Licensing and the Sharing of Knowledge in Research Joint Ventures*

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We consider a three-stage model of research and development (R & D) to capture some key elements of research joint ventures (RJVs). In the last of the three stages, firms compete in the product market. In the second stage, the firms simultaneously choose unobservable R & D levels. In the first stage, the firms can share some or all of their knowledge with other firms in the RJV. We examine the ability of two simple licensing mechanisms to ensure both efficient sharing of knowledge and efficient R & D effort levels. *Journal of Economic Literature* Classification Numbers: D21, D82, L20.   © 1992 Academic Press, Inc.

1. INTRODUCTION

Research joint ventures (RJVs) are now commonplace throughout the world. Among the many ongoing RJVs in the United States are Bell Communications Research, founded in 1984 by the seven regional telephone companies, and the Microelectronics and Computer Technology Corporation (MCC), formed in 1982 to conduct research related to information

technology. A primary goal of these and other RJVs is to exchange knowledge and develop basic expertise that the founders of the RJV can then employ in their own research and development processes and in subsequent marketplace activities.[1] Surprisingly, most economic models of research and development (R & D) have not explicitly addressed the possibility that firms might share important inputs, such as knowledge, before embarking on their own R & D programs.[2] The primary focus of this paper is on the design of rules an RJV might adopt to motivate the sharing of productive knowledge among its members, knowing that they will subsequently compete in both an R & D contest to reduce production costs and, ultimately, in the product market. We concentrate on simple licensing schemes that could be readily implemented in practice. We ask when these simple schemes can provide the ideal incentives for both the sharing of knowledge and R & D effort levels (i.e., when the schemes can ensure a first-best outcome).

The licensing arrangements must overcome an obvious reluctance a firm will have to share its knowledge: sharing enhances the likely performance of competitors, and thereby can reduce one's own chances of winning the R & D contest. Indeed, it has been noted that the individual founders of MCC were initially reluctant to send their best researchers to work at the RJVs facilities,[3] perhaps fearing that their best people would reveal more expertise than they received. Overcoming the reluctance of a participant in an RJV to share superior knowledge can be a delicate matter. To illustrate, one might attempt to motivate the sharing of knowledge by: (a) promising particularly large rewards for winning the R & D contest to the firm that shared its superior knowledge in the first stage; and (b) imposing large licensing fees on winners of the second-stage contest if they received the knowledge in the first stage. Although such a licensing structure might successfully motivate the desired sharing of knowledge in the first stage, it might distort the R & D activities undertaken by participants in the second stage of "the game." In particular, the firm that initially provided knowledge to its competitors might undertake too much R & D effort, while the competitors might undertake too little effort relative to the social optimum.

We find that this potential conflict can be resolved with relatively simple licensing arrangements, provided firms who initially received knowledge can be charged an entry fee for the right to subsequently engage in the R & D race. Although such entry fees may appear anticompetitive on

the surface, they can be instrumental in alleviating the tension between providing rewards for the sharing of knowledge and motivating the first-best levels of R & D effort.

This basic tension can be insurmountable, however, when such entry fees are prohibited (perhaps because of antitrust concerns or because the resources of firms are limited). When all rewards for sharing knowledge must be delivered via licensing fees derived from the profits earned in final-stage product-market competition, a first-best outcome may no longer be feasible. The license fees required to induce the sharing of knowledge at the first stage can distort second-stage R & D effort away from first-best levels. Nevertheless, there will be situations where this ideal outcome can still be ensured, even with the additional restrictions on the licensing mechanism. We characterize such situations, showing, for example, that a first-best outcome will be feasible when a successful second-stage innovation is sufficiently likely or when the number of competitors is sufficiently large. These and other related findings are presented in Section 4.[4]

First, though, the basic elements of our model are described in Section 2. This section also contrasts our paper with others in the literature and defines the two licensing structures we analyze. Section 3 defines a first-best outcome and identifies some important properties of the licensing structures under consideration. Conclusions are drawn in Section 5, where we suggest extensions of our model. All formal proofs are relegated to the Appendix.

2. THE MODEL

There are three distinct stages in our model. In the first stage, N firms have the opportunity to share their private knowledge with other firms. Greater knowledge enhances one's abilities in the second-stage research and development (R & D) contest. During the second stage, firms employ their knowledge independently in an attempt to successfully achieve an innovation of known social value, V. For simplicity, the R & D process for each firm is modeled as an "all-or-nothing" phenomenon: either the innovation of value V is achieved, or nothing is achieved. Although other interpretations are possible, we will treat a successful innovation as a reduction in the constant marginal cost of producing a homogeneous product from c_H to c_L. With inelastic demand at output Q, the social value

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[4] As will become apparent, our focus is entirely upon motivating the sharing of knowledge among members of an RJV. We do not address the issue of spillovers of research findings to firms that are not members of the RJV. It is argued in [27] that at least in the computer and semiconductor industries, these spillovers are of limited importance. The key determinant of competitive success in these industries is a firm's lead time in the R & D process.

of the innovation is $[c_H - c_L] Q = V$. The third stage of our model consists of Bertrand competition among firms who decide to produce.

At the start of the first stage, each of the N risk-neutral firms incurs a fixed cost which enables it to privately observe its knowledge endowment. Each firm's knowledge endowment is modeled as an independent realization of a random variable, $\tilde{n}$, with density $f(n) > 0$ $\forall n \in [\underline{n}, \bar{n}]$ and corresponding distribution function $F(n)$. Unless stated otherwise, the distribution of $\tilde{n}$ is assumed to have no mass points. A higher realized value of n corresponds to a higher level of knowledge, in a sense to be made precise below.[5] After observing privately their own knowledge realization, the firms have an opportunity to simultaneously make public some or all of their knowledge. Revealed knowledge can be verified costlessly; thus, a firm cannot exaggerate its knowledge level. However, a firm could conceal some of its private knowledge rather than share all of it with competitors. Whether knowledge is shared fully depends upon the anticipated reward for disclosure.

