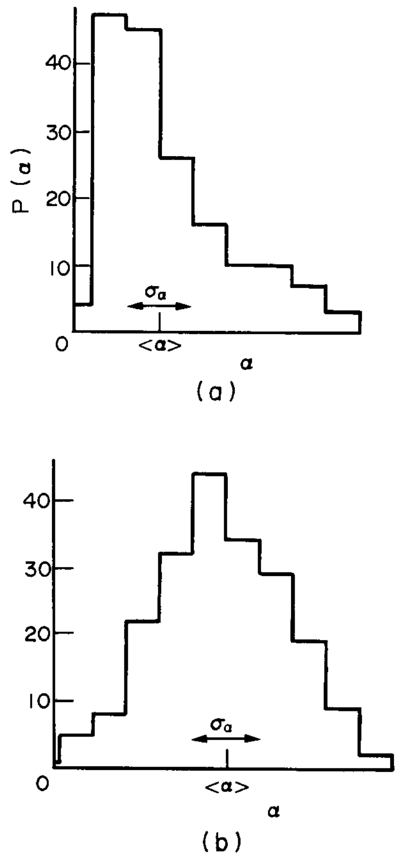

Fig.1. Typical distribution of α calculated for ensembles of N=200 chains. Each chain has a length L=2000 sites. (a) $p=0.5, c=0.1, \langle\alpha\rangle\cdot L=1.4$; (b) $p=0.5, c=0.3, \langle\alpha\rangle\cdot L=11.3$.

Fig.2 shows the power law dependence of $\sigma_\alpha/\langle\alpha\rangle$ on $\langle\alpha\rangle\cdot L$:

$$\frac{\sigma_\alpha}{\langle\alpha\rangle} \sim (\langle\alpha\rangle\cdot L)^{-\omega} \qquad (2)$$

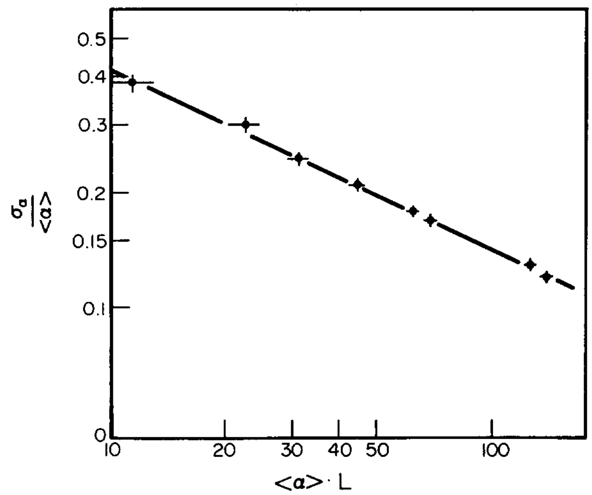

Fig.2. Dependence of $\sigma_\alpha/<\alpha>$ on $<\alpha>\cdot L$. The values of parameters that were used to calculate the points are: L=2000,4000,8000; c=0.1,0.2,0.3; p=0.5.

with $\omega=0.47 \pm 0.07$. The relation $\sigma_\alpha \sim L^{-\frac{1}{2}}$ was found in Ref. 5. Combining this result with scaling considerations we find $\omega=\frac{1}{2}$.

If the statistical behaviour of α is described by a Gaussian distribution (with the appropriate parameters <α> and σ_α), then it is easy to show that for large L

$$\frac{\sigma_R^2}{<R>^2} \sim \exp\,(\sigma_\alpha^2 L^2) \qquad (3)$$

Inserting (2) into (3) and substituting $\omega=\frac{1}{2}$ it is now obvious that

$$\log \frac{\sigma_R}{<R>} = K<\alpha>L \qquad (4)$$

where the coefficient K depends only on the location in the band and not on the randomness of the system. This phenomenon has a great impact on experiments since it indicates the irreproducibility of the measurements of R[8]. It also indicates that a direct calculation of <R> is impossible. Fig. 3 depicts the results of such averaging. For constant c,p and L we have calculated the average resistance $<R>_N$ of N such chains. Gradually increasing the size of ensemble up to N=400, we followed the behaviour of $<R>_N$. The curve is very irregular (note the logarithmic scales, and the fact that the small jumps in $<R>_N$ for large N correspond to a single chain with a huge resistance that appeared in the ensemble).

