Cognitive Complexity of Boolean Problems

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Outline

- Boolean concepts learning.
- Math complexity vs. Cognitive Complexity
- Feldman's complexity and it's criticism
- Regularity as cognitive simplicity
- Experiments
- Future Research Framework
- Conclusions

Mathematical Complexity of Boolean Concepts

- Information complexity - Shannon

Most Functions Are Hard, But We Don't Have Any Bad Examples

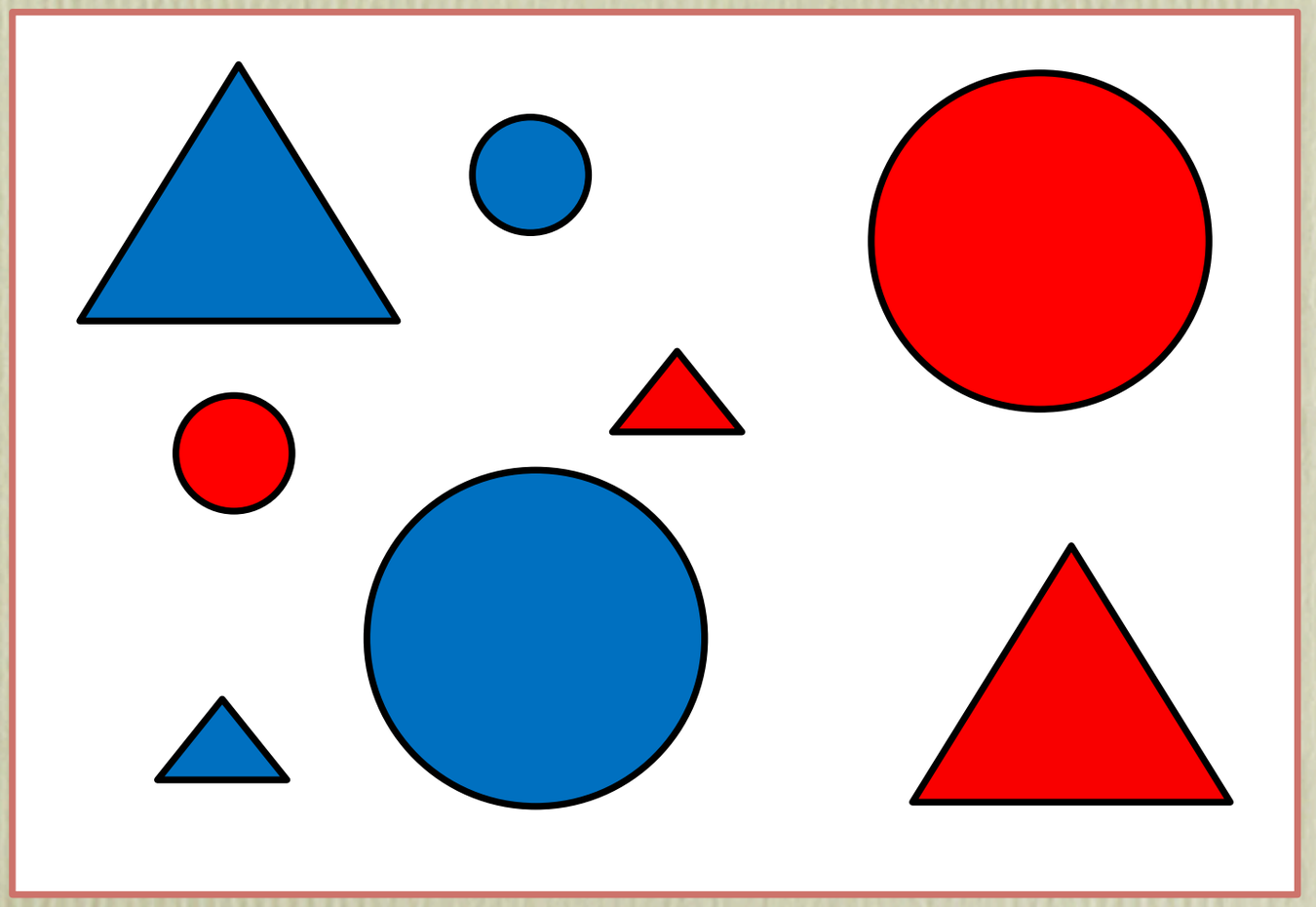
- Algorithmic complexity - Kolmogorov

Complexity of algorithm producing Boolean Concept

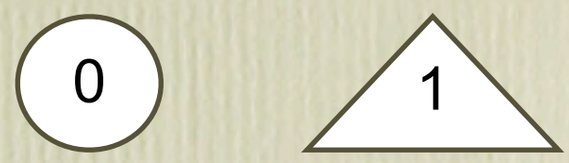
Cognitive complexity of Boolean Concepts

- Shepard, Hovland and Jenkins (1961) - *NPN classes define classes of equal complexity*
- Nosovsky (1962) - *AND-OR non-symmetry*
- Feldman (2000) - *complexity - the number of literals in the minimal Boolean expression*

Boolean concepts

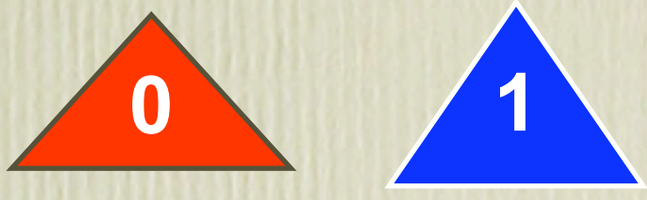


x - Shape



y - Size

z - Color



Boolean concepts

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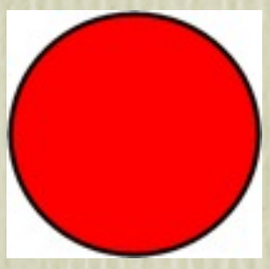
101

001

110



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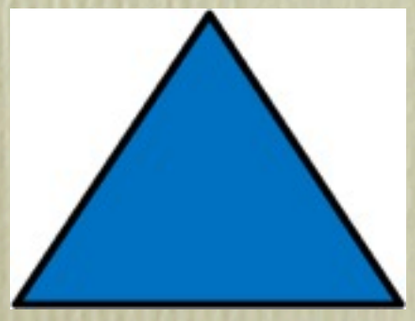

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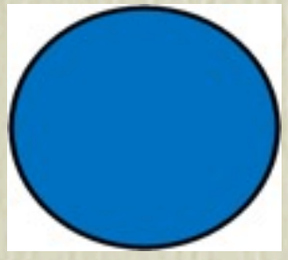

