# Questions in Decision Theory

## Itzhak Gilboa

June 15, 2011

• Pascal and Bernoulli

Image: A math a math

- Pascal and Bernoulli
- Ramsey and deFinetti

< A

- Pascal and Bernoulli
- Ramsey and deFinetti
- von Morgenstern-Neumann

- Pascal and Bernoulli
- Ramsey and deFinetti
- von Morgenstern-Neumann

# • Savage

- Pascal and Bernoulli
- Ramsey and deFinetti
- von Morgenstern-Neumann
- Savage
- Anscombe-Aumann

•  $F = X^S = \{f \mid f : S \to X\}$ 

Image: A matrix and A matrix

• 
$$F = X^S = \{f \mid f : S \to X\}$$

•  $P1 \succeq$  is a weak order

Image: A matrix

- $F = X^S = \{f \mid f : S \to X\}$
- $P1 \succeq$  is a weak order
- **P2**  $f_{A^c}^h \succeq g_{A^c}^h$  iff  $f_{A^c}^{h'} \succeq g_{A^c}^{h'}$

- $F = X^S = \{f \mid f : S \to X\}$
- $P1 \succeq$  is a weak order
- **P2**  $f_{A^c}^h \succeq g_{A^c}^h$  iff  $f_{A^c}^{h'} \succeq g_{A^c}^{h'}$
- **P3**  $x \succeq y$  iff  $f_A^x \succeq f_A^y$

- $F = X^S = \{f \mid f : S \to X\}$
- $P1 \succeq$  is a weak order
- **P2**  $f_{A^c}^h \succeq g_{A^c}^h$  iff  $f_{A^c}^{h'} \succeq g_{A^c}^{h'}$
- **P3**  $x \succeq y$  iff  $f_A^x \succeq f_A^y$
- **P4**  $y_A^x \succeq y_B^x$  iff  $w_A^z \succeq w_B^z$

- $F = X^S = \{f \mid f : S \to X\}$
- $P1 \succeq$  is a weak order
- **P2**  $f_{A^c}^h \succeq g_{A^c}^h$  iff  $f_{A^c}^{h'} \succeq g_{A^c}^{h'}$
- **P3**  $x \succeq y$  iff  $f_A^x \succeq f_A^y$
- **P4**  $y_A^x \succeq y_B^x$  iff  $w_A^z \succeq w_B^z$
- **P5** ∃ *f* ≻ *g*

- $F = X^S = \{f \mid f : S \to X\}$
- P1  $\succeq$  is a weak order
- **P2**  $f_{A^c}^h \succeq g_{A^c}^h$  iff  $f_{A^c}^{h'} \succeq g_{A^c}^{h'}$
- **P3**  $x \succeq y$  iff  $f_A^x \succeq f_A^y$
- **P4**  $y_A^x \succeq y_B^x$  iff  $w_A^z \succeq w_B^z$
- **P5** ∃ *f* ≻ *g*
- **P6**  $f \succ g \exists$  a partition of S,  $\{A_1, ..., A_n\} f_{A_i}^h \succ g$  and  $f \succ g_{A_i}^h$

Savage's Theorem

Assume that X is finite. Then ≿ satisfies P1-P6 if and only if there exist a non-atomic finitely additive probability measure µ on S (=(S, 2<sup>S</sup>)) and a non-constant function u : X → ℝ such that, for every f, g ∈ F

$$f \succeq g$$
 iff  $\int_{S} u(f(s)) d\mu(s) \ge \int_{S} u(g(s)) d\mu(s)$ 

Furthermore, in this case  $\mu$  is unique, and u is unique up to positive linear transformations.

• Accuracy vs. beauty/generality

- Accuracy vs. beauty/generality
- Method: experiments, axioms, neurological data?

- Accuracy vs. beauty/generality
- Method: experiments, axioms, neurological data?
- Goal: theoretical models or applied decisions?

- Accuracy vs. beauty/generality
- Method: experiments, axioms, neurological data?
- Goal: theoretical models or applied decisions?
- Descriptive or normative?

