

## **Lecture 1:** Intro – the essential ingredients of atmospheric dynamics.

The atmosphere is thin film of fluid, on a spherical earth, under the influence of gravity, rotation, and differential heating by the sun. The atmosphere is compressible, and thus stratified, and almost inviscid. The oceans are similar, but essentially incompressible and saline.

This course will deal with the dynamics of such a fluid. In principle, to understand the atmosphere-climate system we need to understand all of its main components and how they interact. Dynamics depends on the distribution of heating (the main energy source and driver of motions). The heating depends on the Radiative transfer properties of the atmosphere. Those depend on water vapor. Water vapor depends on the motion of air- the dynamics. A closed cycle – everything is dependent. For climate you need to consider all. Here we will concentrate on the dynamics but will mention basic characteristics of the other parts when necessary.

**Rotation experiment:** rigidity imparted to rotating fluids.

Two cylinders- one rotating and one not.

Make some waves in the tanks and put two dyes of different colors, slightly apart.

Non rotating - see how each color diffuses and how they mix.

Rotating – see formation of long streaks which swirl around and thin filaments intertwine but don't mix till very late in the flow.

*When is rotation important?* Scale analysis: the typical time scale of fluid motion is  $L/U$ , the typical distance a fluid particle moves, divided by the characteristic velocity. We need to compare this to the rotation time scale,  $\tau$ . For rotation to be important we need its time scale to be shorter than the typical flow time scale. Thus, the ratio of  $Ro \equiv \tau_{rot}/\tau_{fm}$  has to be small for rotation to be important.

The Rossby number is thus:  $Ro = U\tau/L$ .

For our tank: we need to take the velocity relative to the tank, and not the solid body rotation rate. If  $U \sim 1 \text{ cm/sec}$ ,  $L \sim 30 \text{ cm}$ ,  $\tau \sim 2 \text{ sec}$ ,  $Ro \sim 0.07$ , which is small.

For the atmosphere: For typical midlatitude systems:  $L \sim 5000 \text{ km}$ ,  $U \sim 10 \text{ m/sec}$ ,  $\tau = 1 \text{ day} \sim 10^5 \text{ sec}$ , which gives  $Ro(\text{atm}) = 0.2$  which is quite small. Rotation is definitely important.

**T1-View of Earth from space over North Pole:** can see swirling patterns in clouds somewhat similar to tank.

For the ocean: For the largest gyres:  $L \sim 1000 \text{ km}$ ,  $U \sim 0.1 \text{ m/sec}$ ,  $\tau = 1 \text{ day} \sim 10^5 \text{ sec}$ , which gives  $Ro(\text{atm}) = 0.01$  which is very small. Rotation is a controlling factor.

### **Low viscosity:**

Lets look now at a classical fluid dynamics experiment, which has been studied first in the lab and later on by computers- a constant flow of a fluid past a cylinder.

**T2-flow past cylinder:** Schematic and pictures of different experiments. The source of differences we will understand in a minute but note the vortices which form, or in some cases a turbulent region and eddies. In some sense similar to the atmosphere.

**T3-water vapor satellite images:** We see similar eddies, and complexity with some order.

**T4-schematic dependence on Re:** The form of the flow will depend on the importance of viscosity, characterized by a nondimensional constant called the Reynolds number (like  $Ro$  for rotation).

$Re=LU/\nu$  where  $\nu$  is called the kinematic viscosity, which depends on the fluid substance and is measured in units of  $m^2/sec$ .

Note:  $\nu/U$  is a viscous diffusivity scale – it has units of meter. Large  $Re$  implies the motion scale is larger than the diffusivity scale, so viscosity is not important.

Values:  $\nu(\text{air})=1.5 \times 10^{-5} m^2/sec$ ,  $\nu(\text{water})=1.1 \times 10^{-6} m^2/sec$ ,  $\nu(\text{honey})=7.4 \times 10^{-5} m^2/sec$ .

For typical midlatitude atmospheric systems:  $Re \approx 3 \times 10^{13}$ . Turbulent

For the largest ocean gyres:  $Re \approx 10^{11}$ . Turbulent

For stirring of honey with a spoon:  $U \sim 1 \text{ cm/sec}$ ,  $L \sim 1 \text{ cm}$ ,  $Re \approx 1.3$ .

