Main equations/relations from Lecture 2:

Buoyancy: $b \equiv F_{tot}/m_p = -g(\rho_p - \rho_E)/\rho_p$

Convection in water:

If $\rho_p < \rho_E$, the parcel is positively buoyant: b>0, and it will accelerate upwards.

If $\rho_p > \rho_E$, the parcel is negatively buoyant: b<0, and it will accelerate downwards.

$d^2(\Delta z)/dt^2 = b \equiv -N^2 \Delta z$ $\quad N^2 \equiv -b/\Delta z = -g/\rho_{E(z1)}\, d\rho_E/dz$ buoyancy frequency.

Energetics view: $PE_{final} - PE_{initial} = -g(\rho_2 - \rho_1)(z2-z1) \approx -g\, d\rho_E/dz\, (z2-z1)^2$

Water heated from below:

$\langle H \rangle = 1/2\, \rho_{ref}\, C(2/3\, g\, \alpha \Delta z)^{1/2}\, \Delta T^{3/2}$

Dry convection in a compressible atmosphere:

→ $\delta Q = CpdT - dp/\rho = CpdT - \rho_E/\rho\, g\, dz \approx CpdT - gdz$

dry adiabatic lapse rate: $\Gamma d = -dT/dz = g/Cp$

For Cp=1005J/Kg/K, $\Gamma d \approx 10$K/km.

Lecture 3:

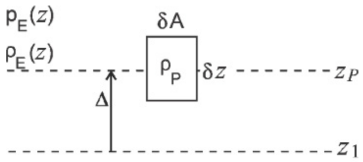

Figure 4.12: A parcel displaced a distance Δ from height z_1 to height z_P. The density of the parcel is ρ_P, and that of the environment, ρ_E.

Continue in lecture 3:

Will a rising parcel (from z1 to z2=z1+δz) be positively or negatively buoyant? We need to compare ρ2 to $\rho_E(z2)$. $\rho 2 = p2/(RT2)$, $\rho_E(z2) = p2/(RT_E(z2))$ where now, unlike for water, $T2 \neq T_E(z2)$

$T2 = T_E1 - \Gamma d\, \Delta z$, $T_E(z2) = T_E(z1) + dT_E/dz\, \Delta z$.

The parcel will be negatively/neutrally/positively buoyant (stable/neutral/unstable) if $\rho_E(z2)$ is smaller/same/larger than ρ2, which will occur if T_E2 is larger/same/smaller than T2, which will occur if dT_E/dz is larger/same/smaller than $-\Gamma d$.

For convection, we need the environmental temperature to drop faster than the adiabatic lapse rate with height.

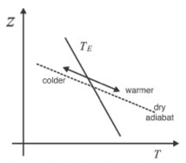

Figure 4.10: The atmosphere is nearly always stable to dry processes. A parcel displaced upwards (downwards) in an adiabatic process moves along a dry adiabat (the dotted line) and cools down (warms up) at a rate that is faster than that of the environment, $\partial T_E/\partial z$. Since the parcel always has the same pressure as the environment, it is not only colder (warmer) but also denser (lighter). The parcel therefore experiences a force pulling it back toward its reference height.

T1-standard T profiles: The observed lapse rate is approximately:

$(T(12)-T(0))/(12-0) = (215-280)/12 = -65/12 \approx -5.41$K/km

Smaller than dry adiabatic, so we expect no dry convection. We will see later that moisture is crucial for convection to occur in the atmosphere- when air cools and water condenses, it releases heat, reducing the parcels cooling as it rises, making it more buoyant.

