# History as a Coordination Device*

Rossella Argenziano† and Itzhak Gilboa‡

October 2010

**Abstract**

Coordination games often have multiple equilibria. The selection of equilibrium raises the question of belief formation: how do players generate beliefs about the behavior of other players? This paper takes the view that the answer lies in history, that is, in the outcomes of similar coordination games played in the past, possibly by other players. We analyze a simple model in which a large population plays a game that exhibits strategic complementarities. We assume a dynamic process that faces different populations with such games for randomly selected values of a parameter. We introduce a belief formation process that takes into account the history of similar games played in the past, not necessarily by the same population. We show that when history serves as a coordination device, the limit behavior depends on the way history unfolds, and cannot be determined from a-priori considerations.

## 1 Introduction

Games with strategic complementarities typically exhibit multiple equilibria. The game theoretic literature has witnessed many attempts to select equilibria based on the parameters of the game. The equilibrium selection literature includes many notions that are defined by the game itself (see van

*We are grateful to Dirk Bergemann, Don Brown, Dino Gerardi, Stephen Morris, Barry O'Neill, Ady Pauzner, Alessandro Pavan, Colin Stewart and Nicolas Vieille for comments and references. Gilboa gratefully acknowledges support from the Israel Science Foundation (Grant No. 975/03). This paper is part of the Polarization and Conflict Project CIT-2-CT-2004-506084 funded by the European Commission-DG Research Sixth Framework Programme.

†Department of Economics, University of Essex. rargenz@essex.ac.uk

‡HEC, Paris, and Tel-Aviv University. tzachigilboa@gmail.com

Damme (1983)), such as the risk-dominance criterion. Other types of considerations attempted to embed the game in a dynamic process (Young (1993), Kandori, Mailath, and Rob (1993), Burdzy, Frankel and Pauzner (2001)) or in incomplete information set-up (Carlsson and van Damme (1994)).

It is noteworthy that risk dominance has emerged as the preferred selection criterion based on quite different types of considerations. On the other hand, the literature on strategic complementarities arising from network externalities tends to favor Pareto dominant equilibria over risk dominant ones (see Katz and Shapiro (1986)). This suggests a more agnostic view, according to which the parameters of the game cannot, in general, predict equilibrium selection. It appears that game theoretic considerations could be used to impose certain restrictions on the possible outcomes, but the actual selection of an equilibrium is often left to history, chance, institutional details, or other unmodeled factors.

In this paper we are interested in a dynamic process, according to which large populations are called upon to play a simple coordination game. In the first stage, each player chooses either a low or a high action. In the second stage, nature chooses a low or a high outcome, and nature's move depends on the set of players choosing the high action. Consider the decision of a single player in this game. The optimal action to take depends on his assessment of the probability of a high outcome. We maintain that this assessment would and should be based on the results of past instances of similar games. These games may have been played by the same population or by others. Each past game might differ from the current one by one parameter at most, which is a proxy for the difference in the expected payoff of the two strategies. Both the nature of the game and the identity of the population playing it should be taken into account in the evaluation of the similarity of past games to the present one. But ignoring these past games would hardly seem a rational way of generating beliefs.[1]

[1] The belief formation process may be embedded in a meta-game, which will also have

Consider for example the following "revolution" game played by a large population. In stage 1, each player $i \in [0,1]$ chooses whether to participate in a revolutionary attempt, or to opt out. In stage 2, nature chooses a move in $\{F, S\}$, which stand for *Failure* and for *Success* of the revolution, respectively. Nature's move depends on the set of players who have chosen to participate. After each player determined her choice of participation and nature determined the success of the revolution, the game is over. The payoff of each player depends only on her own choice of participation, and on nature's move. The payoff function $u = u_i$ for every $i \in [0,1]$ is given by the following matrix:

$$\begin{array}{ccc} & S(uccess) & F(ailure) \\ Participate & 1 & 0 \\ Opt\ out & \frac{x+1}{2} & x \end{array} \tag{1}$$

where $x \in [0,1]$ is the parameter of the game.

The interpretation of this matrix is as follows. The worst thing that can happen to an individual in this game is to participate in a failed coup. The result is likely to involve imprisonment, exile, decapitation, and the like. This worst payoff is normalized to 0. The best thing that can happen to an individual is that she participates in a revolution that succeeds. In this case she is a part of a (presumably) better and more just society. This payoff is normalized to 1.

