

## Rational Choice Theory

### Practice Exam

1. Explain why

a.  $p \rightarrow q$  is equivalent to  $\neg q \rightarrow \neg p$

but

b.  $p \rightarrow q$  is *not* equivalent to  $q \rightarrow p$ .

2. Show that, in the presence of discounts for quantity (that is, the price per unit goes down as you buy large quantities) the feasible set of the consumer is not convex.

3. Discuss the reasons for which competitive equilibria might not be efficient in the interaction between a patient and a physician.

4. Assume that a decision maker exhibits the following preferences (where  $P = (p_1, x_1; p_2, x_2; \dots; p_n, x_n)$  is a lottery that guarantees  $x_i$  euros with probability  $p_i$ ):

$$(500, 1) \sim (0, 0.4; 1000, 0.6)$$

$$(300, 1) \sim (0, 0.5; 500, 0.5).$$

If this decision maker satisfies the von Neumann-Morgenstern axioms, can you say whether she will prefer

$$(0, 0.2; 300, 0.3; 1000, .5)$$

to

$$(500, 1)$$

?

5. Compute all Nash equilibria in the following game:

	L	R
T	(4, 2)	(1, 1)
B	(0, 1)	(3, 5)

## Solutions to Practice Exam

Problems 1-3 are taken from the notes and their solutions have been posted. You don't need to be as detailed in your answer, though. It would suffice to say, for instance:

1. Assume that  $p \rightarrow q$ . Then, if  $q$  is false, we know that  $p$  is false (otherwise,  $p$  is true and then also  $q$ ). That is,  $\neg q \rightarrow \neg p$ . The other direction follows the same logic.

2. If I get 50% discount after a quantity of 100 of each product, the prices are  $p_1 = p_2 = 1$  and income is  $I = 200$ , the points  $(0, 300)$  and  $(300, 0)$  are in the budget set but  $(150, 150)$  isn't.

3. In this interaction, the physician has information that the patient does not. Hence we have asymmetric information, and we know that this is a reason for a competitive equilibrium not to be Pareto efficient. For example, the physician may prescribe more or less tests than necessary, depending on his financial incentive, and the patient may not know the quality of the medical treatment she gets.

The solutions for the other problems are:

4. Let's normalize the vNM function to be

$$\begin{aligned}u(1000) &= 1 \\u(0) &= 0\end{aligned}$$

the indifference  $(500, 1) \sim (0, 0.4; 1000, 0.6)$  implies

$$u(500) = 0.4u(0) + 0.6u(1000) = 0.6$$

and  $(300, 1) \sim (0, 0.5; 500, 0.5)$  implies

$$\begin{aligned}u(300) &= 0.5u(0) + 0.5u(500) \\&= 0.5 * 0 + 0.5 * 0.6 \\&= 0.3\end{aligned}$$

Hence the expected utility of  $(0, 0.2; 300, 0.3; 1000, .5)$  is

$$\begin{aligned} & 0.2u(0) + 0.3u(300) + 0.5u(1000) \\ = & 0.2 * 0 + 0.3 * 0.3 + 0.5 * 1 \\ = & 0.59 \end{aligned}$$

which is lower than the expected utility of  $(500, 1)$  (the latter being  $u(500) = 0.6$ ). That is, the decision maker will prefer 500 for sure to the lottery  $(0, 0.2; 300, 0.3; 1000, .5)$ .

5. Clearly, (T,L) and (B,R) are the only two pure strategy Nash equilibria. There is also a mixed strategy Nash equilibrium. To find it, let  $p$  be the probability of T (for player I) and  $q$  the probability of L (for player II).

For player I to mix between the two strategies we have to have

$$4 * q + 1 * (1 - q) = 0 * q + 3 * (1 - q)$$

or

$$\begin{aligned} 4q &= 2(1 - q) \\ q &= \frac{1}{3} \end{aligned}$$

and for player II to mix we have to have

$$\begin{aligned} 2 * p + 1 * (1 - p) &= 1 * p + 5 * (1 - p) \\ p &= 4(1 - p) \\ p &= \frac{4}{5} \end{aligned}$$

Thus, the third equilibrium is mixed and it is given by  $((\frac{4}{5}, \frac{1}{5}), (\frac{1}{3}, \frac{2}{3}))$ .