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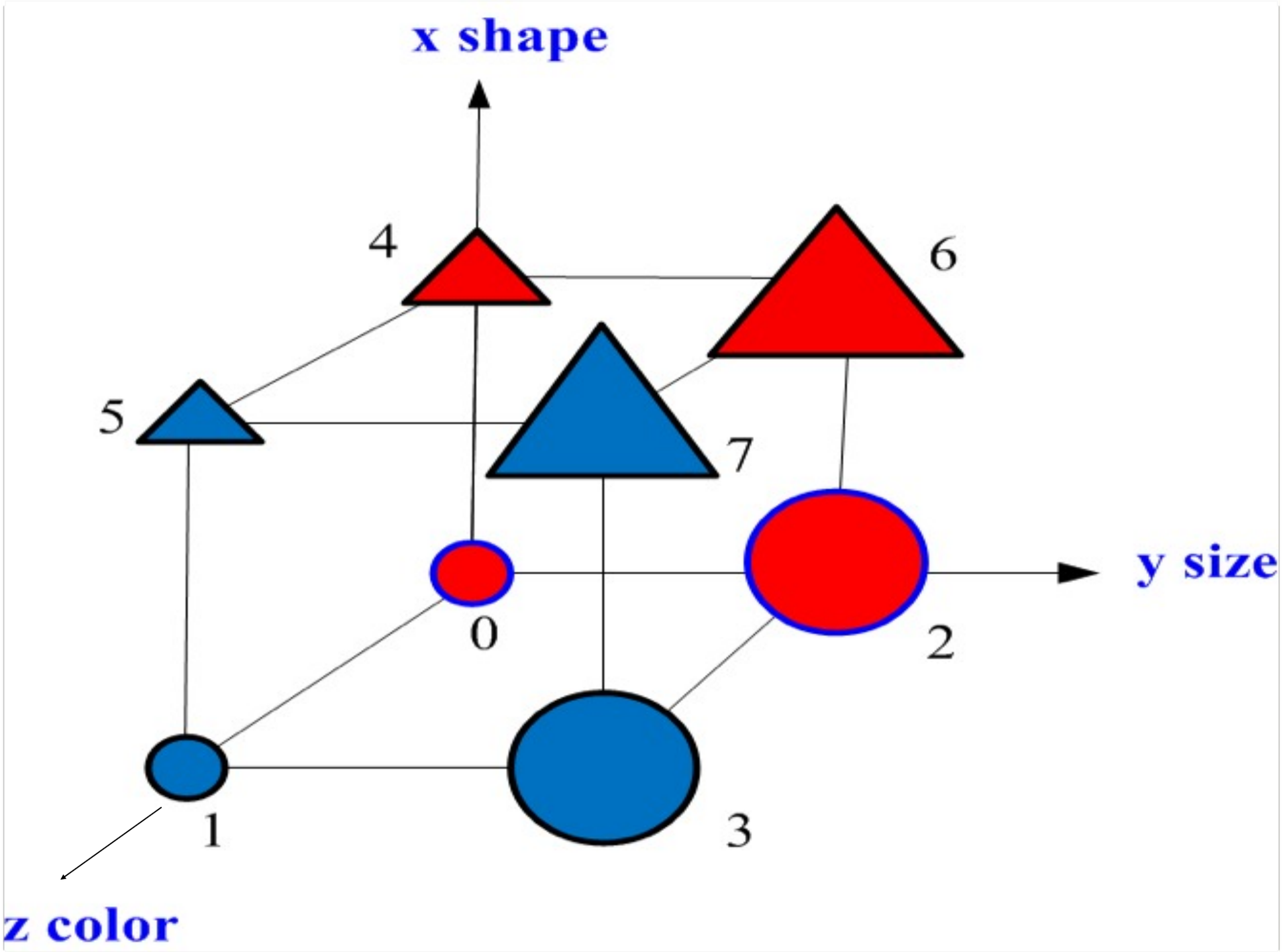
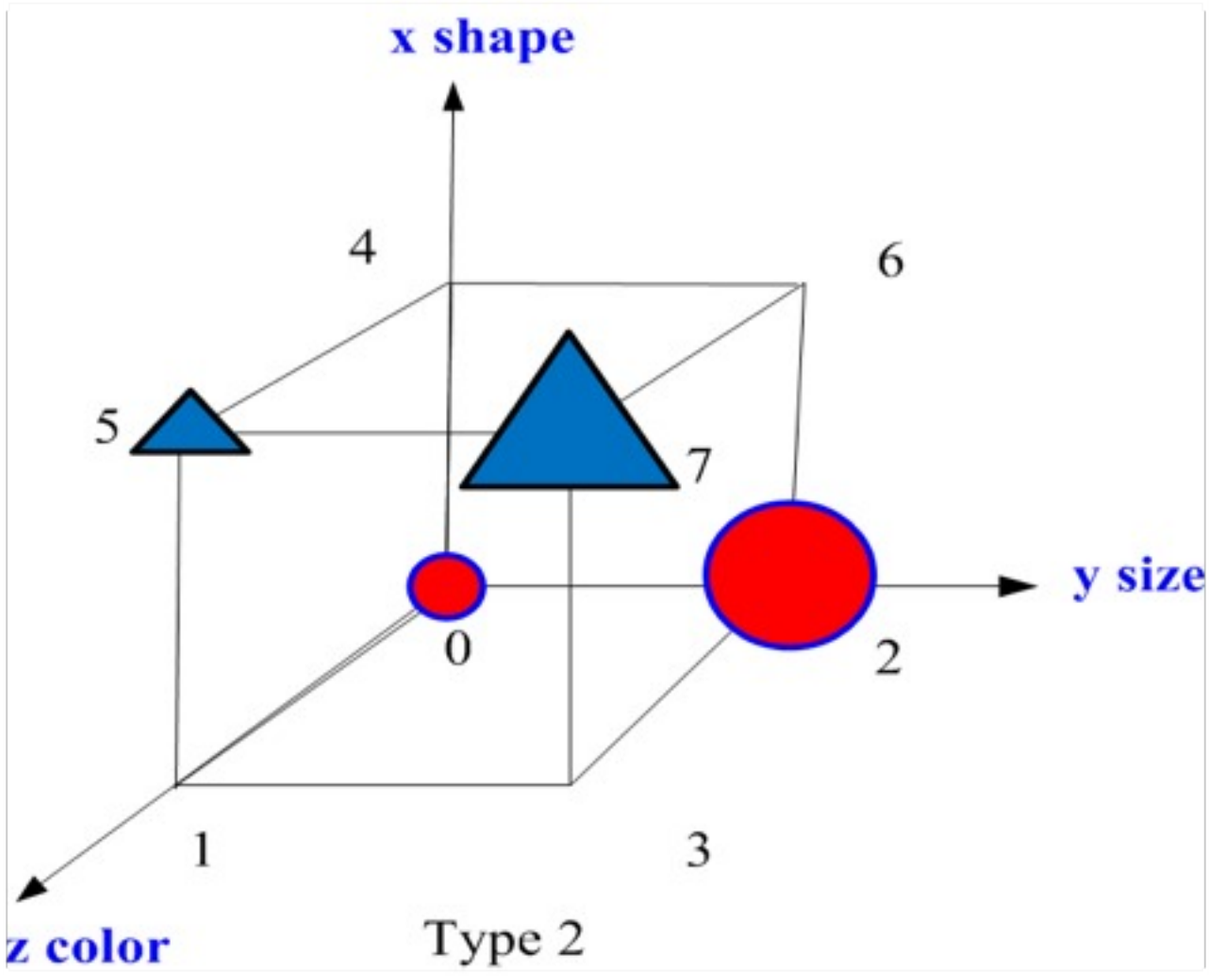
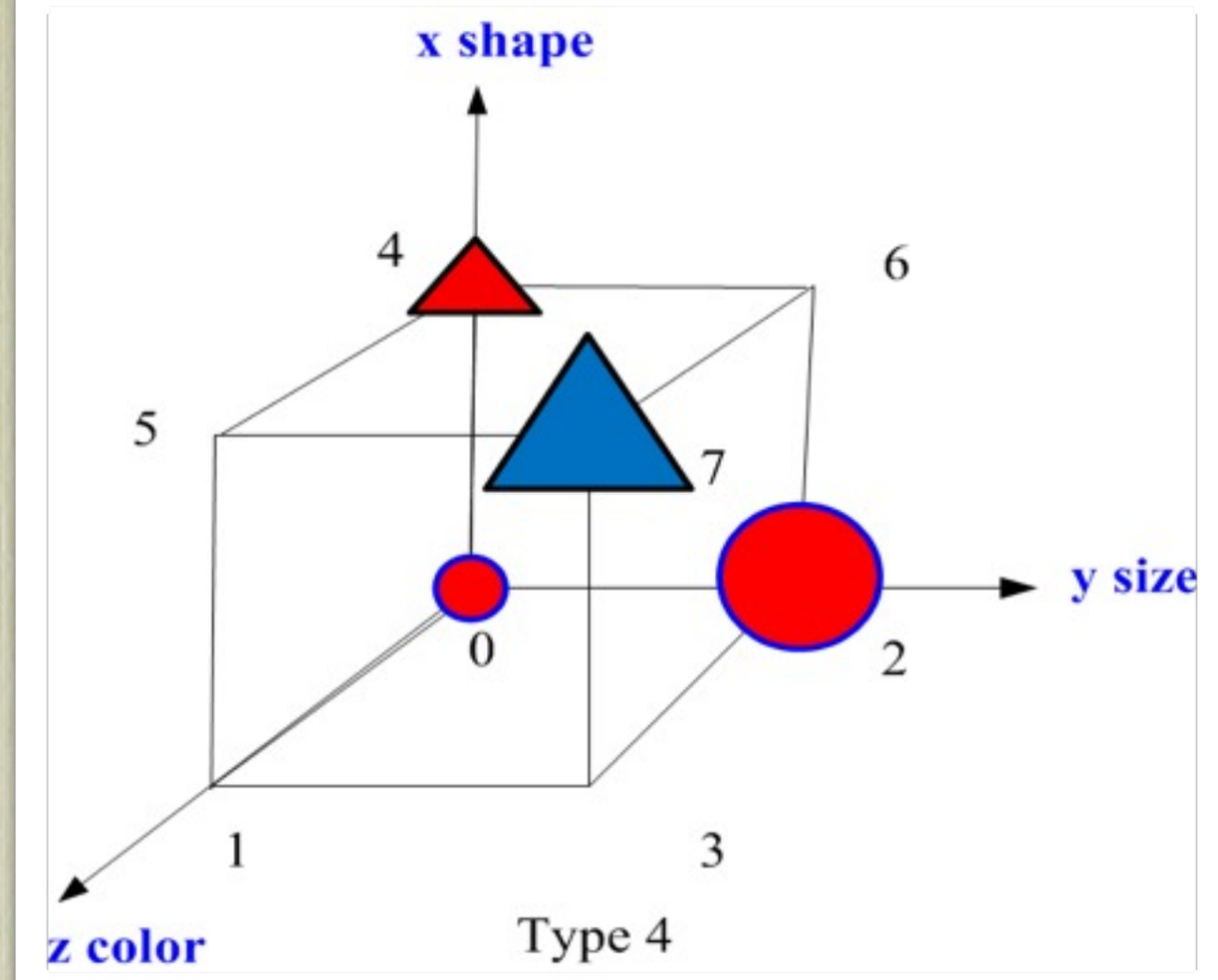
Boolean cube



