1.7 Exercises

1. George says, "I wish to live in a peaceful world. Therefore, I favor policies that promote world peace."

a. Explain why this statement violates the separation of feasibility and desirability.

b. Assume that George thinks that, if a peaceful world is impossible, he is not interested in living anymore, and, furthermore, he doesn't care about anything else that might happen in this world, to himself or to others. Explain why, under these assumptions, George's statement is compatible with rationality.

2. Mary's doctor told her that he is worried about her medical examinations. Mary realizes that she has so;e probability of developing cancer, but she said, "This is too awful to think about, so I simply decided to ignore it." Do you find Mary's approach rational? What would you like to ask her doctor in order to judge how rational Mary is?

2.6 Exercises

1. Assume that apart from preferences between pairs of alternatives $x \succsim y$ or $y \succsim x$, more data is available, such as (i) the probability that x is chosen out of $\{x, y\}$; or (ii) the time it takes the decision maker to make up her mind between x and y; or (iii) some neurological data that show us the strength of preference between x and y. Consider different representation of preferences, corresponding to (i)-(iii), which will also restrict the set of utilities one can ascribe to the decision maker.

2. Assume that $X = \mathbb{R}^2$ and that, due to some axioms, you are convinced that your utility function should be of the form

$$u(x_1, x_2) = v_1(x_1) + v_2(x_2).$$

Discuss how this additional structure may help you to estimate your own utility function, and contrast this case with the (end of the dialog) we started out with.

3. Assume that a consumer has preferences

$$u(x_1, x_2) = x_1^{\alpha} x_2^{\beta}$$

for some parameters $\alpha, \beta > 0$. Explain why these preferences are of the type discussed in (2).

3.4 Exercises

1. You are organizing an inter-disciplinary conference, and you wish to have a good mix of psychologists, economists, and sociologists. There are many scientists of each type, but the cost of inviting them grows with distance – it is relatively inexpensive to invite those that are in your city, but it gets expensive to fly them from remote countries. State the problem as a constrained optimization problem. Is this a convex problem? What do the first order conditions tell you?

2. Provide an example of a consumer problem in which the optimal solution does not satisfy the first order conditions. (Hint: you may use two goods and use a simple budget set such as that defined by $x_1 + x_2 \leq 100$.)

3. Assume that you have to allocate a given amount of time among several friends. Unfortunately, since they live far away, you can't meet more than one friend at the same time. Let x_i the amount of time you spend with friend i. Formulate the problem as a constrained optimization problem. Is it convex?

4. Show that, in the presence of discounts for quantity (that is, the price per unit goes down as you buy large quantities) the feasible set of the consumer is not convex.

iomatization that yields a unique numerical representation can be useful in calibrating the representation in simple problems in order to use it in complex ones.

4.6 Exercises

1. A concave utility function can explain why people buy insurance with a negative expected value. And a convex utility function can explain why they buy lottery tickets, whose expected value is also negative. But how would you explain the fact that some people do both simultaneously?

2. Assume that you are indifferent between getting 700 euro and getting 1000 euro with probability 80% (and otherwise nothing). Assume also that you are indifferent between getting 300 euro and getting 700 euro (not 1000 this time!) with probability 60% (and otherwise nothing). Consider lottery A, which gives you 1000 euro with probability 2/3 (and otherwise nothing), and lottery B, which gives you a 50%-50% bet between 300 and 700 euro. If you follow von-Neumann-Morgenstern's theory, you should:

a. Prefer A to B

b. Prefer B to A

c. Be indifferent between A and B

d. One cannot tell based on the data.

3. Mary likes the von Neumann Morgenstern's axioms and she would like to make decisions in accordance with these axioms. By careful introspection, she has decided that she would be indifferent between

a. 400 € for sure and a 50% of obtaining 100 € (otherwise – nothing); and

b. 600 € for sure and a 80% of obtaining 100 € (otherwise – nothing).

Marie is offered a bet among (0€, 400€, 600€, 1000€) with equal chances (25% each) for a cost of 400€. Should she prefer the bet or should she prefer to keep her 400€ ?

5.5 Exercises

1. Explain what's wrong with the claim "Most good chess players are Russian; therefore a Russian is likely to be a good chess player".

2. When you sail along the shores of the Mediterranean, it seems that much more of the shoreline has hills and cliffs than one would have imagined. One theory is that God created the Earth with the tourism industry in mind. The other is that this is an instance of biased sampling. Explain why.

[Hint: Assume that the Earth is unidimensional, and that its surface varies in slope. To be concrete, assume that the surface is made of the segments connecting $((0,0),(90,10))$ and $((90,10),(100,100))$ (where the first coordinate denotes distance and the second – height). Assume that the height of the water is randomly determined according to a uniform distribution over $[0,100]$. Compare the probability of the shore being at a point of a steep slope to the probability you get if you sample a point at random (uniformly) on the distance axis.]