Note: The experiments shown are with no rotation. Earth fluids are turbulent rotating fluids.

### **Thermally driven – baroclinic fluid:**

In the cylinder experiment shown above, as well as in the rotating-non rotating tank experiment which we did, the energy for the turbulence and waves came from the kinetic energy of the flow past the cylinder, or from the mechanical stirring of the fluid by hand. In the atmosphere and in large part in the ocean, the energy source is thermal (wind driven ocean circulations are not a directly thermally driven). The resemblance of the structures is thus superficial.

In those experiments, and in most of classical fluid dynamics and hydrodynamics theory and experiments, the density is assumed to be constant, or a function of pressure only, meaning it is not a function of temperature:  $\rho = \rho(p)$ . Such a fluid is called *barotropic*.

What would happen if the atmosphere and ocean were barotropic? We would not have convection, and we would not be able to move anything with heating.

**T5a-schematic of barotropic flow:** Assume an atmosphere with this structure.

Gravity will result in pressure increasing downwards (a hydrostatic balance which we will learn about later on), and for the fluid to be gravitationally stable, density has to decrease with height as well, so lighter fluid will always be above heavier fluid, and the fluid will reach a steady state.

**T5b-schematic of baroclinic flow:** If, on the other hand density is also a function of temperature  $\rho = \rho(p, T)$ , then even if the pressure surfaces are horizontal, density can locally change by heating, for example heating of the surface. We can have light fluid created under heavy fluid, with convection ensuing. Turbulence is in general thermally driven.

Note: Our rotating bowl experiment – the vertical sheets formation are a characteristic of barotropic fluids. For those, one can show that the flow does not change in the direction of the rotation axis. For baroclinic flows, we will find that the flow change in the direction of the rotation axis is proportional to the horizontal temperature gradient (thermal wind balance).

### **Atmospheric aspect ratio (depth-horizontal extent ratio) and topography:**

The atmosphere is very thin. The action of gravity on it, causes the density to decrease exponentially with height with a scale height of 7km (we will see in a minute).

About 80% of the mass below 10km.

**T6-atmosphere thickness and some constants:** This gives a sense of how thin the atmosphere is. This allows us to ignore vertical changes in gravity. Also in many cases we can ignore the earth's curvature. Later on we will see also that it

makes the vertical motion in general much smaller than the horizontal and the flow is *quasi-2D*.

### **Characteristics of air – dry and moist:**

**Pressure:** force per unit area. It is independent of direction, even though to define it need to assume a surface.

Units: Pascal  $1 \text{ Pa} = \text{N/m}^2 = \text{kg /m/sec}^2 = \text{J/m}^3$

For the atmosphere we use hPa=100Pa, which is 1 milibar.

Ideal gas: a gas of point particles. Obeys the *ideal gas law*. Holds if the intermolecular forces' effect is negligible compared to the particle's kinetic energy, and the diameter of the molecules is negligible compared to the distance between the molecules. These conditions generally fail at low temperatures and high pressures.

Ideal gas law: relates the pressure, temperature and density of an ideal gas. A particular form of the equation of state.

Pressure is inversely proportional to volume, if temperature is held fixed.

Pressure is proportional to temperature if the volume is held fixed.

$$p = \rho RT$$

$$R = R_g / m_a = 8314.3 / 28.97 = 287 \text{ J/Kg/K}$$

$R_g$ - the universal gas constant.

$m_a$ - the mean molecular weight of the gas

### **Moisture:**

The ideal gas law can be applied to individual components of the air mixture. In particular, for water vapor:

$$p = p_d + e \text{ dalton's law}$$

$$p_d = \rho_d R_d T$$

$$e = \rho_v R_v T$$

$$R_v = 8314.3 / 18.02 = 461.39 \text{ J/Kg/K}$$

$p_d$ - partial pressure of the dry air

$e$  – partial pressure of the water vapor.