Complexity of 4 elements recognition



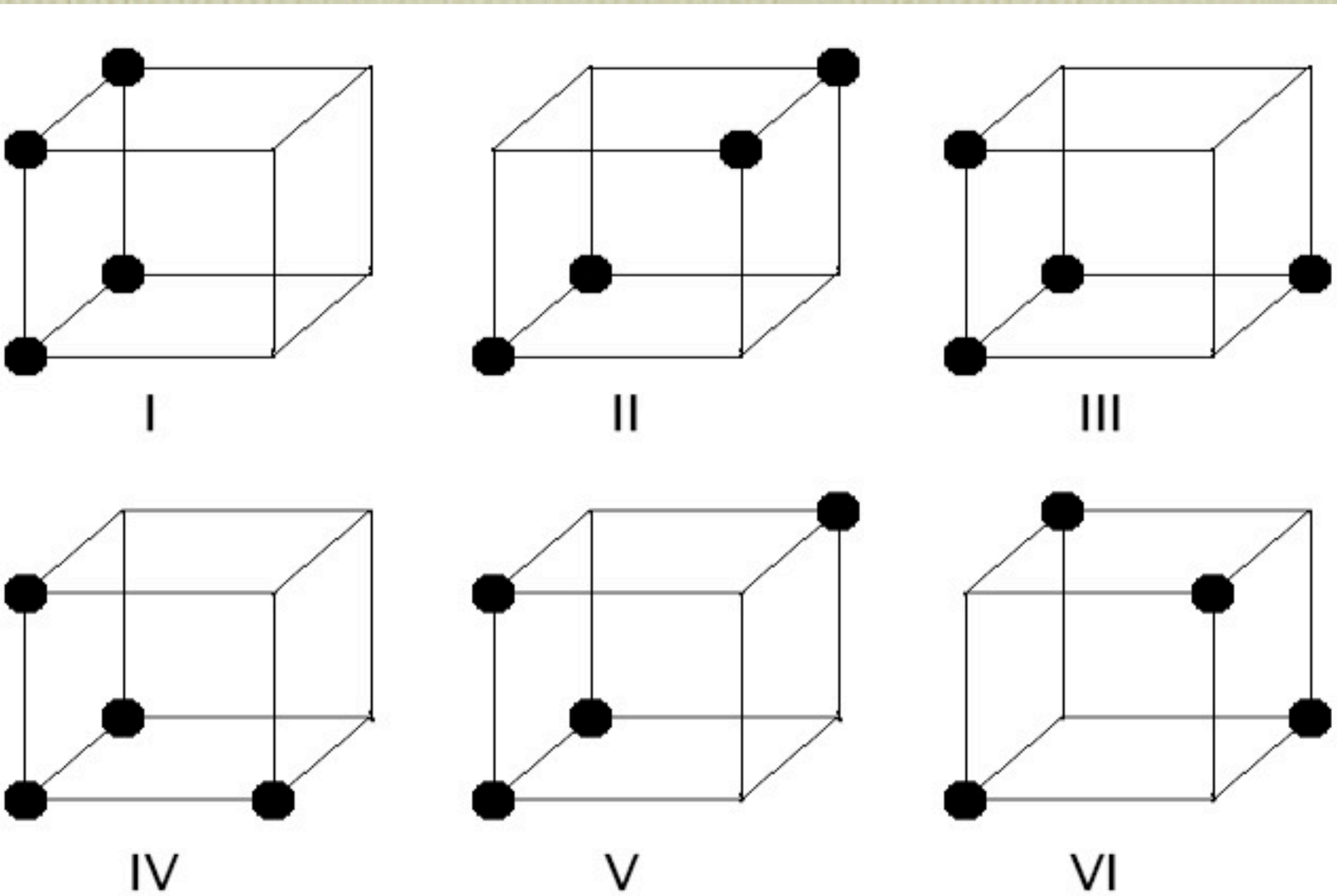


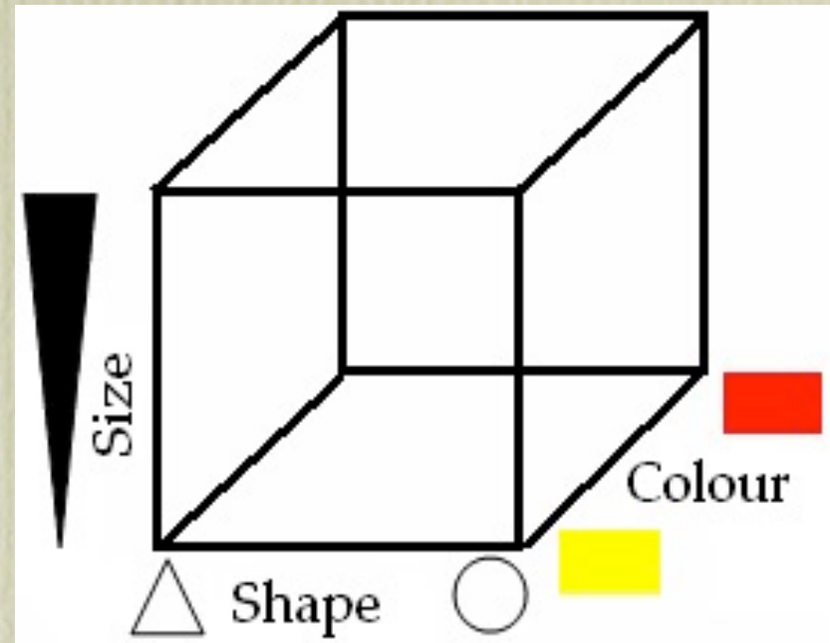
What is more complex for humans?

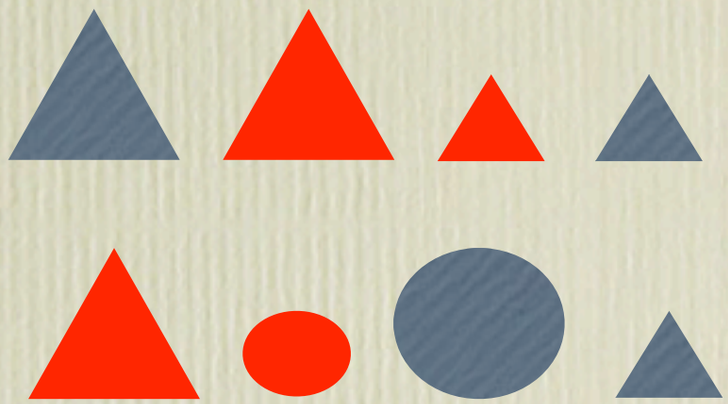
$$xz + \overline{x}\,\overline{z}$$

$$\overline{y}\,\overline{z} + xyz + \overline{x}y\overline{z}$$

Difficulty of Boolean concept learning







Feldman (2000), *Nature*

Minimal Boolean complexity

Length of the shortest logical expression that captures the concept

I < II < III, IV, V, < VI

1 4 6 6 6 10

II: (a and b) or (not a and not b)

VI: (a and ((not b and c) or (b and not c))) or (not a and ((not b and not c) or (b and c)))

Problems with minimal complexity

Ad hoc choice of connectives (and, or, not)

Include xor: I < II < III, IV, V < VI

1 2 4 5 3 3

Not minimal! (Mathy & Bradmetz, 2004; Vigo, 2006)

Corrected: I < II < III, IV, V < VI

1 4 4 5 6 10

No psychological mechanism for forming minimal descriptions

Feldman's Complexity Issues

- Problem of Boolean minimization (Vigo, 2003)

Feldman's heuristics failed to find the correct minimal descriptions for certain concepts. People cannot really minimize

- Problems of basis

To justify the set of primitive connectives that are allowed in formulating descriptions. A standard defense of not, and, and or, is that they have been conventional since the work of Neisser and Weene (1962). But, why should we exclude XOR?

- Problems of functional properties

Individuals can carry out the task of categorizing instances of a concept without attempting to formulate its minimal description. Simple example: symmetric functions

- Boolean Approximation problems

People usually approximate their solutions

- Representation problems

Cubes, sets, diagrams, cards, DDs etc.

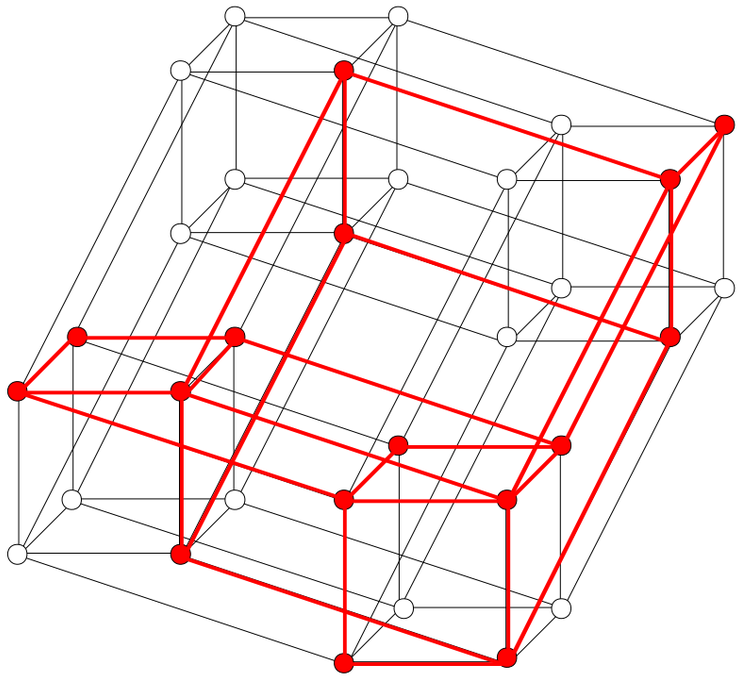
- Different tasks problems

Whether the approach works for different types of tasks. Recognition vs. Reverse Engineering.