• Rationality

Image: A math a math

- Rationality
- Probability

Image: A mathematical states and a mathem

- Rationality
- Probability
- Utility

Image: A mathematical states and a mathem

- Rationality
- Probability
- Utility
- Rules and analogies

- 一司

- Rationality
- Probability
- Utility
- Rules and analogies
- Group decisions

• Older concept: "Rational Man" should do...

- Older concept: "Rational Man" should do...
- In neoclassical economics: only consistency

- Older concept: "Rational Man" should do...
- In neoclassical economics: only consistency
- An even more subjective view: which consistency?

- Older concept: "Rational Man" should do...
- In neoclassical economics: only consistency
- An even more subjective view: which consistency?
- Rationality as robustness

- Older concept: "Rational Man" should do...
- In neoclassical economics: only consistency
- An even more subjective view: which consistency?
- Rationality as robustness
- Weaknesses (?): subjective, empirical, not monotonic in intelligence

- Older concept: "Rational Man" should do...
- In neoclassical economics: only consistency
- An even more subjective view: which consistency?
- Rationality as robustness
- Weaknesses (?): subjective, empirical, not monotonic in intelligence

Defense

• A decision maker is defined by two relations  $(\succeq^*, \succeq^{\hat{}})$ 

- A decision maker is defined by two relations  $(\succeq^*, \succeq^*)$
- $\bullet \ensuremath{\succsim^*}$  can convince "any reasonable decision maker" that it is right

- A decision maker is defined by two relations ( $\succeq^*$ ,  $\succeq^{\hat{}}$ )
- $\bullet \succsim^* \mathsf{can}$  convince "any reasonable decision maker" that it is right
- $\gtrsim$  cannot be convinced that it is wrong

- A decision maker is defined by two relations ( $\succsim^*$ ,  $\succsim^{\hat{}}$ )
- $\bullet \succsim^* \mathsf{can}$  convince "any reasonable decision maker" that it is right
- $\succsim$  cannot be convinced that it is wrong
- Clearly,  $\succeq^* \subset \succeq^{\hat{}}$

Classical and Bayesian Statistics

• Classical: attempts to be objective, no intuition

Classical and Bayesian Statistics

- Classical: attempts to be objective, no intuition
- Bayesian: attempts to incorporate intuition and hunches

Classical and Bayesian Statistics

- Classical: attempts to be objective, no intuition
- Bayesian: attempts to incorporate intuition and hunches
- Classical for making a point (to others)
Classical and Bayesian Statistics

- Classical: attempts to be objective, no intuition
- Bayesian: attempts to incorporate intuition and hunches
- Classical for making a point (to others)
- Bayesian for making a decision (for oneself)

• What is the probability of

- < A

- What is the probability of
- A coin coming up Head?

- What is the probability of
- A coin coming up Head?
- A car being stolen?

- What is the probability of
- A coin coming up Head?
- A car being stolen?
- A surgery succeeding?

- What is the probability of
- A coin coming up Head?
- A car being stolen?
- A surgery succeeding?
- A war erupting?

• Relying on remarkable foundations (Ramsey, de Finetti, Savage, Anscombe-Aumann)

- Relying on remarkable foundations (Ramsey, de Finetti, Savage, Anscombe-Aumann)
- Yet problematic:

- Relying on remarkable foundations (Ramsey, de Finetti, Savage, Anscombe-Aumann)
- Yet problematic:
- Descriptively: people violate axioms (Ellsberg)

- Relying on remarkable foundations (Ramsey, de Finetti, Savage, Anscombe-Aumann)
- Yet problematic:
- Descriptively: people violate axioms (Ellsberg)
- Normatively: completeness?

- Relying on remarkable foundations (Ramsey, de Finetti, Savage, Anscombe-Aumann)
- Yet problematic:
- Descriptively: people violate axioms (Ellsberg)
- Normatively: completeness?
- Back to rationality: if it's so rational, why isn't it objective?