3. Comment on the claim, "Some of the greatest achievements in economics are due to people who studies mathematics. Therefore, all economists should better study mathematics first".

4. Trying to understand why people confuse $P(A|B)$ with $P(B|A)$, it is useful to see that qualitatively, if A makes B more likely, it will also be true that B will make A more likely:

a. Show that, for any two events A, B,

$$\begin{aligned} P(A|B) &> P(A|B^c) \\ iff \quad P(A|B) &> P(A) > P(A|B^c) \end{aligned}$$

iff

$$\begin{aligned} P(B|A) &> P(B|A^c) \\ iff \quad P(B|A) &> P(B) > P(B|A^c) \end{aligned}$$

where A^c is the complement of A. (Assume that all probabilities involved are positive, so that all the conditional probabilities are well-defined.)

b. If the proportion of Russians among the good chess players is higher than their proportion overall in the population, what can be said?

5. Consider Problem 4 above, and explain how many prejudices in the social domain may result from ignoring base probabilities.

6. Consider a regression line relating the height of children to that of their parents. We know that its slope should be in $(0, 1)$. Now consider the following generation, and observe that the slope should be again in $(0, 1)$. Does it mean that, due to the "regression to the mean", all the population will converge to a single height?

6.6 Exercises

1. In order to determine a unique utility function for each individual, to be used in the summation of utilities across individuals, it was suggested to measure individual's vNM utility functions (for choice under risk), and to set two arbitrary outcomes to given values (shared across individuals). Discuss this proposal.

2. The "Eurovision" song contest uses a scoring rule, according to which each country ranks the other countries' songs and gives them scores according to this ranking. It has been claimed that the scores given favor standard songs over more innovative ones. Does this claim make sense? Is it more convincing when the score scale is convex or concave?

3. It turns out that, for two particular individuals, Pareto domination defines a complete relation. (That is, for every two distinct alternatives, one Pareto dominates the other.) Assume also that

$$u(X) = \{(u_1(x), u_2(x)) \,|x \in X\}$$

is convex. What can you say about the utility functions of these individuals?

4. Assume that individual i has a utility function u_i. For $\alpha = (\alpha_1, ..., \alpha_n)$ with $\alpha_i > 0$, let

$$u_\alpha = \sum_{i=1}^{n} \alpha_i u_i.$$

Show that if x maximizes u_α for some α, it is Pareto efficient.

5. Is it true that every Pareto efficient alternative maximizes u_α for some α? (Hint: for $n = 2$, consider the feasible sets

$$X = \{(x_1, x_2) \,|\sqrt{x_1} + \sqrt{x_2} \leq 1; \quad x_1, x_2 \geq 0\}$$

and

$$X = \left\{(x_1, x_2) \,\middle|x_1^2 + x_2^2 \leq 1; \quad x_1, x_2 \geq 0\right\}$$

where $u_i = x_i$.)

6. Show that, under approval voting, it makes sense for each individual to approve of her most preferred alternative(s), and not to approve of the least preferred one(s) (assuming that the voter is not indifferent among all alternatives).

7.8 Exercises

1. Assume that the prisoner's dilemma is player T times between two players. Show that playing D is not a dominant strategy, but that the only Nash equilibria still result in consecutive play of D.

2. Consider the following story. In a certain village there are n married couples. It is the case that, should one married woman be unfaithful to her husband, all other men are told about it immediately, but not the husband. This fact is commonly known in the village. The law of the land is that, should a husband know that his wife has been unfaithful to him, he must shoot her to death on the same night. But he is not allowed to hurt her unless he knows that for sure.

One day a visitor comes to the village, gets everyone to meet in the central square, and says "There are unfaithful wives in this village". He then leaves.

That night, and the following one, nothing happens. On the third night, shots are heard.

a. How many shots where heard on the third night?

b. What information did the visitor add that the village inhabitants did not have before his visit?

3. Consider an extensive for game, and show how a player might falsify common knowledge of rationality (by deviating from the backward induction solution). Show an example in which it may be in the player's best interest to do so.

4. Compute the mixed strategies equilibria in games 6,7,8 above.

5. A computer sends a message to another computer, and it is commonly known that it never gets lost and that it takes 60 seconds to arrive. When it arrives, it is common knowledge (between the two computers) that it has, indeed, been sent and arrived. Next, a technological improvement was introduced, and the message can now take any length of time between 0 and 60 seconds. How long after the message was sent will it be commonly known that it has been sent?

8.5 Exercises

1. Discuss the reasons for which equilibria might not be efficient in the following cases:

a. A physician should prescribe tests for a patient

b. A lawyer assesses the probability of success of a legal battle

c. A teacher is hired to teach a child.

2. The dean has to decide whether to give a department overall budget for its activities, or to split it among several activities such as "conferences", "visitors", and so forth. Discuss pros and cons of the two options.

3. Consider the student-course assignment problem. Show that for every n it is possible to have examples in which n is the minimal number of students that can find a Pareto-improving re-allocation of courses.