

Decisions Theory in Practice

Problems and Exercises

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Suggested Reading

On the foundations of theories of rational choice:

Hammond, J.S., R. Keeney, and H. Raiffa, (1998) *Smart Choices: A Practical Guide to Making Better Decisions*. Harvard Business School Press.

Binmore, K. (2009) *Rational Decisions*. Princeton University Press.

Gilboa, I. (2010) *Rational Choice*. MIT Press.

On the psychology of choice and violations of classical theories:

Bazerman M. H. and D. Moore (2006), *Judgment in Managerial Decision Making*. Wiley.

Ariely, D. (2008), *Predictably Irrational*. MIT Press.

Thaler, R. H. and C. R. Sunstein (2008), *Nudge: Improving Decisions About Health, Wealth, and Happiness*. Yale University Press.

An integrative approach:

Gilboa, I. (forthcoming) *Making Better Decisions: Decision Theory in Practice*. Blackwell-Wiley.

Chapter 1: Judgment and Choice Biases I

Problems – Group A

Problem 1.1

A 65-year old relative of yours suffers from a serious disease. It makes her life miserable, but does not pose an immediate risk to her life. She can go through an operation that, if successful, will cure her. However, the operation is risky. 30% of the patients undergoing it die. Would you recommend that she undergoes it? _____

Problem 1.2

You are given \$1,000 for sure. Which of the following two options would you prefer?

- a. to get additional \$500 for sure;
- b. to get another \$1,000 with probability 50%, and, with probability 50%, nothing (and be left with the first \$1,000).

Problem 1.3

You go to a movie. It was supposed to be good, but it turns out to be boring. Would you leave in the middle and do something else instead?

Problems – Group B

Problem 1.4

A 65-year old relative of yours suffers from a serious disease. It makes her life miserable, but does not pose an immediate risk to her life. She can go through an operation that, if successful, will cure her. However, the operation is risky. 70% of the patients undergoing it survive. Would you recommend that she undergoes it?

Problem 1.5

You are given \$2,000 for sure. Which of the following two options would you prefer?

- a. to lose \$500 for sure;
- b. to lose \$1,000 with probability 50%, and otherwise – to lose nothing.

Problem 1.6

Your friend had a ticket to a movie. She couldn't make it, and gave you the ticket "instead of just throwing it away". The movie was supposed to be good, but it turns out to be boring. Would you leave in the middle and do something else instead?

Chapter 2: Judgment and Choice Biases II

Problems – Group A

Problem 2.1

Linda is 31 years old, single, outspoken, and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and she participated in antinuclear demonstrations.

Rank order the following eight descriptions in terms of the probability (likelihood) that they describe Linda:

- a. Linda is a teacher in an elementary school.
 - b. Linda works in a bookstore and takes yoga classes.
 - c. Linda is active in a feminist movement.
 - d. Linda is a psychiatric social worker.
 - e. Linda is a member of the League of Women Voters.
 - f. Linda is a bank teller.
 - g. Linda is an insurance salesperson.
 - h. Linda is a bank teller who is active in a feminist movement.
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Problem 2.2

In four pages of a novel (about 2,000 words) in English, do you expect to find that more than ten words of $_ _ _ _ n _$ (seven-letter words that have the letter n in the sixth position)?

Problem 2.3

What is the probability that, in the next 2 years, there will be a cure for AIDS?

Problem 2.4

What is the probability that, during the next year, your car would be a “total loss” due to an accident?

Problem 2.5

Which of the following causes more deaths:

- a. Digestive diseases
- b. Motor vehicle accidents

Problem 2.6

A newly hired engineer for a computer firm in Melbourne, Australia, has four years of experience and good all-around qualifications.

Do you think that her annual salary is above or below \$65,000? _____

What is your estimate of the salary? _____

Problem 2.7

You have a ticket to a concert, which cost you \$50. When you arrive at the concert hall, you find out that you lost the ticket. Would you buy another one (assuming you have enough money in your wallet)? _____

Problem 2.8

Which of the following two options do you prefer?

- a. Receiving \$10 today.
- b. Receiving \$12 a week from today.

Problems – Group B

Problem 2.9

Linda is 31 years old, single, outspoken, and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and she participated in antinuclear demonstrations.