- Relying on remarkable foundations (Ramsey, de Finetti, Savage, Anscombe-Aumann)
- Yet problematic:
- Descriptively: people violate axioms (Ellsberg)
- Normatively: completeness?
- Back to rationality: if it's so rational, why isn't it objective?
- The Bayesian approach is good at representing knowledge, poor at representing ignorance

• Exist in simple cases (iid)

- Exist in simple cases (iid)
- Can be defined with identicality, as long as causal independence is retained

- Exist in simple cases (iid)
- Can be defined with identicality, as long as causal independence is retained
- Rule-based approaches: logit

- Exist in simple cases (iid)
- Can be defined with identicality, as long as causal independence is retained
- Rule-based approaches: logit
- Case-based approaches: empirical similarity

- Exist in simple cases (iid)
- Can be defined with identicality, as long as causal independence is retained
- Rule-based approaches: logit
- Case-based approaches: empirical similarity
- But none extends to the cases of wars, stock market crashes...

Alternatives to the Bayesian approach

• Schmeidler (1989): non-additive probabilities (capacities)

Alternatives to the Bayesian approach

- Schmeidler (1989): non-additive probabilities (capacities)
- Integration by Choquet's integral

Alternatives to the Bayesian approach

- Schmeidler (1989): non-additive probabilities (capacities)
- Integration by Choquet's integral
- Maxmin EU: there exists a set of probabilities C such that

$$V(f) = \min_{P \in C} \int_{S} u(f(s)) dP(s)$$

Other multiple-priors models

• Nau, Klibanoff-Marinacci-Mukerji: "smooth preferences"

 $\varphi: \mathbb{R} \to \mathbb{R}$  $\int_{\Delta(S)} \varphi\left(\int u(f) \, dp\right) d\mu$ 

Other multiple-priors models

• Nau, Klibanoff-Marinacci-Mukerji: "smooth preferences"

$$\varphi: \mathbb{R} \to \mathbb{R}$$
$$\int_{\Delta(S)} \varphi\left(\int u(f) \, dp\right) d\mu$$

• Maccheroni-Marinacci-Rustichini: "variational preferences"

$$V(f) = \min_{P \in \Delta(S)} \left\{ \int_{S} u(f(s)) dP(s) + c(P) \right\}$$

#### Incomplete relation

• Bewley:

$$\begin{array}{rcl} f &\succ & g \\ & & iff \\ \forall p &\in & C \\ \int_{S} u\left(f\left(s\right)\right) dP\left(s\right) &> & \int_{S} u\left(g\left(s\right)\right) dP\left(s\right) \end{array}$$

Gilboa ()

June 15, 2011 15 / 18

• • • • • • • •

2

#### Incomplete relation

• Bewley:

$$\begin{array}{rcl} f &\succ & g \\ & & iff \\ & \forall p &\in & C \\ \int_{S} u\left(f\left(s\right)\right) dP\left(s\right) &> & \int_{S} u\left(g\left(s\right)\right) dP\left(s\right) \end{array}$$

• Fits the "objective rationality" notion

#### Incomplete relation

Bewley:

$$\begin{array}{rcl} f &\succ & g \\ & & iff \\ \forall p &\in & C \\ \int_{S} u\left(f\left(s\right)\right) dP\left(s\right) &> & \int_{S} u\left(g\left(s\right)\right) dP\left(s\right) \end{array}$$

- Fits the "objective rationality" notion
- Can be combined with the maxmin criterion as "subjective rationality"

• What is utility and how is it related to well-being or happiness?

- What is utility and how is it related to well-being or happiness?
- Measurement of well-being and its relation to money

- What is utility and how is it related to well-being or happiness?
- Measurement of well-being and its relation to money
- The paraplegics and lottery winners

- What is utility and how is it related to well-being or happiness?
- Measurement of well-being and its relation to money
- The paraplegics and lottery winners
- Problems of measurement

- What is utility and how is it related to well-being or happiness?
- Measurement of well-being and its relation to money
- The paraplegics and lottery winners
- Problems of measurement
- All happy families... ?

• In the context of probability

- In the context of probability
- Statistics

- In the context of probability
- Statistics
- Moral argumentation

- In the context of probability
- Statistics
- Moral argumentation
- Recent model unifying the two, as well as Bayesian

Group decisions

• Do groups make better decisions than do individuals?

Group decisions

- Do groups make better decisions than do individuals?
- "Truth wins" vs. risk/uncertainty aversion
Group decisions

- Do groups make better decisions than do individuals?
- "Truth wins" vs. risk/uncertainty aversion
- Aggregation of opinions/judgment aggregation