- Nonstandard solutions problems

Bridge Circuit: Example of nonstandard solution

Bridge Circuits

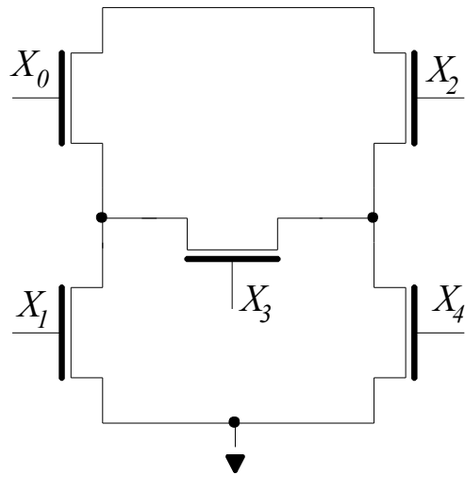


$$F(X_0, X_1, X_2, X_3, X_4) =$$
$$X_0X_1 + X_2X_4 + X_0X_3X_4 + X_1X_2X_3$$

2

Bridge Circuits

Bridge implementation for our example



5 transistors

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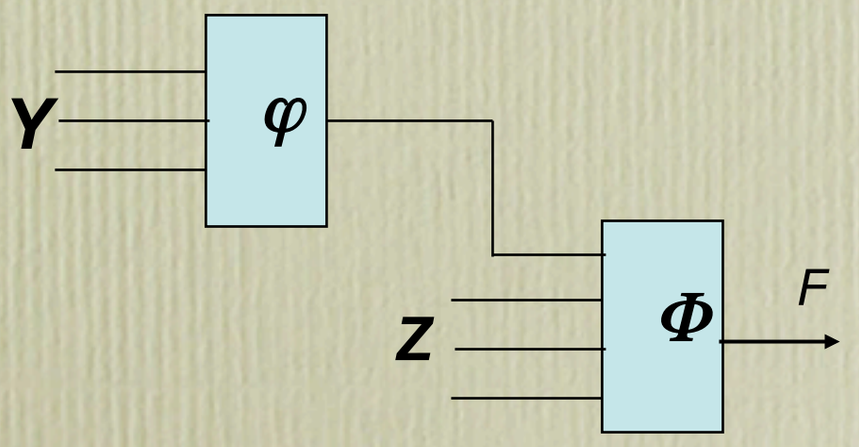
Decomposition: classic example of nonstandard solution

$$F(X) = \overline{x}_0\overline{x}_1x_2x_3 + \overline{x}_0\overline{x}_1x_2x_4 + \overline{x}_0\overline{x}_1x_3x_4 +$$
$$+x_0x_1\overline{x}_2\overline{x}_3 + x_0x_1\overline{x}_2\overline{x}_4 + x_0x_1\overline{x}_3\overline{x}_4$$

$$F(X) = (x_2x_3 + x_2x_4 + x_3x_4)\overline{x}_0\overline{x}_1 +$$
$$+\overline{(x_2x_3 + x_2x_4 + x_3x_4)}x_0x_1$$



$$F(X) = F(x_0, x_1, x_2, x_3, x_4) = \Phi(x_0, x_1, \varphi(x_2, x_3, x_4)) =$$
$$= \varphi\overline{x}_0\overline{x}_1 + \overline{\varphi}x_0x_1$$
$$\varphi = x_2x_3 + x_2x_4 + x_3x_4$$

Feldman's Improvement

• Feldman (2006) developed a new theory of complexity, which rests on a formalization of "algebraic" complexity rather than the length of minimal descriptions.

• In this updated model, the complexity of a concept is driven by its decomposition into a set of underlying regularities.

• The basic idea is that any concept can be decomposed into a series of more basic concepts. Thus, any concept can be decomposed into a set of underlying rules, each of differing degrees of complexity, depending on the number of variables that they instantiate.

But still...

- What kinds of regularities individuals are able to recognize?

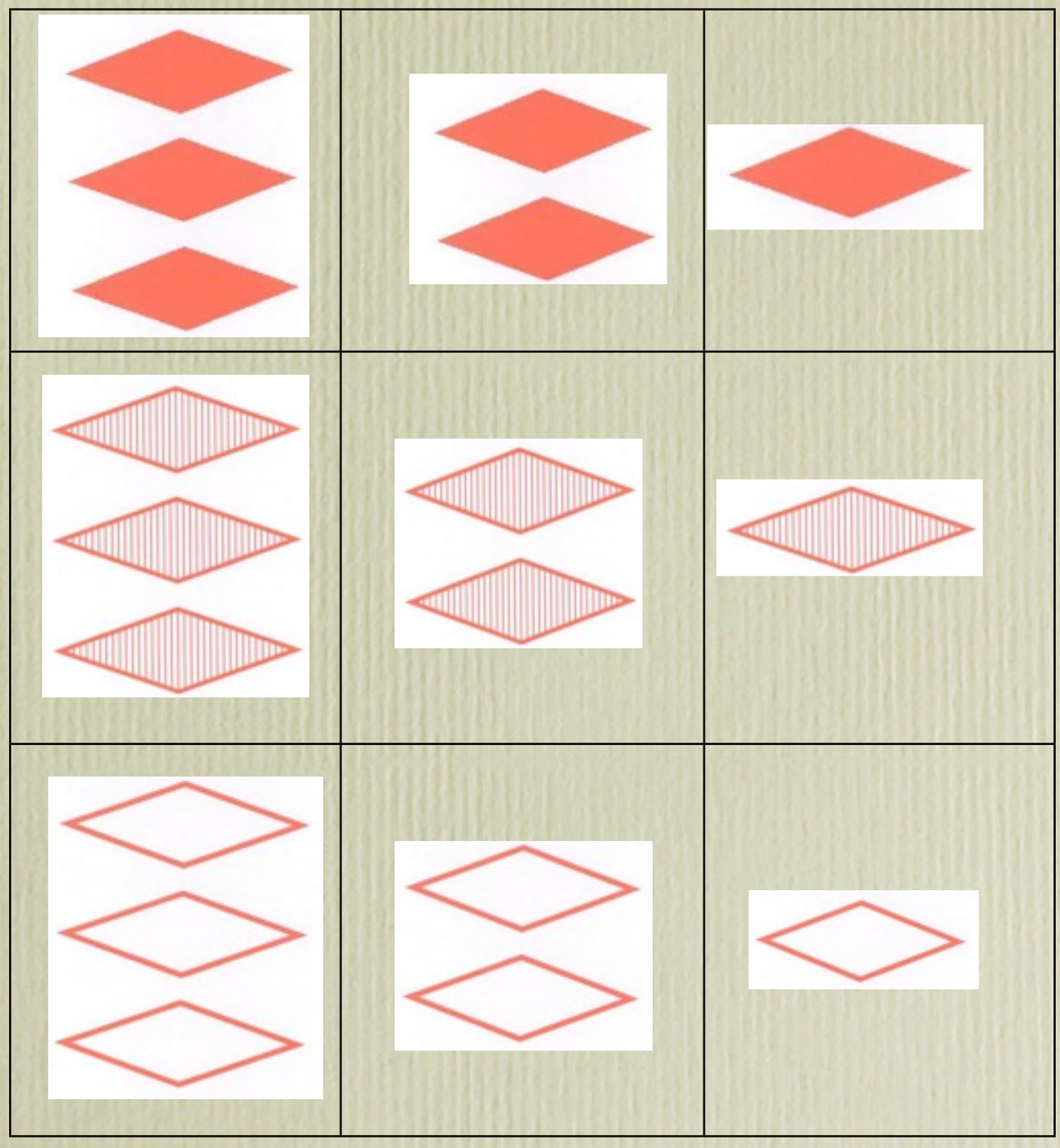
- How these regularities are connected with known theoretical results of Boolean algebra and Logic Design?

- Are the original and/or nonstandard Boolean solutions inspired by human learning and/or by math consideration?