An individual who decides to participate in the revolution therefore decides to bet on its success with the extreme payoffs of 0 and 1. Between these extreme payoffs lie the payoffs for an individual who decides to opt out, foregoing the chance of being part of the revolution. The payoff of such an individual still depends on the outcome of the revolutionary attempt. Should this attempt fail, such an individual would get $x$, which is a measure of the well-being of the people in the status quo. If, however, the revolution

a flavor of a coordination game. We assume, however, that people have a fundamental tendency to expect the future to be similar to the past. To quote Hume (1748), "From similar causes we expect similar effects."

succeeds, even the individuals who were passive will benefit from the new regime. However, not being part of the revolutionary forces, they would not reap the benefits of revolution in its entirety. Their payoff would only equal the arithmetic average between the full benefit, 1, and the status quo, $x$.

Consider the decision problem of a potential rebel. Imagine that rumors have been spreading that the revolution would start tonight. She can ignore the rumors and go to sleep, or take to the streets. For simplicity, assume that this is a one-shot, binary decision. The potential rebel sits at home and attempts to assess the probability that the revolution would succeed. How would she do that? Suppose that it is common knowledge in the population that revolution games of the type above have been played in the past. We believe that the history of such similar games played in the past should affect the beliefs of the potential rebel.

In this paper, we present a simple belief formation process for a class of games that includes the "revolution" example above. The process is such that the probability assigned to a high outcome in the current game is the weighted empirical frequency of high outcomes in past games, where the weights are given by a similarity function that takes into account the differences between past games and the current one. We find that beliefs that are history-dependent may lead to different behavior, depending on the way history unfolds.

The rest of this paper is organized as follows. We first discuss related literature. Section 2 describes the stage game. We devote Section 3 to modeling the way players generate beliefs given history. Section 4 describes the dynamic process and provides the main result of the paper. Finally, Section 5 concludes.

### 1.1 Related literature

Our paper is closely related to the equilibrium selection literature discussed in the introduction. It also relates to the literature on coordination games of regime change. The conceptualization of a revolution as a coordination game dates back to Schelling (1960) at the latest. There exist alternative conceptualizations in the political science literature, such as Muller and Opp (1986), who emphasize the public good aspect of a revolution. Yet, the coordination game model of a revolution has been the subject of many studies. Lohmann (1994) studied the weekly demonstrations in Leipzig and the evolution of beliefs along the process. More recently, Edmond (2008) studied information manipulation in games of regime change, whereas Angeletos, Hellwig, and Pavan (2007) focus on a learning process by which individuals playing such games form beliefs. As in Lohmann (1994) and Angeletos, Hellwig, and Pavan (2007), we study the evolution of beliefs in a game that is played repeatedly. However, as opposed to these papers, our game is played by a new population at every stage. Thus, our focus is on the generation of prior beliefs (over other players' actions), based on similar games, rather than on the update of already existing prior beliefs by Bayes's law. Closely related to the belief formation process that we study is the process studied by LiCalzi (1995), which looks at the case where players give the same similarity weight to the outcome of all the games in a given class. Jehiel (2005) introduces a solution concept in which players form beliefs about their opponents' behavior by grouping nodes in which the opponents play into analogy classes. Finally, Steiner and Stewart (2008) study similarity-based learning in games and show that contagion can lead to unique long-run outcomes.

## 2 The Stage Game

We describe a symmetric two-stage extensive form game $G_x$ depending on a parameter $x \in X \equiv \{x^1, ..., x^J\}$ for $J > 2$. The cardinal values of the parameter $x$ will be of no import, but their order will. There is a continuum of players $[0, 1]$. In stage 1 all players move simultaneously. The set of moves for each player $i$ is $S_i = \{0, 1\}$. In stage 2, after each player determined her move in $\{0, 1\}$, nature chooses an outcome $\nu \in \{0, 1\}$. Nature's move depends on the set of players choosing 1 in stage 1, $A \subset [0,1]$. Specifically, if $A$ is Lebesgue-measurable, we assume that nature chooses $\nu = 1$ with probability $\varphi(\lambda(A))$ where $\varphi$ is strictly increasing, with $\varphi(0) = 0$ and $\varphi(1) = 1$, and $\lambda$ stands for Lebesgue's measure. If $A$ is non-measurable, the probability of nature choosing $\nu = 1$ can be defined arbitrarily. At equilibrium, the set $A$ will be measurable.