Varying c and p, we checked the dependence of <α> on the randomness of the chain (namely, on the variance of the potential strength, which, in our case, is $c^2p(1-p)U^2$). These results were consistent with $<\alpha>\sim c^2p(1-p)$. This is in agreement with analytical results in Refs. 4 and 5 which treated similar systems. Two examples of such dependence are shown in Fig.4.

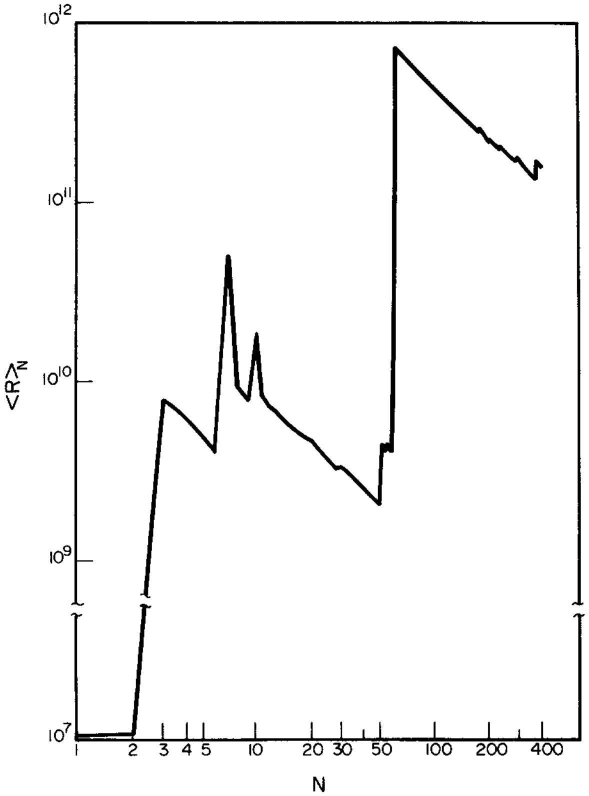

Fig.3. Dependence of the average resistance $<R>_N$ on the ensemble size N.

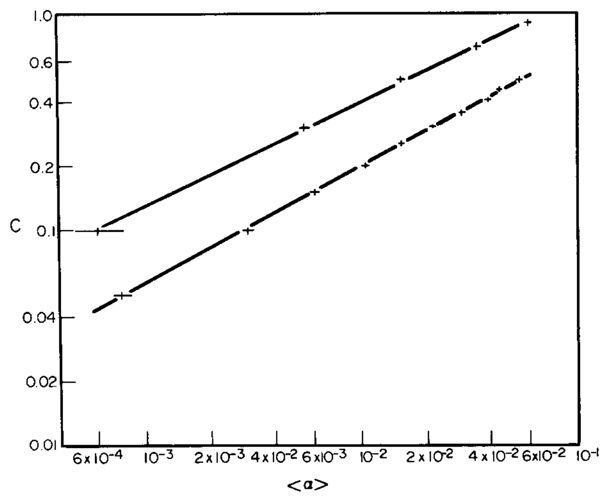

Fig.4. Dependence of <α> on c.200 chains were averaged to calculate each point. For the upper curve p=0.5, L=8000, U=π, k=π/2 were taken. For the lower curve p=0.5, L=10000, U=3π/4, k=π/2 were taken.

