Feb 23, 2011. Homework due March 9th

Middle Atmosphere Dynamics, Homework 1

1. The Quasi-Geostrophic equations: Starting from the definitions of u_g, v_g (use a streamfunction $\psi = \phi/f_o$), the geostrophic momentum aproximation (the meterial derivative follows the geostrophic flow - $\frac{d}{dt} \equiv D_g$), the hydrostatic relation, the assumptions that $S = S(z)$, small isentropic slopes, and the scaling assumptions of small R_o and small $\frac{\beta L}{f_o}$, derive the following equations from the full log-p equations presented in class:

a) The horizontal momentum equations:

$$D_g u_g - \beta y v_g - f_o v_a = G_x$$

$$D_g v_g + \beta y u_g - f_o u_a = G_y$$

b) The QG temperature equation:

$$D_g T + w_a S = Q$$

c) Use the 3 QG equations you derived in a,b, along with the continuity equation, the thermal wind relations, (and the assumptions above) to derive the QG PV equation:

$$D_g q = \chi$$

where

$$q = f_o + \beta y + \nabla^2 \psi + \frac{1}{\rho}\frac{\partial}{\partial z}\left(\frac{\rho f_o^2}{N^2}\frac{\partial \psi}{\partial z}\right)$$

$$\chi = \frac{\partial G_y}{\partial x} - \frac{\partial G_x}{\partial y} + \frac{1}{\rho}\frac{\partial}{\partial z}\left(\frac{\rho f_o}{S} Q\right)$$

2. The wave equation for varying N^2: For the idealized vertical wave propagation equation derived in class:

$$\nabla^2 \psi + \frac{1}{\rho}\frac{\partial}{\partial z}\left(\epsilon \frac{\partial \psi}{\partial z}\right) = 0$$

where $\epsilon = \frac{{f_o}^2}{N^2}$ assume a normal mode solution of the form: $\psi = Re\left(\Psi(y,z)e^{ik(x-ct)}\right)$ and show that if ϵ varies with z, the transformation of variables $\varphi = \sqrt{\rho\epsilon}\,\Psi$ yields a wave equation of the form

$$a\varphi_{zz} + \varphi_{yy} + {n_r}^2 \varphi = 0$$

Find a and n_r^2 (which can be functions of height).

3. The attached figure shows the seasonal evolution of 1mb, wave 1 amplitude for both hemispheres.

a) Explain the seasonal cycle which you see, and the relative phasing between the two hemispheres.

b) There is a mid-winter minimum in wave activity in the Southern Hemisphere which is not see in the Northern Hemisphere. What is a plausible explanation for this?

1 mb AMP. OF ST. WAVES (WN = 1)

(a)