**T7-saturation of air over water:** The amount of water vapor varies a great deal due to evaporation and condensation. Water condenses when the air is saturated. Think of a box with air above water. Water molecules leave the water and enter into the air, and at the same time water vapor molecules from the air hit the water surface. The rate at which the latter happens depends on the partial pressure of the water vapor. Equilibrium is reached when the rates of these two processes is the same, and the air is then saturated. If there is less water in the air it will accumulate and if there is more it will condense. The saturation vapor pressure  $e_s$  depends on pressure and temperature in a relation called the Clausius-Clapeyron equation:

$$\frac{de_s}{dT} = \frac{Le_s}{R_v T^2}$$

This is not a very simple relation to work with. There are above 100 approximate simplifying formulae. I will choose one particularly simple one here, for illustrative purposes:

$e_s = Ae^{\beta T}$  with  $A=6.11$  hPa, and  $\beta=0.067^\circ\text{C}^{-1}$ , and  $T$  is in  $^\circ\text{C}$ . **Shown in T7-saturation**

This essentially exponential relation has some important implications:

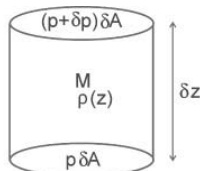
- Moisture concentration decays rapidly with height, because temperature decreases with height. Most of the moisture is in the lowest 4 km.
- There is much more moisture in the tropics than at higher latitudes.
- Very large spatial gradients.
- Precipitation occurs when air is cooled by convection
- In past climates- warm climates were moist and cold climates (ice ages) were arid.

### **The vertical structure of the atmosphere:**

Temperature:

**T8-vertical T and O<sub>3</sub> profiles:** The vertical structure of temperature is determined to a large extent by radiation transfer, thus we will only mention the very basics, and not go into any detail. Heat sources are the surface, ozone, and molecular oxygen, which absorb solar radiation at the ground, at 50 and at 80 km, and emit IR back to the atmosphere.

**Hydrostatic balance:** Lets look at a static parcel of air. The pressure will depend on the weight of the fluid column above. The resulting balance between gravity and the pressure forces results in *hydrostatic balance*:



**Figure 3.5:** A vertical column of air of density  $\rho$ , horizontal cross-sectional area  $\Delta A$ , height  $\Delta z$ , and mass  $M = \rho \Delta A \Delta z$ . The pressure on the lower surface is  $p$ , the pressure on the upper surface is  $p + \Delta p$ .

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The mass of the parcel:  $M = \rho \Delta A \Delta z$ .

The pressure at the lower side is  $p$ , and at the upper side,  $p + \Delta p$ , with  $\Delta p = \frac{\partial p}{\partial z} \Delta z$

The forces acting on the parcel (upward positive):

- gravity:  $F_g = -gM = -g\rho\Delta A\Delta z$
- pressure force at top:  $F_t = -(p + \Delta p)\Delta A$
- pressure force at bottom:  $F_b = p\Delta A$

In balance we require the net force to be zero:  $F_g + F_t + F_b = 0$

We are ignoring vertical accelerations. This is a good approximation in most cases, but not in small scale vigorous systems (convection, tornadoes, thunderstorms, deep polar convection in oceans).

Assuming balance holds, we get:  $\Delta p + g\Delta z = \frac{\partial p}{\partial z} \Delta z + g\Delta z = 0$

$$\rightarrow \frac{\partial p}{\partial z} = -\rho g \quad \text{hydrostatic balance}$$

Integrating from height  $z$  to infinity, and using the fact that pressure goes to zero there, we get:

$$p(z) = g \int_z^{\infty} \rho(z) dz$$

The integral term is the mass of a unit area of the atmospheric air column. The mass of the air column, and of the atmosphere, can thus be deduced from the surface pressure. The total atmospheric surface pressure is the mass of the atmosphere divided by the surface area of the earth.

To get  $p(z)$  we need to know how density varies with height.

This we get from the ideal gas law:

$$\frac{\partial p}{\partial z} = -\rho g = -\frac{gp}{RT}$$

But we now need to know  $T(z)$ , which is determined from the radiation and heating processes.

However, unlike  $p$  and  $\rho$ , which vary by orders of magnitude in the atmosphere from the surface to 100km,  $T$  varies from about 200-280K, which is around 15% from 240K. So by assuming it is constant we can get an idea of how  $p$  and  $\rho$  vary.

#### **Isothermal atmosphere:**

$$\frac{\partial p}{\partial z} = -\rho g = -\frac{gp}{RT_0} \equiv -\frac{p}{H} \quad \text{with the scale height } H = RT_0/g. \text{ For } T_0 = 250\text{K, which is}$$

close to the *emission temperature* of the Earth,  $H = 7.32\text{km}$ .