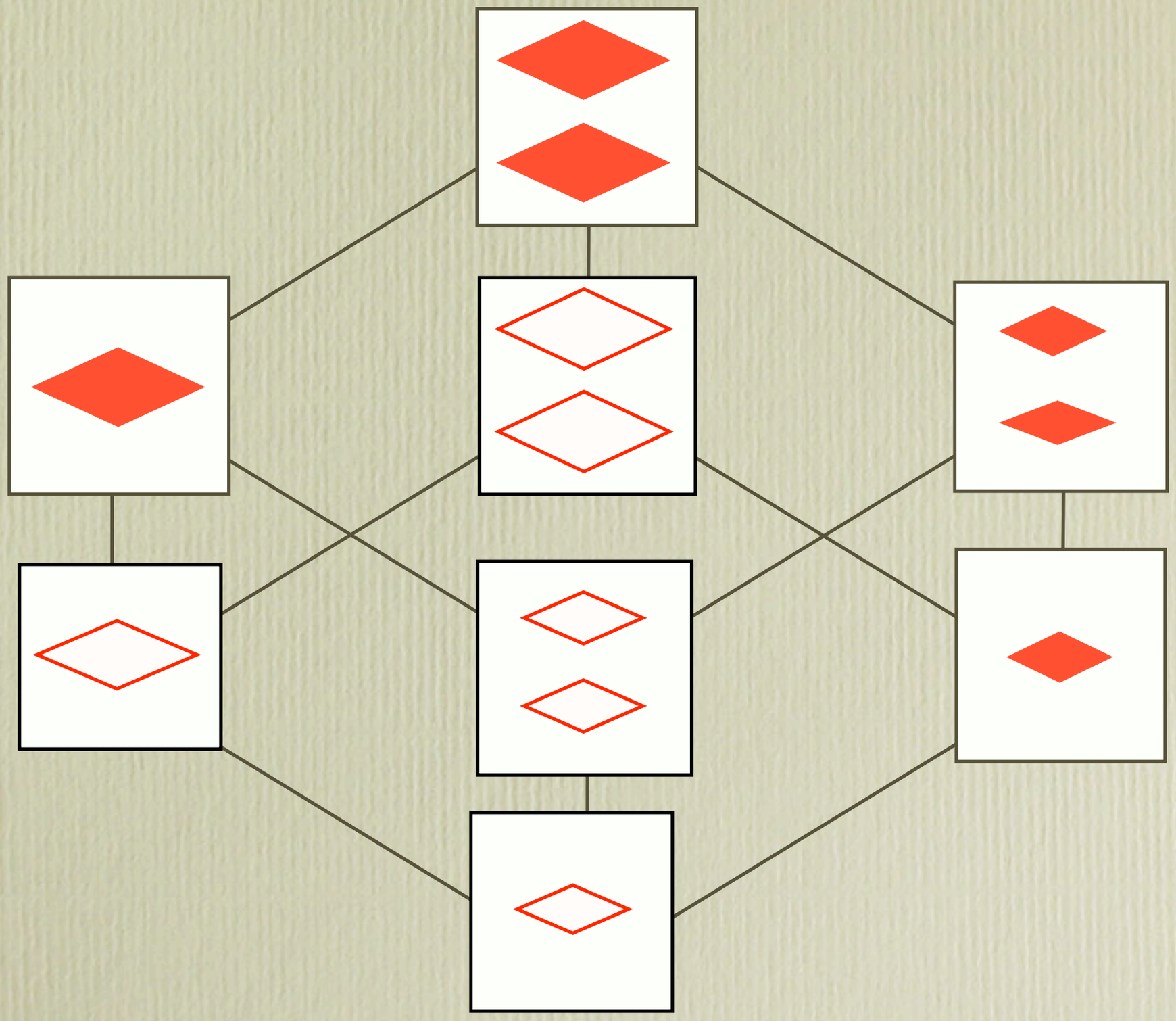
Our study

- Population: a group of young children
- Use the Set game® cards to answer the questions:
    - Does the Feldman's complexity reflect an ability of young children to solve logic problems?
    - Which kinds of regularities can be recognized by children?
    - Does recognition of specific regularities support children's success in solving logic problems (decrease the cognitive complexity)?

Set game®



Boolean cube of cards for Set game



Research Methodology

- We propose a specific regularity function that measures cognitive complexity of the task

- We study experimentally, how students recognize regularity within the shapes' characteristics and how this recognition supports children in solving logic problems

- Our research hypothesis was that despite of high Feldman's complexity, children are able to solve logic problems successfully by recognizing some regularities

Set cards characteristics

- Size
- Fill
- Number

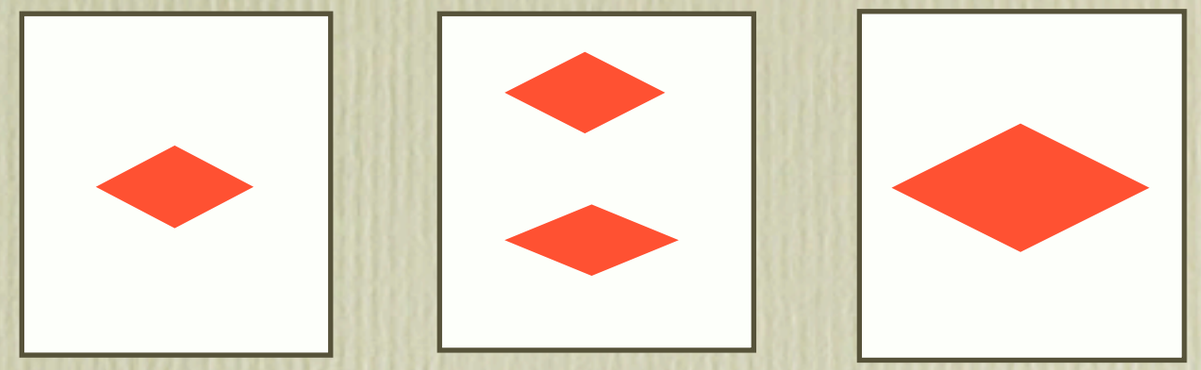
Set cards regularities

- Homogeneity
- Difference
- Monotony
- Symmetry

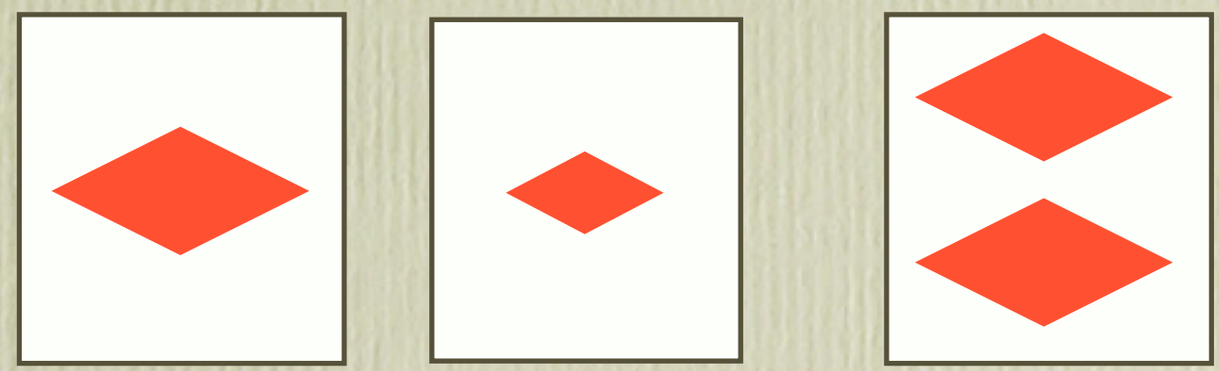
The regularities can be recognized in each of the characteristics