Potential temperature:
We need to find a different temperature variable which is conserved under adiabatic vertical displacements.
For an adiabatic process, $C_p dT = RT dp/p$, $dT/T = R/C_p\, dp/p = \kappa\, dp/p$; $\kappa = R/C_p = 2/7$ for a perfect diatomic gas.
$d(\ln T) - \kappa d(\ln p) = 0 \rightarrow T/p^{\kappa} = \text{const}$
Define: $\theta = T(p_o/p)^{\kappa}$, which is conserved under dry adiabatic conditions. $p_o = 1000$hPa
Potential temperature – the temperature a parcel will have if brought to pressure p_o adiabatically.
Differentiation gives: $d\theta/\theta = dT/T - \kappa dp/p = 0$ in adiabatic processes.
Note also: $d\theta/\theta = \delta Q/(C_p T) \equiv d\eta/C_p$, η is entropy. $\rightarrow \eta = C_p \ln\theta + \text{const}$
We can deduce the temperature that a rising/sinking parcel will have at a given pressure from its potential temperature, which is constant, and the pressure it is at.

The stability condition in terms of potential temperature:
The parcel will be negatively/neutrally/positively buoyant (stable/neutral/unstable) if $d\theta_E/dz$ is larger/same/smaller than 0.
T4-climatological tropical T, θ and θe profiles: Shows the profiles are stable to dry convection. We know there is convection- moist.
T5-convection of warmed surface air, from T and θ perspectives: A warmed parcel will rise to the level at which its temperature is similar to the environment.

- Convection mixes potential temperature.

Stable conditions- gravity waves: What happens to displacements under stable conditions?
Look at the equation of motion again:

$d^2(\Delta z)/dt^2 = b = -g(\rho_p - \rho_E)/\rho_p = g(T_p - T_E)/T_p = g(\theta_p - \theta_E)/\theta_p$ where we used the ideal gas law and the fact that the parcel and environmental pressures are the same.
For small Δ under stable conditions, $\theta_p(z1 + \Delta z) = \theta_E(z1)$,
$\theta_E(z1 + \Delta z) = \theta_E(z1) + d\theta_E/dz\, \Delta z$ so $b = -g/\theta_E\, d\theta_E/dz\, \Delta z \equiv -N^2 \Delta z$
and we get oscillatory solutions: $\Delta z = \Delta_1 \cos(Nt) + \Delta_2 \sin(Nt)$ which are gravity waves. Similar to surface waves where the density difference is between water and overlying air. Have also in atmosphere and ocean.
N^2 is the ***buoyancy frequency*** or ***Brunt Vaisala frequency***.
From figure T1 we estimate: p(12km)=1000 exp(-12/7.3)=193hPa.
$\theta(12km) = 215(1000/193)^{2/7} = 344K$. $N^2 = 9.81\ m/sec^2/295K\ (320K)/12km = 1.25\times10^{-4} s^{-2}$.
This corresponds to a period of $2\pi/N \approx 9$min (for vertical oscillations – slanted ones have $N\cos\alpha$ with the α the angle to the vertical).
T6-gravity waves clouds: Lenticular clouds schematic and photo
If the air is close to saturation then the lifting due to the waves results in condensation. We can estimate the wave length: $2\pi/N \cdot U \approx 9\text{min} \cdot 10\text{m/sec} \approx 5\text{km}$, 10km for U20m/sec.
T7-gravity waves clouds: Island wake wind–driven waves.

Extra stable conditions- temperature inversions.
T8-inversion schematic: two types. Surface cooling- can be reinforcing cause convection mixes surface cooling but inversion prevents this vertical mixing.
T9-inversion effect- photos: air polution

Moist convection:
When rising air cools it can reach saturation, leading ot condensation. This will release latent heat → more unstable.
Humidity measures- need to derive a humidity measure which his conserved following th flow of an air parcel.

- *Specific humidity*: mass of vapor to mass of air, per unit volume. $q=\rho_v/\rho$ where ρ_v is the density of water vapor and $\rho=\rho_v+\rho_d$ is the density of air
- *Saturation specific humidity*: $q^*=(e_s/R_vT)/(p/RT)=R/R_v\, e_s(T)/p$

Note: q* increases with height due to pressure decrease, decreases more strongly with height since e_s decreases strongly with T.

- *Relative humidity*: H(%)=100q/q*. H is around 80% near surface, so condensation occurs quite easily when air rises.