Rank order the following eight descriptions in terms of the probability (likelihood) that they describe Linda:

- a. Linda is a teacher in an elementary school.
 - b. Linda works in a bookstore and takes yoga classes.
 - c. Linda is active in a feminist movement.
 - d. Linda is a psychiatric social worker.
 - e. Linda is a member of the League of Women Voters.
 - f. Linda is a bank teller.
 - g. Linda is an insurance salesperson.
 - h. Linda is a bank teller who is active in a feminist movement.
-

Problem 2.10

In four pages of a novel (about 2,000 words) in English, do you expect to find that more than ten words of the form _ _ _ _ *ing* (seven-letter words that end with *ing*)?

Problem 2.11

What is the probability that, in the next 2 years, there will be a new genetic discovery in the study of apes, and a cure for AIDS?

Problem 2.12

What is the probability that, during the next year, your car would be a “total loss” due to:

- a. an accident in which the other driver is drunk?
- b. an accident for which you are responsible?
- c. an accident occurring while your car is parked on the street?
- d. an accident occurring while your car is parked in a garage?
- e. one of the above? _____

Problem 2.13

Which of the following causes more deaths:

- a. Digestive diseases
- b. Motor vehicle accidents

Problem 2.14

A newly hired engineer for a computer firm in Melbourne, Australia, has four years of experience and good all-around qualifications.

Do you think that her annual salary is above or below \$135,000? _____

What is your estimate of the salary? _____

Problem 2.15

You are going to a concert. Tickets cost \$50. When you arrive at the concert hall, you find out that you lost a \$50 bill. Would you still buy the ticket (assuming you have enough money in your wallet)? _____

Problem 2.16

Which of the following two options do you prefer?

- a. Receiving \$10 50 weeks from today.
- b. Receiving \$12 51 weeks from today.

Exercises (Chapters 1 and 2 combined)

1. Some people are afraid of flights. They are often surprised to learn that many more people lose their lives in motor vehicle accidents (on the ground) than in flights. Why are their evaluations of these numbers inaccurate? And does it follow that flying is less dangerous than driving?
2. Jim and Joe are students who live on small scholarships. They go to an all-you-can-eat restaurant and pay \$8.95 for the meal. Joe is unexpectedly told that, being the 100th customer of the day, he gets his money back (and gets to eat at no charge). Other things being equal, do you think that Joe will consume the same amount of food as will Jim?
3. Magazines often offer their new customers subscription over an initial period at a very low cost. Provide at least two reasons for which this may be a smart way to attract customers.
4. Credit card companies used to offer students loans at enticing rates. Presumably, this was an example of voluntary trade among adults, which should be allowed in a free market. Provide a reason for which such offers may be restricted by law.
5. In most countries, a driver who wishes to join an organ donation program has to make an explicit choice to do so. There is a proposal to make every driver an organ donor unless they opt out.¹ Do you think that this proposal might have an effect on the number of organ donors? If so, which psychological effect might be responsible for this?
6. Mary noticed that, when she gets an unexpected bonus from her employer, she allows herself to buy goods she didn't plan to buy, and often ends up spending an amount of money larger than her bonus. What psychological effect is related to this phenomenon, and what goes wrong in her decision making?

Chapter 3: Consuming Statistical Data

Problems

Problem 3.1

A newly developed test for a rare disease has the following features: if you do not suffer from the disease, the probability that you test positive (“false positive”) is 5%. However, if you do have the disease, the probability that the test fails to show it (“false negative”) is 10%.

You took the test, and, unfortunately, you tested positive. The probability that you have the disease is: _____

Problem 3.2

You are going to play the roulette. You first sit there and observe, and you notice that the last five times it came up “black.” Would you bet on “red” or on “black”? _____

Problem 3.3

A study of students’ grades in the US showed that immigrants had, on average, a higher grade point average than did US-born students. The conclusion was that Americans are not very smart, or at least do not work very hard, as compared to other nationalities.

What do you think?

Problem 3.4

In order to estimate the average number of children in a family, a researcher sampled children in a school, and asked them how many siblings they had. The answer, plus one, was averaged over all children in the sample to provide the desired estimate.

Is this a good estimate?

Problem 3.5

A contractor of small renovation projects submits bids and competes for contracts. He noticed that he tends to lose money on the projects he runs. He started wondering how he can be so systematically wrong in his estimates.