Example: regularities of Size

Monotony



Symmetry



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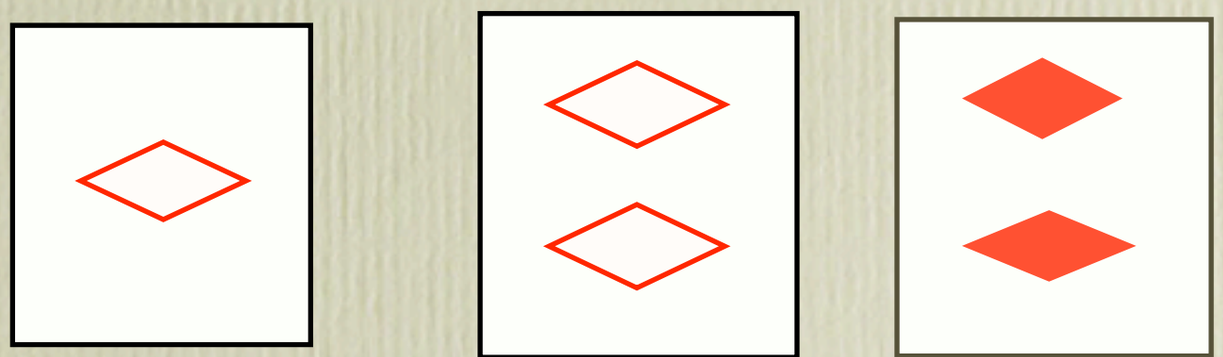
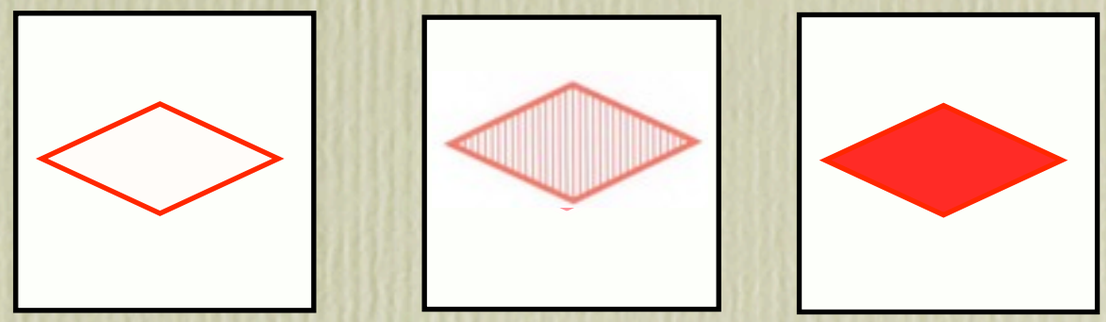
Homogeneity