Then  $p = p_0 e^{-z/H}$ . Alternatively, we can express  $z$  in terms of  $p$ :  $z = -H \ln(p/p_0)$ . This called log-pressure height. In an isothermal atmosphere, the log-pressure height is equal the actual geometric height. In a non-isothermal atmosphere it is close but not exactly equal.

#### **When temperature varies with height:**

When  $T = T(z)$ , we can still define a scale height  $H(z) = RT(z)/g$  and then:

$$\frac{1}{p} \frac{\partial p}{\partial z} = \frac{\partial \ln p}{\partial z} = -\frac{1}{H(z)}$$

$$p(z) = p_s \exp\left(-\int_0^z \frac{z'}{H(z')} dz'\right)$$

Note that  $H_v = R_v T/g$  is larger for vapor than for air since  $R$  is larger, but the observed scale height for vapor is smaller than for air. Why? Because  $e_s$  varies sharply with temperature.

We can also get the height of a pressure surface by integrating from the surface upwards:

$$dz = -H(p) d \ln(p) = -RT(p) d \ln(p)$$

$$z(p) = R \int_p^{p_s} \frac{T(p)}{g} \frac{dp}{p}$$

Note: A given pressure surface is higher when the air below it is warmer.

**T9-pressure for isothermal and non-isothermal atmospheres:** The difference is small. Where  $T(z) < T_0$ ,  $H(z) < H_0$ , and  $\frac{\partial p}{\partial z} > \frac{\partial p}{\partial z}$  (isothermal)

**T10- T, H and pressure for non isothermal, and pressure for isothermal.**

**Density:** We use the ideal gas law:  $\rho = p/(RT)$

For an isothermal atmosphere:  $\rho = p_s/(RT_0)e^{-z/H} = \rho_s e^{-z/H}$ .

Use this to calculate the mass of air above height  $z$ . Plug into  $p(z) = g \int_z^{\infty} \rho(z) dz$  and

get:  $p(z)/p(0) = \exp(-z/H)$ .

**T11-exp(-z/7km):** about 75% of the atmosphere is below 10km, 90% below 16km.

For a non isothermal atmosphere:

$$\rho(z) = \frac{p_s}{RT(z)} \exp\left(-\int_0^z \frac{z'}{H(z')} dz'\right)$$

**Geopotential and geopotential height:**

The gravitational force per unit mass acting on an air parcel is  $\mathbf{g}$  which is a vector acting vertically- towards Earth's center. (more about this later).

Can write the gravitational force as a potential:  $\nabla\phi = -\mathbf{g}$ .

Since  $\mathbf{g} = -g\mathbf{k}$  – a downward pointing vector, this gives:  $d\phi/dz = g$ .

The geopotential  $\phi(z)$  is the work required to raise a unit mass to height  $z$  from the

surface:  $\phi(z) = \int_0^z g dz$

Note that  $d\phi = g dz = -dp/\rho = -RT \ln(p)$  where we used the ideal gas law and the hydrostatic balance relation.

This is similar to the expression we got for the height of a pressure surface, only divided by  $g$ .

We define the *geopotential height* to be the height corresponding to  $dz = d\phi/g_0$  where  $g_0$  is the gravitational acceleration at the surface.

$$z_\phi(p) = \frac{R}{g_0} \int_p^{p_s} T(p) \frac{dp}{p}$$

In the case of an isothermal atmosphere, this is just the log-pressure height we saw above (with the gravitational acceleration held fixed approximation).

In the HW we will see that taking  $g$  to be constant is a good approximation in the troposphere, but much less so in the stratosphere and in particular the mesosphere.

**Summary:** We discussed the main characteristics of the atmospheric flow – rotating thin layer of inviscid compressible fluid on a sphere, under the action of gravity.

We learnt the ideal gas law for dry air and water vapor, and discussed the vertical structure of the various fields- temperature (heating), pressure (gravity and hydrostatic balance), and density (follows from pressure, temperature and ideal gas law). Discussed the vertical structure of these for an isothermal and non isothermal atmosphere, and introduced the geopotential and geopotential height.