Can you explain that?

Problem 3.6

Comment on the following.

[At a restaurant] Ann: I hate it. It's just like I told you: they don't make an effort anymore.

Barbara: They?

Ann: Just taste it. It's really bad food. Don't you remember how it was the first time we were here?

Barbara: Well, maybe you're tired.

Ann: Do you like your dish?

Barbara: Well, it isn't bad. Maybe not as good as last time, but...

Ann: You see? They first make an effort to impress and lure us, and then they think that we're anyway going to come back. No wonder that so many restaurants shut down after less than a year.

Barbara: Well, I'm not sure that this restaurant is so new.

Ann: It isn't?

Barbara: I don't think so. Jim mentioned it to me a long time ago, it's only us who didn't come here for so long.

Ann: So how did they know they should have impressed us the first time and how did they know it's our second time now? Do you think the waiter was telling the chef, "Two sirloins at no. 14, but don't worry about it, they're here for the second time"?

Problem 3.7

Studies show a high correlation between years of education and annual income. Thus, argued your teacher, it's good for you to study: the more you do, the more money you will make in the future.

Is this conclusion warranted?

Problem 3.8

In a recent study, it was found that people who did not smoke at all had more visits to their doctors than people who smoked a little bit. One researcher claimed, "Apparently, smoking is just like consuming red wine – too much of it is dangerous, but a little bit is actually good for your health!"

Do you accept this conclusion?

Problem 3.9

Comment on the following.

Charles: I don't use a mobile phone anymore.

Daniel: Really? Why?

Charles: Because it was found to be correlated with brain cancer.

Daniel: Com'n, you can't be serious. I asked an expert and they said that the effect is so small that it's not worth thinking about.

Charles: As long as you have something to think with. Do as you please, but I'm not going to kill myself.

Daniel: Fine, it's your decision. But I tell you, the effects that were found were insignificant.

Charles: Insignificant? They were significant at the 5% level!

Problem 3.10

Comment on the following.

Mary: My skin is killing me. Look how red it is.

Paula: Yeah, it's really bad. Why don't you take something?

Mary: I tried everything. Nothing works.

Paula: Nothing?

Mary: I'm telling you, I tried anything I could put my hands on.

Paula: Look, maybe I can help you. I know this guy who works for BigMed, you know, the drug company.

Mary: Sure I know, they're big.

Paula: Well, they are in the final phase of testing an ointment, and I think it's precisely for this type of rash. They need volunteers for the test – why don't you join the study? They even give you all kinds of skin products as a gift.

Mary: I don't need any gifts. If it can help, I have enough of an incentive to take it, believe me. But what if it's going to be worse?

Paula: It won't. They're a serious company and the product has already passed many tests.

Mary: So was it approved by the FDA [Food and Drug Administration]?

Paula: No, they're still testing it, that's the point of the test.

Mary: I don't get it. It's either or: if you're so sure it's OK, why isn't it approved? If it's not yet approved, it's probably not yet OK.

Paula: It's never 100% sure to be 'OK', as you put it. A drug can be approved and then still kill people. It's all a matter of probabilities and statistics.

Mary: What does it help me that you call this probability? Again: either the probability is low enough so that it can be approved, or it's not low enough and then I don't want to take it.

Paula: Which probability?

Mary: The probability that something bad might happen. I don't know what, but they are testing something, aren't they?

Paula: It's up to you, of course. It's your skin and it's your decision. But we always take risks, when we board planes and when we play squash. All I'm saying is that, given BigMed's reputation, this is a very reasonable risk to take, and it's a pity to go on suffering.

Mary: Well, then, given BigMed's reputation, why are they still testing it instead of the FDA just approving it?

Exercises

1. A home owner who has a mortgage and who is not going to default may miss a payment on a particular month with probability 2.8%. (One who defaults obviously misses the payment for sure). If Mr A missed a payment, what is the probability that he is going to default?
 - a. 2.8%
 - b. $2.8\%/[2.8\%+1]$
 - c. $1/[2.8\%+1]$
 - d. Cannot be determined.
 - e. Can be determined, but differs from (a)-(c).