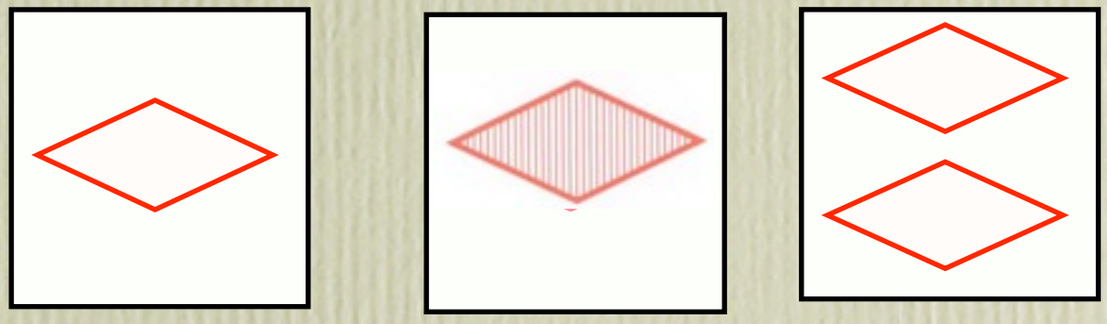


Example: of regularities of Fill

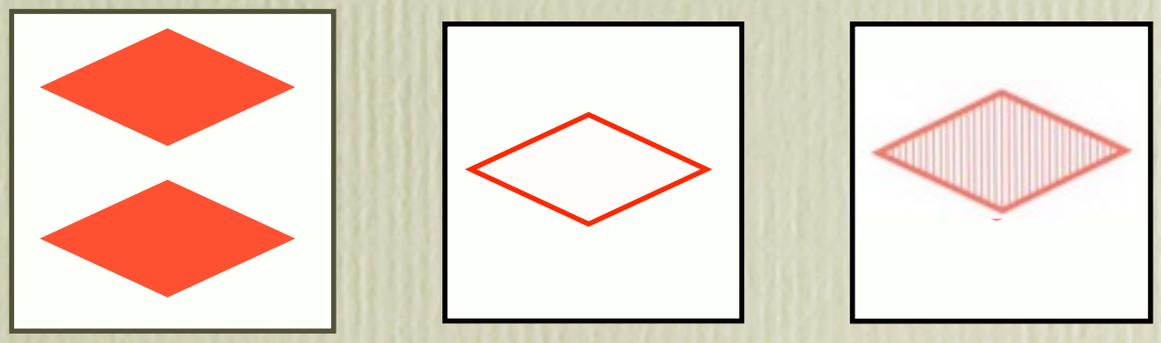
Monotony



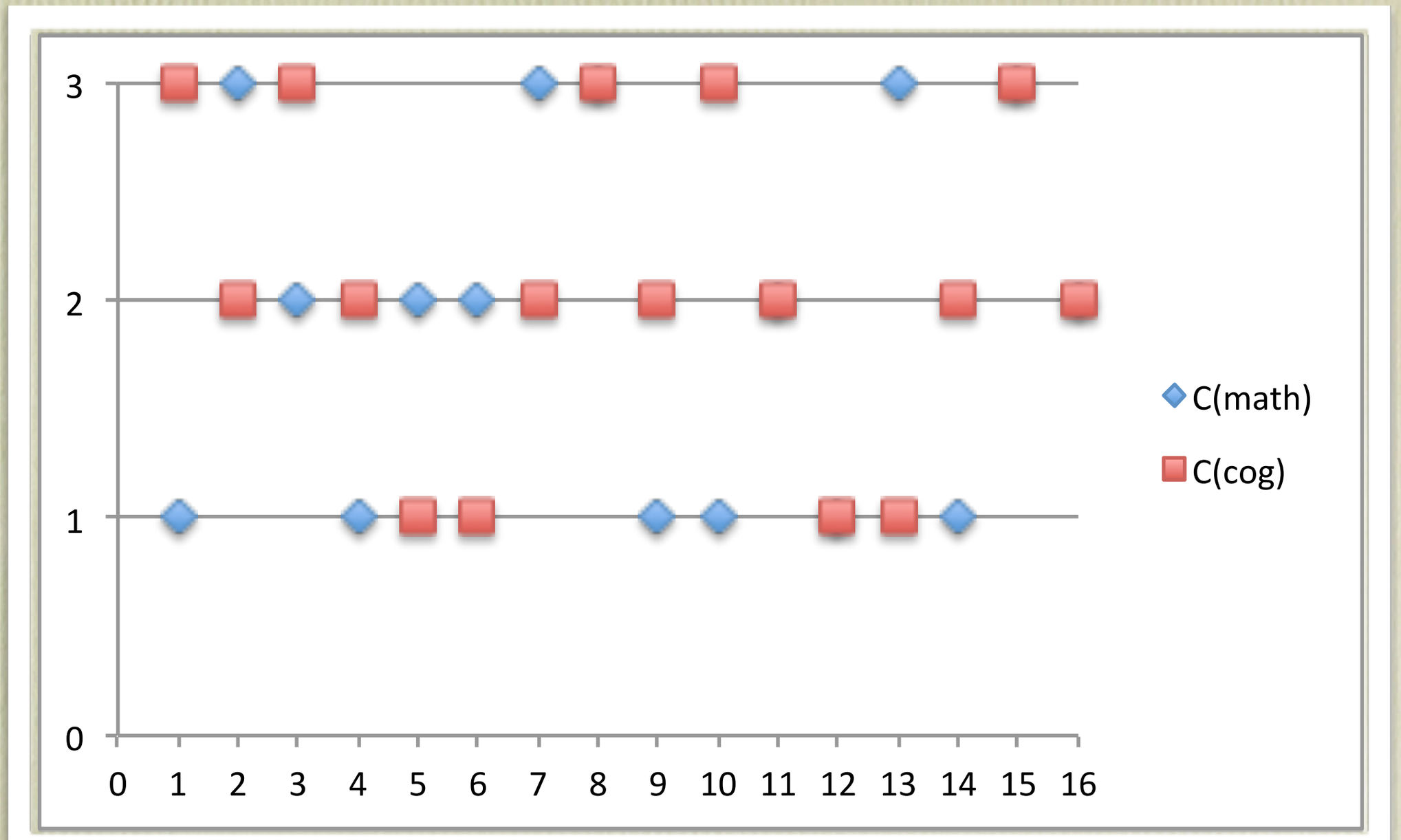
Symmetry



Difference



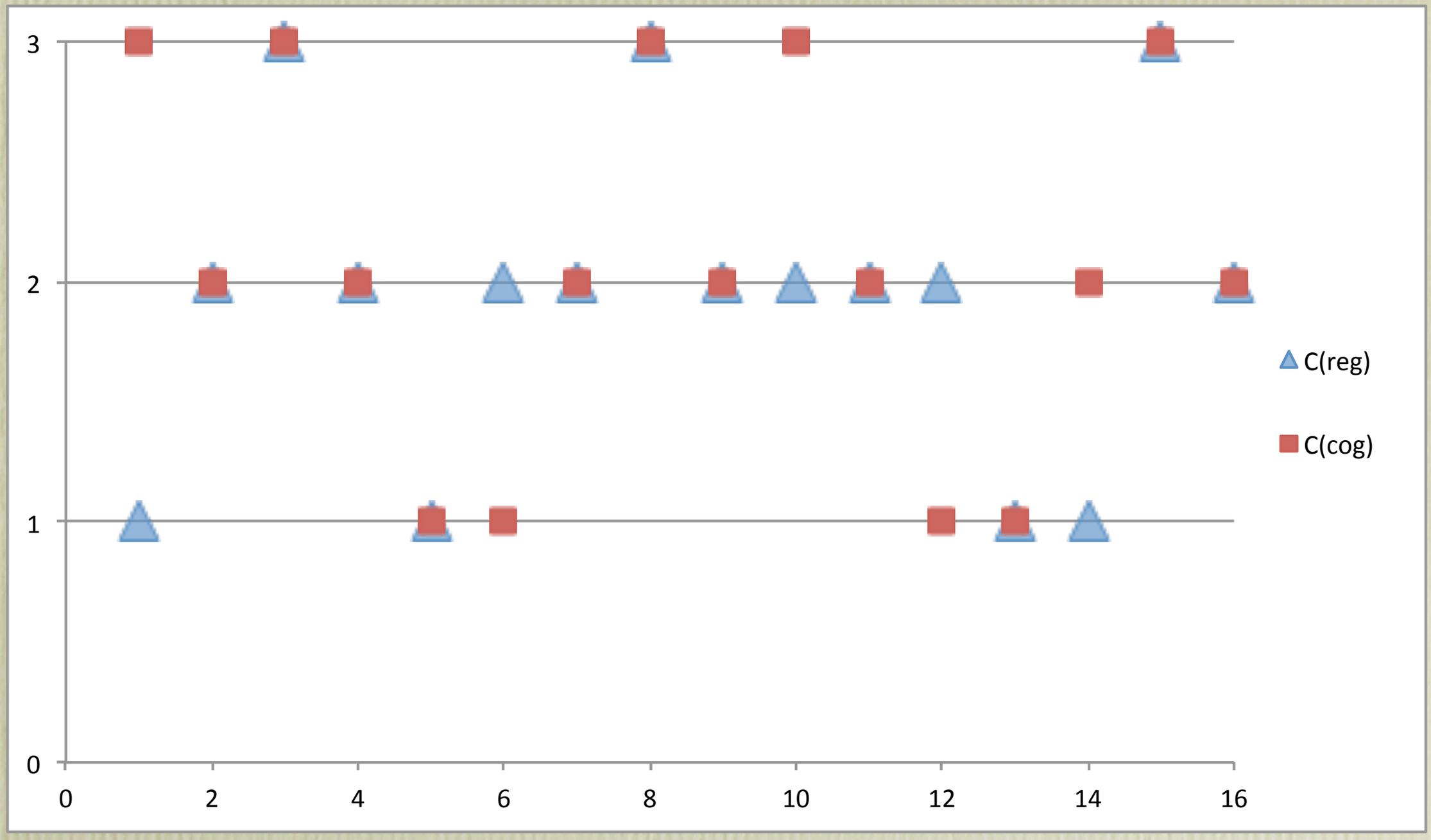
Feldman vs. Cognitive Complexities



C(math) - choice according to the Feldman's complexity

C(cog) - choice according to the cognitive complexity

Regularity vs. Cognitive Complexities



C(math) - choice according to the Regularity complexity

C(cog) - choice according to the cognitive complexity

Research findings

- We study ways students understand Boolean concepts by comparing the math and the cognitive complexities of Boolean concepts
- We have proposed a function of regularity as a measure of cognitive complexity
- The main findings of the study:
    - The most recognized type of regularity is the monotony
    - The most recognized characteristic of set cards is the number
    - In a large number of examples, no correlation has been indicated between the Feldman's complexity and the cognitive complexity
    - Recognition of regularities supports problem solving

Directions of Future Research in Boolean Concept Learning

- Properties of Boolean functions as Cognitive Regularities. Known properties (monotony, linearity etc., and new human oriented properties).
- Study of Different types of Boolean tasks in Boolean Concept Learning. (Example: Recognition vs. Reverse Engineering)
- Study of Boolean Approximation by Humans. (How people recognize Boolean concepts if the requirement of exactness is relaxed?)

Any combination of the above directions is of considerable interest.

Conclusions

- We have analyzed the main results obtained in the field, related to studies of the concept of cognitive complexity. We have defined three groups of factors, which open the three new directions of further research. The three directions have allowed formulating a model of the subject matter of Boolean Concept Learning

- We therefore form the space of research in the new subject matter by taking into account a) properties of Boolean functions, b) various types of Boolean problems and c) techniques of approximating the Boolean functions by humans

- The phenomenon of Boolean Concept Learning has a perspective to be studied not only by using empirical methods of the conventional cognitive science, but also by analytical methods. This may bring scientists from the field of classical Logic Design to a research in the Boolean Concept Learning