The lines correspond to a power law dependence

$$\langle\alpha\rangle \sim c^{\beta} \qquad (5)$$

with $\beta=2.10\pm0.20$ (the upper line) and $\beta=1.86\pm0.15$ (the lower line) which are consistent with $\beta=2$.

For $k=\pi$ (i.e., zone boundary) the resistance of a non random chain is proportional to L^2. At this special point an analytical calculation of the resistance of a random chain can also be performed easily[9], showing the same dependence on the length. Obviously this means $\langle\alpha\rangle=0$. The mapping of Fig.5 demonstrates the decrease of $\langle\alpha\rangle$ (divergence of the localization length) for any value of U.

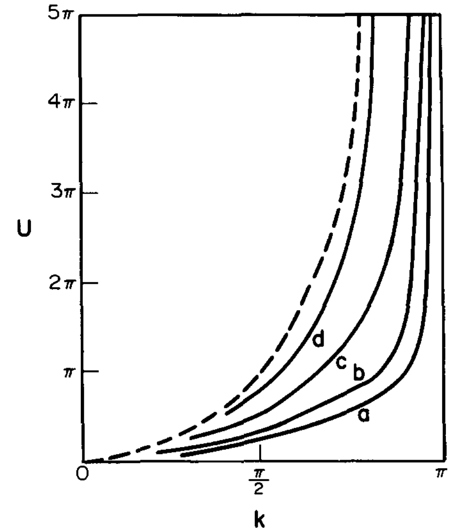

Fig.5. Lines of constant $\langle\alpha\rangle$ inside the first allowed band. $\alpha_a=2\cdot10^{-4}$, $\alpha_b=4\cdot10^{-4}$, $\alpha_c=1\cdot10^{-3}$, $\alpha_d=1\cdot10^{-3}$. The mapping was done for c=0.1, p=0.5 and the length of the chains was determined by the condition $\langle\alpha\rangle\cdot L>100$. Dashed line stands for the lower boundary of the first allowed band

Finally we have studied, for the first time, the behaviour of $\langle\alpha\rangle$ (k) as k approaches π from below. It was found that close enough to the zone boundary

$$\langle\alpha\rangle \sim (\pi-k)^{\gamma} \qquad (6)$$

Fig.6 depicts such a dependence for $U=\pi$. For the range $2\cdot10^{-3}<(\pi-k)/\pi<0.2$ we found $\gamma=1.001\pm0.014$, which suggests $\gamma=1$. To ensure small variance of the results we have chosen L to be such that $\langle\alpha\rangle\cdot L\gtrsim100$. Thus the numerical calculations were performed using extremely long chains (up to 24,000,000 sites); each case was repeated for 6 different chains.

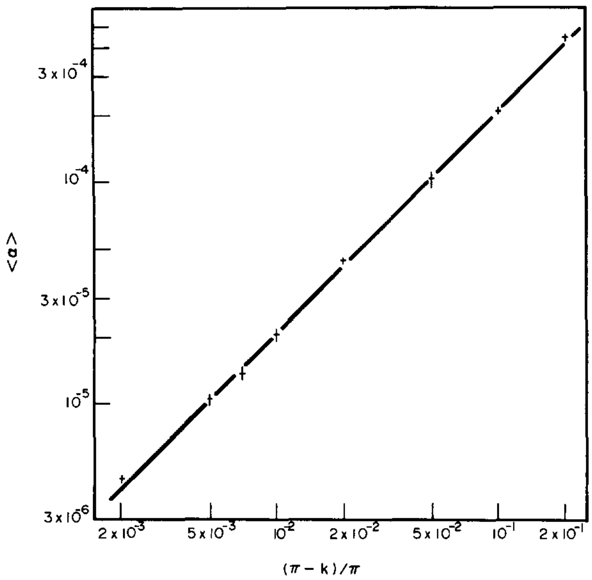

Fig.6. Dependence of $\langle\alpha\rangle$ on $(\pi-k)/\pi$ for $U=\pi$ near the zone boundary.

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