2. A leading newspaper followed up on the inflation rate predictions by several economists. It has selected the five with the best record, and asked them to predict the inflation in the current year. At the end of the year, it appeared that they were not so successful. The journalist concluded that we must be living in a very tumultuous period, where even top experts cannot make good predictions. This conclusion is
 - a. Erroneous, and it reflects the journalist's anchoring bias.
 - b. Reasonable, because the journalist can't tell the inflation rate either.
 - c. Erroneous, as this might be a case of regression to the mean.
 - d. Quite likely, though the journalist may still be exposed to an availability bias.

3. “Most journalists I met were superficial. Next time I see someone superficial, I’m going to ask them if they are journalists.” Which statement would you endorse?
- a. It’s not enough to know that most journalists are superficial – maybe most people are superficial anyway. One has to look at the comparison between superficial people among journalists and among non-journalists.
 - b. Even if most journalists are superficial, it doesn’t mean that most superficial people are journalists.
 - c. Assuming that there are many more superficial people in the population than there are journalists, the percentage of superficial among the journalists must be larger than the percentage of journalists among the superficial.
 - d. All of the above.
 - e. None of the above.
4. Suppose that fashion models tend to be stupid more than the rest of the population. In this case
- a. We can conclude that the fashion industry tends to hire stupid people for modeling.
 - b. We can conclude that the life of a model tend to dull the mind.
 - c. We can conclude that the fashion industry chooses its models according to some criteria that correlate negatively with intelligence.
 - d. All of the above (All are warranted conclusions).
 - e. None of the above.

5. Your friend has a car repair shop, specializing in transmission systems. You told him that you consider buying a car of make A, which is not very popular. His reaction was, “Don’t get near them – I fix their transmission all the time. In fact, they’re 90% of my business!” What can you say based on your friend’s experience?
- a. That, if you buy a car of make A, you’ll have 90% probability of transmission problems.
 - b. That, if you buy a car of make A, you’ll be more likely to have transmission problems than not.
 - c. That, if you buy a car of make A, you’ll be more likely to have transmission problems than if you buy a car made by another make.
 - d. All of the above.
 - e. None of the above.
6. A certain genetic disease is recessive, which implies that a child might have it only if both parents are carriers of the disease. The probability of each person being a carrier is 2%. One of two prospective parents took a test and was found to be a carrier. Before the second took the test, the doctor said, “Oh, don’t worry: I have seen people who were carriers of the disease in my life, but I’ve never seen two parents being carriers!” Do you support the doctor’s view?
7. We wish to estimate the expectation μ of a random variable X . We ask two statisticians, one classical and the other Bayesian, to do the job. The difference between them will be that
- a. The Bayesian one will have a guess about μ even before taking the sample.
 - b. The Bayesian one will not take a sample at all.
 - c. The classical one will generate a confidence interval, but she will not truly think that it contains the parameter μ .
 - d. The classical one will prefer counter-intuitive answers.
 - e. All of the above.

8. The difference between confidence intervals and hypotheses tests is that
- a. The confidence level is a probability only a priori, before taking the sample, whereas significance is a probability also after the sample has been taken.
 - b. Significance looks at the difference between values of the unknown parameters, and not just at the probabilities of type I and type II errors.
 - c. Confidence intervals are general-purpose estimation technique, whereas each hypothesis test is tailored to a particular statement.
 - d. All of the above.
 - e. None of the above.

Chapter 4: Decisions under Risk

Problems

In each of the following problems you are asked to choose between two lotteries. A “lottery” gives you certain monetary prizes with given probabilities.

For instance,

A:	\$0	.5
	\$1,000	.5

A is a lottery that gives you \$0 with probability 50%, and \$1,000 otherwise.

A “sure” prize will be represented as a lottery with probability 1, say:

B:	\$500	1
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B is a “lottery” that gives you \$500 for sure.

Please denote your preferences between the lotteries by

$$A \prec B \quad \text{or} \quad A \succ B$$

(or $A \sim B$ if you are indifferent between the two).