After each player determined her choice and nature determined the outcome (by the probability $\varphi(\lambda(A))$), the game is over. The payoff of each player depends only on her own choice and on nature's move. However, ex-ante, the game exhibits strategic complementarities: the expected payoff of a strategy $s_i \in \{0,1\}$ is strictly increasing in the measure of players taking this strategy

Assume, then, that an individual $i$ attempts to estimate the expected utility of playing 1 versus 0 for a given game $x \leq x^J$. Suppose that individual $i$'s belief over the measure of other individuals who choose 1 is given by a measure $\mu_{i,x}$ over (the Lebesgue $\sigma$-algebra on) $[0,1]$. That is, for every Lebesgue-measurable set $B \subset [0,1]$, individual $i$ assigns probability $\mu_{i,x}(B)$ to the event that the measure of individuals who eventually choose 1 (with or without herself) lies in $B$. Specifically, the subjective probability of individual $i$ that nature will choose 1 in the game $x$ is

$$\widehat{p}_{i,x} = \int_{[0,1]} \varphi(p) d\mu_{i,x}(p).$$

We assume that for every $x \in X$ is there exists a unique $\bar{p}_x \in [0,1]$ such that playing 1 is optimal if and only if player $i$ believes that nature will choose 1 with probability larger or equal to $\bar{p}_x$.

Given beliefs $\mu_{i,x}$ , player $i$'s expected payoff from playing 1 in game $G_x$ is greater (smaller) than her expected payoff from playing 0 iff $\widehat{p}_{i,x} > \bar{p}_x$ ($\widehat{p}_{i,x} < \bar{p}_x$). For simplicity we assume that in case of a tie, $\widehat{p}_{i,x} = \bar{p}_x$, player $i$ will play 0.[2]

[2] While this assumption will prove immaterial, it simplifies analysis because a random

We assume that $\bar{p}_x$ is strictly increasing in $x \in X$. That is, the games are assumed to be ordered according to the difference in the expected payoff of the two strategies. We further assume that $\bar{p}_{x^1} = 0$ and $\bar{p}_{x^J} = 1$. That is, in the game $G_{x^1}$, strategy 1 is dominant, whereas in $G_{x^J}$ – strategy 0 is.

At equilibrium, all players will have the same beliefs, hence $\widehat{p}_{i,x} = \widehat{p}_x$, i.e., it is independent of $i$. Therefore, at equilibrium all players will either play 1 or 0. This implies that a player who has beliefs $\mu_{i,x}$ and who is aware of the entire process, can follow the same reasoning we do and conclude that the probability of nature choosing 1 is, in fact, either 0 or 1, rather than $\widehat{p}_{i,x}$. To accommodate these players, define $\widehat{p}_{i,x}$ as the player's naive beliefs, and the beliefs that result from our analysis – as the player's sophisticated beliefs. Due to strategic to strategic complementarities, an act that is optimal with respect to the naive beliefs will also be optimal with respect to the sophisticated beliefs.

It is important to note that if naive beliefs were to be ignored, and players were to have only sophisticated beliefs, then any assignment of 0 or 1 to the games $G_{x^1}, ..., G_{x^J}$ could be a consistent set of equilibrium beliefs. However, such a model would not describe the process by which beliefs are formed. The naive belief formation process is the topic of the next section.[3]

## 3 Belief formation process

Our approach to the belief formation question is history- and context-dependent. Specifically, we assume that games of the type $G_x$ above are being played over and over again, by different populations $[0, 1]$, and for different values of $x$. The history of similar games played in the past, which is assumed to be common knowledge, determines the beliefs $\widehat{p}_x$ of the individuals in question.

More concretely, we assume that time is discrete and that the game $G_x$

tie-breaking rule requires some additional assumption about the law of large numbers applying to a continuum of i.i.d random variables.

[3] For a discussion of modeling the formation of rational beliefs, see Gilboa, Postlewaite, and Schmeidler (2010).

is played in every period by a new generation of players. We further assume that at the beginning of each period $t$ nature selects a value for $x_t \in X \equiv \{x^1, ..., x^J\}$ in an i.i.d. manner, according to a known discrete distribution. Thus the process is determined by a probability vector $(p_1, ..., p_J)$.

Let $H_t = \left((x_\tau, \nu_\tau)_{\tau=1}^{t-1}\right)$ be the history at the beginning of period $t$, where, for $\tau < t$, $x_\tau \in X$ denotes the game played at period $\tau$, and $\nu_\tau \in \{0, 1\}$ denotes its outcome. In each period $t$, all the players are assumed to observe the same history $H_t$. Before playing, they observe the game $G_{x_t}$ and form an expectation on the probability of a success that is based on the similarity between the current game and previous games that ended, respectively, with a success or a failure.