T10-convection of warmed surface under moist conditions: Air rises dry-adiabatically, until it reaches H=100 at the *condensation level*, after which it rises moist adiabatically (temperature decreases more slowly and air reaches higher than under pure dry conditions).

When water condenses, the 1st law: $\delta Q= C_p dT - dp/\rho + L dq =0$ for adiabatic process.
$C_p dT + g dz + L dq = d(C_pT + gz+Lq)=0$
CpT+gz- *dry static energy*; cpT+gz+Lq – *moist static energy*
In ascent, air is saturated, so q=q*(p,T), $dq^*=\frac{\partial q^*}{\partial p}dp+\frac{\partial q^*}{\partial T}dT=-\frac{q^*}{p}dp+\beta q^* dT$

where we used $\frac{\partial q^*}{\partial p}=-\frac{R}{R_v}\frac{e_s}{p^2}=-\frac{q^*}{p}$ and $\frac{\partial q^*}{\partial T}=\frac{R}{R_v}\frac{1}{p}\frac{\partial e_s}{\partial T}=\frac{R}{R_v}\frac{\beta e_s}{p}=e_s q^*$

→ $C_p dT - dp/\rho + L\beta q^* dT - q^*/p\, dp=0$
$(C_p+ L\beta q^*)dT=(1+Lq^*\rho/p)dp/\rho=-(1+Lq^*\rho/p)g dz$

→ $--\frac{dT}{dz}=\Gamma_s=\Gamma_d\left[\frac{1+Lq^*/RT}{1+L\beta q^*/C_p}\right]$ the *saturated adiabatic lapse rate*, also *pseudo-adiabatic lapse rate*

$\Gamma s= \Gamma s(q^*(T,p)) \approx 3K/km – 10(K/km)$ (moist tropical lower troposphere to upper troposphere which is dry cold and low pressure).

T11-typical dry, moist and observed T profiles: dry largest lapse rate, moist lapse rate starts small and joins dry at top, and observed is in between. Wet adiabat warmer than surroundings till around 10km.

Stability condition in a saturated atmosphere: $dT/dz<-\Gamma s$ for convection.
Since $\Gamma s < \Gamma d$, we have conditional instability - conditional upon the air being saturated.

In tropics, dT/dz≈- Γs.

Equivalent potential temperature θe: a potential temperature that is conserved in moist processes and is mixed by moist convection, just like θ is mixed in dry convection.

CpdT - dp/ρ + Ldq =CpdT - RTdp/p + Ldq =0
d(lnT) - κd(lnp) = - L/(CpT)dq = d(lnθ)
d(lnθ)≈-d[Lq/(CpT)] (we took L/(CpT) into the parenthesis, since q varies much more than T.
Integrate from q=0 to q, with θ being the dry potential temperature:

$$\theta_e = \theta e^{\frac{Lq}{C_p T}}$$

θe equals θ when the air is dry q=0). It is conserved under both moist and dry processes. If there is no condensation, q is conserved, and both θ and θe are conserved. If condensation occurs, q→q* and only θe is conserved:
d(lnθ)≈-d(ln[exp(Lq/CpT)]) → d(ln[θ exp(Lq/CpT)])=0 θ exp(Lq/CpT)=const

Convection tends to mix away vertical gradients of θe.
T4-climatological tropical T, θ and θe profiles: θe is roughly constant with height in the tropics.

Convection in the atmosphere:
Most of the convection in the atmosphere is moist. Downwelling parcels are not saturated since the descending air warms. How come they are not positively buoyant? Radiative cooling. This is a much slower process than latent heat release, hence descent is much slower than ascent, and thus occurs over a much larger area.

Types:
Convective clouds: Cumulus (Cu)- small, fair weather, no rain; Cumulonimbus (Cb)- associated with thunderstorms, rain, hail.
T12- Schematic of the two cloud types

Two main types of convection: moderate and relatively shallow, topped by Cumulus clouds (non precipitating), and deep convection in precipitating Cumulonimbus (Cb) clouds .