Problem 4.1

A:	\$0.5	B:	\$500	1
	\$1,000.5			

Problem 4.2

A:	\$0.2	B:	\$500	1
	\$1,000.8			

Problem 4.3

A:	\$2,000.5	B:	\$1,000	.5
	\$4,000.5		\$5,000	.5

Problem 4.4

A:	\$2,000.5	B:	\$1,000	.4
	\$4,000.5		\$5,000	.6

Problem 4.5

A: \$0 .2 B: \$3,000

\$4,000 .8

Problem 4.6

A: \$0 .2 B: \$400 .6

\$400 .6 \$500 .4

\$1,000 .2

Problem 4.7

A: \$0 .1 B: \$400 .5

\$400 .5 \$500 .5

\$1,000 .4

Problem 4.8

A:	\$2,000 .2	B:	\$1,000 .2
	\$4,000 .2		\$5,000 .2
	\$6,000 .6		\$6,000 .6

Problem 4.9

A:	\$2,000 .2	B:	\$1,000 .16
	\$4,000 .2		\$5,000 .24
	\$6,000 .6		\$6,000 .6

Problem 4.10

A:	\$0 .8	B:	\$0 .75
	\$4,000 .2		\$3,000 .25

Exercises

1. Assume that you are indifferent between getting \$700 and getting \$1000 with probability 80% (and otherwise nothing). Assume also that you are indifferent between getting \$300 and getting **\$700** (not 1000 this time!) with probability 60% (and otherwise nothing). Consider lottery A, which gives you \$1000 with probability $\frac{2}{3}$ (and otherwise nothing), and lottery B, which gives you a 50%-50% bet between \$300 and \$700 dollar. If you follow von-Neumann-Morgenstern's theory, you should:
 - a. Prefer A to B
 - b. Prefer B to A
 - c. Be indifferent between A and B
 - d. One cannot tell based on the data.

2. Mary likes the von Neumann Morgenstern's axioms and she would like to make decisions in accordance with these axioms. By careful introspection, she has decided that she would be indifferent between

\$400 for sure and a 50% of obtaining \$1,000 (otherwise – nothing);

and also between

\$600 for sure and a 80% of obtaining \$1,000 (otherwise – nothing).

Mary is offered a bet among (\$0, \$400, \$600, \$1,000) with equal chances (25% each) for a cost of \$400. Should she prefer the bet or should she prefer to keep her \$400?

3. A state lottery sells tickets for a cost of \$1 each. The ticket has a probability of $1/(2,400,000)$ of winning \$1,000,000, and otherwise – nothing.
 - a. What is the expected profit of the state from each ticket sold?
 - b. In the hope of increasing profits, the state considers to increase the award to \$2,000,000 and to reduce the probability of winning to $1/(4,800,000)$. A statistician said that it's not worth the trouble, because the expected profit remains precisely the same. What do you think?
4. It is often argued that the value function in Kahneman and Tversky's Prospect Theory is convex in the domain of losses, that is, that individuals behave in a risk loving way when it comes to losses. How can this be reconciled with the fact that people buy insurance (where premia exceed expected losses)?

Additional Topic: Well-Being and Happiness

Problems

Problem 5.1

Mary works in your public relations office. She is doing a good job and you're pleased with her performance. About six months ago, she hired a new employee, named Jane, who turned out to be a born talent. They get along fine.

Mary's direct boss just quit, and you're looking for someone for the job. You don't think that Mary is perfect for it. By contrast, Jane seems a great fit. But it may be awkward to promote Jane and make Mary her subordinate. A colleague suggested that you go ahead and do this, but give both of them a nice raise to solve the problem.

What percentage of a salary raise do you think will solve the problem?

Problem 5.2

Robert and John went to school together, and they got married roughly at the same time. They lived in the city and enjoyed it very much.

Robert and his wife have not had any children. John and his wife had a first child after one year, and, two years later, they had the second one, now eight months old. As a result, John had to move to suburb, took a mortgage to buy a large house and he feels financially strained.

Robert is on a ski vacation with his wife, while John is at home. He can't even dream of a ski vacation with the two children, to say nothing of the expense. In fact, John would be quite happy just to have a good night sleep.

Do you think that Robert is happier than John?

Solutions

Solutions to odd-numbered exercises are given below.