Let there be two matrices of non-negative numbers $s^+, s^- : X \times X \to \mathbb{R}_+$ with the following interpretation. $s^+ (x_\tau, x_t)$ measures the degree of support that a past game $x_\tau$, resulting in $\nu_\tau = 1$, gives to the outcome 1 at the new game $x_t$. Similarly, $s^- (x_\tau, x_t)$ measures the degree of support that a past game $x_\tau$, resulting in $\nu_\tau = 0$, gives to the outcome 0 at the new game $x_t$. These degrees of support generate naive beliefs as follows. Denote $S_t = \{\tau < t \,|\, \nu_\tau = 1\}$ and $F_t = \{\tau < t \,|\, \nu_\tau = 0\}$, and set

$$\widehat{p}\,(H_t, x_t) = \frac{\displaystyle\sum_{\tau \in S_t} s^+ (x_\tau, x_t)}{\displaystyle\sum_{\tau \in S_t} s^+ (x_\tau, x_t) + \sum_{\tau \in F_t} s^- (x_\tau, x_t)} \tag{2}$$

Note that if the functions $s^+, s^-$ are identically 1, the expression above is simply the relative frequency of 1's in the history $H_t$. The formula (2) allows different past games to have different weight in the evaluation of probabilities at the current period. Thus, it can be viewed as a generalization of empirical frequencies to weighted empirical frequencies.[4]

[4] The idea of generating beliefs in a game based on past empirical frequencies is at the heart of "fictitious play", dating back to Robinson (1951). Extending empirical frequencies to similarity-weighted empirical frequencies was suggested and axiomatized in Billot, Gilboa, Samet, and Schmeidler (2005), and Gilboa, Lieberman, and Schmeidler (2006).

Specifically, we assume that games with a lower index $x$ are commonly perceived as a-priori more likely to result in 1 than are games with a higher index $x'$. (This is in line with the assumption that the difference in the expected payoff of strategies 1 and 0 is strictly decreasing in $x$.) Thus, a result of 1 in $G_x$ is less surprising than the same result in a game $G_{x'}$. Hence, a result of 1 in $G_x$ lends weaker support to the same result in the current game than would the result 1 in a game $G_{x'}$. Formally, assume that $s^{+}(x,y)$ is strictly increasing in its first argument and strictly decreasing in its second argument. Similarly, we also assume that $s^{-}(x,y)$ is strictly decreasing in its first argument and strictly increasing in its second argument. An implication of these assumptions is that $\widehat{p}(H_t, x_t)$ is strictly decreasing in its second argument.

The formula (2) is not well-defined for the first period, $t = 1$. Also, it allows $\widehat{p}(H_t, x)$ to be 0 or 1, if history contains only 0-outcomes or one 1-outcomes, respectively. We find such extreme beliefs unwarranted. Hence we use Equation (2) only when history contains both -outcomes or one 1-outcomes. Formally, we assume that $t \geq 3$, and that history contains at least one 0-outcome and at least one 1-outcome, so that $\widehat{p}(H_t, x) \in (0, 1)$.

# 4 The Dynamic Process

We now wish to study the dynamic process in which at every stage $t \geq 1$, $x_t$ is drawn from $X = \{x^1, ..., x^J\}$ according to probabilities $(p_1, ..., p_J)$, beliefs are formed in accordance with equation (2), and the players' behavior in $G_{x_t}$ is chosen by the beliefs $\widehat{p}_t(\cdot)$.

A state of the process is fully summarized by a matrix of relative frequencies

Here we extend the notion of similarity-weighted empirical frequencies to incorporate directional thinking.

$R_t =$

| | $x = x^1$ | $x = x^2$ | ... | $x = x^J$ |
|---|---|---|---|---|
| 1 | $r_{t,11}$ | $r_{t,12}$ | ... | $r_{t,1J}$ |
| 0 | $r_{t,01}$ | $r_{t,02}$ | ... | $r_{t,0J}$ |

where $r_{t,ij}$ is the relative frequency, up to time $t$, of periods in which the game was $G_{x^j}$ and the outcome was $i$.

Consider the following matrices:

$R^0 =$

| | $x = x^1$ | $x = x^2$ | ... | $x = x^J$ |
|---|---|---|---|---|
| 1 | $p_1$ | 0 | ... | 0 |
| 0 | 0 | $p_2$ | ... | $p_J$ |

$R^1 =$

| | $x = x^1$ | $x = x^2$ | ... | $x = x^J$ |
|---|---|---|---|---|
| 1 | $p_1$ | $p_2$ | ... | 0 |
| 0 | 0 | 0 | ... | $p_J$ |

We can finally present our main result.

