

## Middle Atmosphere Dynamics, Homework 2

1. Assume an atmosphere with constant basic state wind and  $N^2$ , with density  $\rho = e^{-z/H}$ , that is bounded by a rigid lid ( $w = 0$ ) at  $z = z_t$ . Solve for linear waves which are forced to have an amplitude of  $\varphi_b$  at the lower boundary ( $z = 0$ ). Calculate  $F_z$ , the vertical component of the EP flux for this case. Explain.

2. Some points about energy and its exchange with the mean flow:

Consider conservative (no damping or heating), barotropic QG motion on a  $\beta$  plane, governed by the momentum equations:

$$D_g u_g - \beta y v_g - f_o v_a = 0$$

$$D_g v_g + \beta y u_g - f_o u_a = 0$$

a) Show that small amplitude perturbations to a zonal mean flow  $U(y)$  satisfy the energy equation:

$$\frac{\partial}{\partial t} \left( \frac{1}{2} (\overline{u'^2} + \overline{v'^2}) \right) + \overline{u'v'} \frac{dU}{dy} = - \frac{\partial}{\partial y} (\overline{v'_a \phi'}) \quad (1)$$

where the notation is the same as used in class.

b) For small amplitude propagating waves of the form:

$$\psi' = \text{Re} \left( \Psi(y) e^{ik(x-ct)} \right)$$

show that the “energy flux”  $\overline{v'_a \phi'}$  is related to the momentum flux by:

$$\overline{v'_a \phi'} = -(U - c) \overline{u'v'} \quad (2)$$

Hint: implement the wave structure by noting that  $\frac{\partial}{\partial t} = -c \frac{\partial}{\partial x}$  and use the  $x$  momentum equation.

c) Using equations 1 and 2 above, show that:

$$\frac{\partial}{\partial t} \left( \frac{1}{2} (\overline{u'^2} + \overline{v'^2}) \right) = (U - c) \frac{\partial}{\partial y} (\overline{u'v'}) \quad (3)$$

What does equation 3 imply about the wave activity budget of steady conservative waves? What does it imply about the energy flux of steady conservative waves? (use equation 2).

d) Show from the energy equation (1), and equation 3, that for conservative steady waves

$$\frac{\partial}{\partial y}(\overline{u'v'} \bar{U} + \overline{v'_a \phi'}) = 0 \quad (4)$$

What does this imply about the energy budget?