Chapters 1-2

1. Flight accidents are typically much more visible than motor vehicle accidents. By availability heuristic, we should expect that they would be over-represented in our memory, and result in an over-estimate of the danger of flying. On the other hand, the mere numbers of fatalities caused by each mode of transportation aren't the relevant statistics either, because we would want to look at these numbers relative to the number of miles driven/flown, or to consider some other measures that would make it more related to the conditional probability that we face in either mode of transportation.
3. One reason may be sheer forgetfulness: once the initial period is up, you may simply forget to cancel the subscription, even if at first you thought you would. If you planned to cancel the subscription but didn't, this may be a form of dynamic inconsistency. The other obvious reason has to do with habit formation, the endowment effect, or the status quo bias: before having the magazine you may not value it as much as you would after having consumed it for a while.
5. It stands to reason that this changing the default choice from "opt out" to "opt in" would have a big effect on the number of people choosing to donate organs. One related phenomenon is the anchoring effect: when a decision is the default, it serves as an anchor. It may also be rational to choose it, if you have an implicit belief that the default choice is the norm in the society you live in, and that it probably makes sense if it were chosen to be the norm. Another effect might be simple forgetfulness or unawareness: many people adopt the default choices without ever checking what they are.

Chapter 3

1. The correct answer is (d): this probability cannot be determined without knowing what percentage of overall home owners with mortgage eventually default.
3. The answer is (d). (a) is true because what we need to compare to make any inference is whether one event (being a journalist) makes the other (being superficial) more likely than it is before knowing anything, and the benchmark of 50% isn't relevant. Clearly, (b) is a point that we made over and over again. Statement (c) is correct, because, if $P(A|B) > P(B|A)$ whenever $P(A) > P(B)$.
5. The answer is (c). (a) is false because the probability of A given B is not the same as the probability of B given A. (b) is false because the type of inference we can draw relates the probability of the event to other probabilities of the same events (as in $P(A|B) > P(A)$) but not to any specific number such as 50%. Finally, (c) is true, because if the car is not very popular, we may assume that its probability in the entire population is less than 90%, and therefore the probability to find it in the car repair shop is higher than in the general population. Thus,

$$P(\text{make A} \mid \text{problem}) = 90\% > P(\text{make A})$$

and therefore

$$P(\text{problem} \mid \text{make A}) > P(\text{problem})$$

and also

$$P(\text{problem} \mid \text{make A}) > P(\text{problem} \mid \text{another make})$$

7. The answer is (a): the Bayesian statistician will indeed have a guess about the unknown parameter even before looking at the sample. But she will take a sample (hence (b) is false). The classical statistician will not generate a confidence interval that she believes is wrong; she will simply avoid stating her beliefs about the specific interval that resulted, saying that she can only quantify beliefs *given* the unknown parameter, and before the sample was taken. Hence (c) is wrong.

Finally, (d) is also wrong: Classical statistics does not look for counter-intuitive answers, it simply attempts to avoid intuition altogether.

Chapter 4

1. If you assign $u(\$1,000)=1$ and $u(\$0)=0$, you find that

$$u(\$700) = 0.8 * u(\$1,000) + 0.2 * u(\$0) = 0.8$$

and then

$$u(\$300) = 0.6 * u(\$700) + 0.4 * u(\$0) = 0.6*0.8 = 0.48$$

Thus, the expected utility of lottery A is

$$2/3 * u(\$1,000) + 1/3 * u(\$0) = 2/3$$

and of lottery B –

$$0.5 * u(\$700) + 0.5 * u(\$300) = 0.5*0.8 + 0.5*0.48 = 0.64$$

Since $2/3 > 0.64$, you will prefer A to B, and the answer is (a).

3. a. The expected profit is the revenue of \$1 minus the expected payoff, which is

$$\$1,000,000 * (1/2,400,000) \cong 0.416$$

that is, about 58.4 cents.

b. The statistician is right in observing that the expected profit for each ticket does not change. However, because lottery buyers do not base their decisions solely on their expected gain, the volume of sales might well change as a result of the new policy. Even if a potential lottery buyer were an expected utility maximizer, as long as his utility function is not linear, two lotteries with the same expected value need not be equally attractive to him. (And we may ignore potential buyers who are expected utility maximizers with linear utility functions because they would

not buy lottery tickets anyway.) Moreover, using Prospect Theory we can expect that the higher award will attract more lottery buyers, but that the difference between the probability of $1/2,400,000$ vs. $1/4,800,000$ will hardly make a difference. Hence, we should not be surprised is the new award structure does indeed attract more buyers and increases profits.
