# Questions in Decision Theory 

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## History

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- P5 $\exists f \succ g$
- P6 $f \succ g \exists$ a partition of $S,\left\{A_{1}, \ldots, A_{n}\right\} f_{A_{i}}^{h} \succ g$ and $f \succ g_{A_{i}}^{h}$


## Savage's Theorem

- Assume that $X$ is finite. Then $\succsim$ satisfies P1-P6 if and only if there exist a non-atomic finitely additive probability measure $\mu$ on $S$ $\left(=\left(S, 2^{S}\right)\right)$ and a non-constant function $u: X \rightarrow \mathbb{R}$ such that, for every $f, g \in F$

$$
f \succsim g \quad \text { iff } \quad \int_{S} u(f(s)) d \mu(s) \geq \int_{S} u(g(s)) d \mu(s)
$$

Furthermore, in this case $\mu$ is unique, and $u$ is unique up to positive linear transformations.

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- Descriptive or normative?


## Main Questions

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- Group decisions


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- Defense


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- Clearly, $\succsim^{*} \subset \succsim^{\wedge}$


## Classical and Bayesian Statistics

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- Bayesian - for making a decision (for oneself)


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- The Bayesian approach is good at representing knowledge, poor at representing ignorance


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Objective probabilities

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- Can be defined with identicality, as long as causal independence is retained
- Rule-based approaches: logit
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- But none extends to the cases of wars, stock market crashes...

Alternatives to the Bayesian approach

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- Integration by Choquet's integral
- Maxmin EU: there exists a set of probabilities $C$ such that

$$
V(f)=\min _{P \in C} \int_{S} u(f(s)) d P(s)
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## Other multiple-priors models

- Nau, Klibanoff-Marinacci-Mukerji: "smooth preferences"

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- Maccheroni-Marinacci-Rustichini: "variational preferences"

$$
V(f)=\min _{P \in \Delta(S)}\left\{\int_{S} u(f(s)) d P(s)+c(P)\right\}
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Incomplete relation

- Bewley:

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f & \succ g \\
\forall p & \in C \\
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- Can be combined with the maxmin criterion as "subjective rationality"


## Utility

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- Measurement of well-being and its relation to money
- The paraplegics and lottery winners
- Problems of measurement
- All happy families... ?

Rules and analogies

- In the context of probability

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- In the context of probability
- Statistics
- Moral argumentation
- Recent model unifying the two, as well as Bayesian


## Group decisions

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- "Truth wins" vs. risk/uncertainty aversion
- Aggregation of opinions/judgment aggregation