Moderate convection: Shallow (few km), capped by Cu coluds which are around 1-2km tall.
T13: Photo of Cu field
They grow in about 15 minutes, which implies w≈2000/(15*60) ≈2.22m/sec.
Characteristic temperature fluctuations in such a cloud field are typically 0.1K.
Estimated vertical heat transport: $H \approx \rho C_p \overline{w'T'} \approx 1x1000x2x0.1 = 200W/m^2$
Quite a large number compared to Radiative fluxes:
T14- Annual mean latitudinal profile of incoming solar and outgoing IR radiation fluxes: We see this value is comparable to outgoing IR flux.

A model of vertical heat transport: The buoyancy of an air parcel rising within a Cu cloud: $b=g(Tp-T_E)/T=g\ \Delta T/T$.
$\Delta PE/\rho=g\ \Delta T/T\ \Delta z$ (recall $\Delta PE=-g\ \Delta\rho\Delta z$, and $\Delta\rho/\rho=\Delta T/T$ from ideal gas law and assuming parcel has environmental pressure).
$\Delta KE/\rho=\Delta PE/\rho \rightarrow 3/2w^2= g\ \Delta T/T\ \Delta z$ and $<H>=1/2\ \rho Cp(2/3\ g\ \Delta z/T)^{1/2}\ \Delta T^{3/2}$ (the same as for the water case, but with α replaced by 1/T).
For $H=200W/m^2$, and Δz=1km we get $\Delta T\approx0.19K$, and $w\approx 2H/(\rho Cp\Delta T)\approx$2m/sec which is reasonable.

Cu fields typically form in boundary layers, when ground is heated during day.
Why are the clouds so shallow? See for example T4 – if parcels rise from the warmed ground to level where they are neutrally buoyant and according to this figure they should rise much more than 2 km from cloud base… Answer: mixing.
T15: Photo of Cu field T profile in cloud- starts with dry adiabatic ascent, then follows moist adiabat, till it mixes with environment, then continues rising following moist adiabat, mixes again, etc. Ascent will stop when moist adiabatic lapse rate equals that of the environment. This is much lower then when the moist adiabatic temperature starting at cloud base equals the environmental temperature. Observed Cu tops are around level where $\Gamma s\approx -dT_E/dz$.

Deep convection: Common in tropics, but not only. Deep Cb clouds.
T16: Photo of Cu field
T17: Photo of Cu field
Tops reach tropopause and form anvils. $w\approx O(10m/sec)$, $\Delta T\approx O(1K)$, $H\approx O(10^4W)$, but these numbers are for a single cloud and they are intermittent in space and time. Main form of vertical heat transport in tropics.
Unlike for Cu clouds, there is very little mixing of the rising parcels with the surroundings, and the vertical velocity anomalies are much stronger than the horizontal ones. Thus, $\Delta KE/\rho\approx 1/2w^2=\Delta PE/\rho\approx g\ \Delta T/T\ \Delta z$ and $w\approx(2g\Delta T/T\ \Delta z)^{1/2}\approx$26m/sec for ΔT=1K and Δz=10km. This value is typical of observed. This is enough to suspend hail stones till they grow to large sizes.

The dynamics are also affected by the background flow, since these clouds are so deep. **T18: Schematic of Cu flow.**

Where does convection occur?
Deep convection is more common in tropical regions of warm SST, and less over the subtropical deserts. Can view it by looking at maps of OLR. LR depends on temperature.
T19: OLR map polar regions and a few tropical spots have similar low values. Low polar clear- low T, but why low tropical OLR? Shows T or emitting layer, which is at cloud top. The higher the cloud top the lower the OLR.
T20: Schematic of OLR

Convection forms when surface is heated afternoon convection, common In the tropics. Also- when upper air is cooled, as in cold front in midlatitudes.

Radiative convective equilibrium: observed to hold everywhere. In tropics it is clear, in midlatitudes less clear that convection sets vertical laps rate.

HW: Marshall and Plumb chapter 4 ex: 1,2,7,10,12