**Theorem 1** *For given $\bar{p}$ and $s^+$, $s^-$ , there exist distributions $(p_1, ..., p_J)$ such that there is a positive probability that $R_t$ converges to $R^0$ and a positive probability that it converges to $R^1$.*

It will be obvious from the proof of the theorem that there is nothing exceptional about the distributions $(p_1, ..., p_J)$ that allow convergence to either of the extreme outcomes. The main condition will be that the probabilities of the extreme games, $p_1, p_J$ be strictly positive but small, relative to the other probabilities (and given the values of $x_k$, the similarity functions, and the function $\varphi(\cdot)$). In particular, the set of distributions $(p_1, ..., p_J)$ contains open sets.

# 5 Conclusion

Ever since the early days of game theory, there has been a quest for a solution concept that would satisfy existence and uniqueness, with robustness and dynamic stability as additional desiderata. The attempt to narrow down the class of potential predictions was motivated by the desire to make game theory more meaningful and powerful, whether interpreted descriptively or normatively. Clearly, even if uniqueness of equilibria cannot be obtained, tighter theoretical predictions would make the theory more useful, and will thereby reduce the need to resort to extra-theoretical reasoning in order to select an equilibrium as a likely or a recommended outcome.

The literature on refinements of Nash equilibrium (see van Damme, 1983) is generally perceived as falling short of pinpointing unique equilibria in games. However, the more recent literature, viewing a game in the context of similar and related games, have resulted in several results that changed the way we think about equilibrium selection (Carlsson and van Damme, 1994, Burdzy, Frankel, and Pauzner, 2001). These results may suggest that, in a sufficiently detailed model, a unique equilibrium prediction would exist.

The present paper is offered as an example, showing that incorporation of additional details into the model may leave the game theoretic prediction ambiguous. We believe that game theoretic analysis is extremely useful, but that, in general, it cannot subsume the need in historical and institutional knowledge. Rather, the formal, mathematical analysis needs to be combined with such knowledge to generate trustworthy predictions.

# 6 Appendix: Proof of theorem 1.

First, observe that the relative frequencies of the columns of $R_t$ are governed only by the selection of $x$, and are independent of the players' behavior.

Under our assumptions, for every history $H_t$ we can predict the outcome of the game played at time $t$ by considering the difference $\widehat{p}(H_t, x_t) - \bar{p}_{x_t}$ (decreasing in $x_t$). If this difference is strictly positive, all players' expectation $\widehat{p}_t(\cdot)$ will be above the critical belief $\bar{p}_{x_t}$. They will therefore all play 1, and Nature will select $\nu_t = 1$ with probability 1. Otherwise, all players will play 0 and Nature will select $\nu_t = 1$ with probability 1.

Recall that

$$\widehat{p}\left(H_t, x^k\right) = \frac{\sum\limits_{\tau \in S_t} s^+\left(x_\tau, x^k\right)}{\sum\limits_{\tau \in S_t} s^+\left(x_\tau, x^k\right) + \sum\limits_{\tau \in F_t} s^-\left(x_\tau, x^k\right)}.$$

The assumption that $t \geq 3$, and that history contains at least one 0-outcome and one 1-outcome implies that $\widehat{p}(H_t, x_t) \in (0,1)$. This in turn implies that the difference $\widehat{p}(H_t, x_t) - \bar{p}_{x_t}$ is strictly positive at $x_t = x^1$ and strictly negative at $x_t = x^J$.

We simplify notation by defining

$$\begin{aligned} A_{kt} &= \sum_{\tau \in S_t} s^+\left(x_\tau, x^k\right) && (3) \\ B_{kt} &= \sum_{\tau \in F_t} s^-\left(x_\tau, x^k\right) && (4) \\ \widehat{p}\left(H_t, x^k\right) &= \frac{A_{kt}}{A_{kt} + B_{kt}} && (5) \\ z_{kt} &= \left(1 - \bar{p}_{x^k}\right) A_{kt} - \bar{p}_{x^k} B_{kt} && (6) \end{aligned}$$

so that

$$\widehat{p}(H_t, x^k) - \bar{p}_{x^k} > 0 \Leftrightarrow z_{kt} > 0.$$

Given that $A_{kt}$ is strictly decreasing in $x^k$, $B_{kt}$ is strictly increasing in $x^k$, and $\bar{p}_x$ is strictly increasing in $x^k$, it follows that $z_{kt}$ is strictly decreasing in $x^k$, hence for history $H_t$ there exists a unique $y_t \in \{x^1, ...x^{J-1}\}$ for which $z_{y_t t} > 0 \geq z_{y' t}$ for any $y'$ in $X$ such that $y' > y_t$.

At time $t$, the expected change in $z_{kt}$ is given by:

$$\begin{aligned} & E\left[z_{k(t+1)} - z_{kt} | y_t = y\right] \\ = \; & E\left[\left(1 - \bar{p}_{x^k}\right) A_{k(t+1)} - \bar{p}_{x^k} B_{k(t+1)} | y_t = y\right] - \left[\left(1 - \bar{p}_{x^k}\right) A_{kt} - \bar{p}_{x^k} B_{kt}\right] \\ = \; & \left(1 - \bar{p}_{x^k}\right) E\left[A_{k(t+1)} - A_{kt} | y_t = y\right] - \bar{p}_{x^k} E[B_{k(t+1)} - B_{kt} | y_t = y] \\ = \; & \left(1 - \bar{p}_{x^k}\right) \sum_{x^j \leq y} p_j s^+\left(x^j, x^k\right) - \bar{p}_{x^k} \sum_{x^j > y} p_j s^-\left(x^j, x^k\right). \end{aligned}$$

This is increasing in $y$, as $s^{+}(\cdot)$ and $s^{-}(\cdot)$ are nonnegative. Also, it is decreasing in $x^k$ because $\bar{p}_x$ and $s^{-}(\cdot)$ are increasing in $x^k$ and $s^{+}(\cdot)$ is decreasing in $x^k$.

Consider histories $H_t$ that contain at least one 1-outcome and a sufficiently long list of 0-outcomes such that $z_{2t} \leq 0$. Such histories have positive probability as long as $p_1, p_J > 0$. Since $z_{kt}$ is strictly decreasing in $x^k$, $z_{2t} \leq 0$ implies $z_{kt} \leq 0$ for $k \in \{2, ..., J\}$.

The expected change in $z_{2t}$ is given by:

$$E\left[z_{2(t+1)} - z_{2t} | z_{2t} \leq 0\right] = (1 - \bar{p}_{x^2})\, p_1 s^{+}\left(x^1, x^2\right) - \bar{p}_{x^2} \sum_{j \geq 2} p_j s^{-}\left(x^j, x^2\right).$$

Let $(p_1, ..., p_J)$ be such that $E\left[z_{2(t+1)} - z_{2t} | z_{2t} \leq 0\right] < 0$. (That is, assume that $p_1 > 0$ is small enough relative to the other $p_k$'s.)

Since $E\left[z_{k(t+1)} - z_{kt} | y_t = y\right]$ is decreasing in $x_t$, $E\left[z_{2(t+1)} - z_{2t} | z_{2t} \leq 0\right] < 0$ implies $E\left[z_{k(t+1)} - z_{2t} | z_{kt} \leq 0\right] < 0$ for $k \in \{2, ..., J\}$. We argue that, given that $z_{2t} \leq 0$, there is a positive probability that $z_{2\tau} \leq 0$ for all $\tau > t$. To see this, observe that, as long as $z_{2\tau} \leq 0$ for $\tau > t$, $z_{2\tau}$ follows a Markov process. The distribution of $z_{2\tau}$ conditional on $z_{2\tau} > 0$ is not guaranteed to be stationary. However, if we replace it by any stationary distribution, we obtain a new process $\{\hat{z}_{2\tau}\}_{\tau > t}$ that is Markovian, with $E\left[\hat{z}_{k(t+1)} - \hat{z}_{2t} | \hat{z}_{2t} \leq 0\right] < 0$ and that is identical to $\{z_{2\tau}\}_{\tau > t}$ as long as the latter is non-positive. Since $\{\hat{z}_{2\tau}\}_{\tau > t}$ has a positive probability of never becoming positive, so does $\{z_{2\tau}\}_{\tau > t}$. This completes the proof that our process has a positive probability of converging to $R^0$.

A symmetric argument shows that there are probabilities $(p_1, ..., p_J)$ for which the process has a positive probability of converging to $R^0$. Moreover, following the arguments above it is clear that one can find such probabilities for which both events occur with positive probability: basically, one has to guarantee only that $p_1, p_J > 0$ are small enough relative to the other $p_k$'s. □

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