After disclosures are made, the second stage begins. During this stage, firms simultaneously undertake an unobservable, immutable R & D effort that is privately costly. Effort by firm i secures a probability of success, P_i. The cost to each firm of implementing success probability, P_i, when its knowledge level is $n \in [\underline{n}, \bar{n}]$ is given by $C(P, n)$.[6] This cost includes a positive fixed cost, C_0, which must be incurred to achieve $P > 0$.[7] For a given level of knowledge, the cost of implementing success probability $P > 0$ is an increasing, strictly convex function of P (i.e., $C_P(P, n) > 0$ and $C_{PP}(P, n) > 0$ $\forall n$, $P > 0$, where subscripts denote the obvious partial derivatives). Higher levels of knowledge reduce the total and marginal costs of implementation (i.e., $C_n(P, n) < 0$ and $C_{Pn}(P, n) < 0$ $\forall n$, $P > 0$).[8] For

[5] One might view this as a process where each of N firms hires a researcher from an infinite pool of observationally equivalent researchers. Eventually, each researcher's ability to conduct R & D becomes known privately to the firm that hired him. Thus, the skill of the hired researcher determines the firm's knowledge in this interpretation. Of course, there are a number of obvious simplifications in this story. First, firms that hire "bad" researchers do not return to the pool of researchers and obtain another "draw." This may be due to large setup costs or long lags between the time a researcher is hired and when he learns his skill level. Second, the researcher's skill becomes known to the firm and effectively becomes the property of the firm. There are a variety of interesting issues concerning the intellectual property of employees that are beyond the scope of this paper.

[6] This cost function, like the density function $f(n)$, is common knowledge.

[7] A strictly positive fixed cost ensures that the socially optimal number of researchers is finite. This fixed cost should be thought of as the cost required to utilize any disclosed knowledge level.

[8] Obviously, this cost structure is a reduced form of a more complex underlying process of knowledge sharing. One such process is the following. Suppose there are a countable infinity of development techniques that can be employed to develop the innovation. Each technique appears the same to firms *ex ante*, in the sense that each costs the same to employ and each

simplicity, we also assume $[1 - P]\, C_P(P, n)$ is strictly monotonic in P for all n. This assumption ensures that at the ideal (first-best) outcome (defined in Section 3), each participating firm chooses the same success probability. [9]

We assume that the knowledge levels of the firms in our model can be ordered in a Blackwell sense [4]. Thus, if firms i and j have knowledge levels n_i and n_j, respectively, with $n_i > n_j$, only firm j can gain from an exchange of knowledge. At best, its knowledge level can be raised to n_i. Although other information structures are conceivable (see Section 5), the Blackwell ordering we adopt seems natural as a first step. [10] This ordering is also consistent with the observation in [14] that some of the more successful joint ventures are those in which one firm provides knowledge to other firms and receives no knowledge in return.

What remains is to describe the two simple licensing mechanisms we consider. The central feature of both mechanisms is that the leading firm (i.e., the firm that reveals the most knowledge at the first stage) is treated more favorably than lagging firms (i.e., firms that do not reveal the most knowledge) in order to provide incentives to share superior knowledge. The differential treatment encompasses the following asymmetric restrictions on a firm's right to employ the innovation it achieves. There are no restrictions on the leading firm. The leading firm never pays a licensing fee

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is thought capable of implementing the innovation with probability q. Each firm that expends a fixed cost, C_0, becomes capable of testing techniques. A "bad" technique (i.e., one that is certain to fail) can be identified with probability one by any test. A "good" technique (i.e., one that will succeed if adopted) will be identified as such by a test with probability $n \in (0, 1)$, and will be erroneously categorized as a bad technique with probability $1 - n$. Thus, a higher n represents a more accurate test. This n represents a firm's basic knowledge level in this example, and a firm can share its knowledge by showing other firms how to increase the accuracy of their test of techniques to probability $d \leqslant n$. Given n (or d, the level of knowledge disclosed), a firm can choose a probability of successful innovation by choosing the maximum number of techniques it will test (t) in the second stage. (Similar arguments apply to sequential sampling.) The cost of testing $t(>0)$ techniques, $C_1(t)$, is a strictly increasing convex function. Straightforward calculations show that in this environment, the indirect cost function for identifying a good technique with probability P, i.e., $C(P, n)$, has the properties described above.

[9] This fact is proved in Lemma 1 below. Notice that this monotonicity condition is satisfied in the sampling interpretation of the cost structure outlined in the preceding footnote.

[10] Notice that this ordering is analogous to that presumed in the related literature on multistage R & D races. (See, for example, [10, 13, 15, 17]). In that literature, a sequence of projects must be completed in a specified order before an innovation is achieved. Firms that have completed more projects can assist the progress of firms that have completed fewer projects, but the reverse is not true. It is precisely this unidimensional aspect of knowledge sharing that the presumed Blackwell ordering implies in our model. Note that in the context of the sampling interpretation developed above, the Blackwell ordering implies that whenever a test with accuracy n_1 identifies a technique as "bad," so will any other test with lower accuracy, $n_2 \leqslant n_1$.

and is always free to employ any innovation it develops. On the other
hand, a lagging firm that is the only lagging firm to succeed at the R & D
stage is permitted to employ its innovation, but is charged a fee of rV for
doing so. [11] (The choice of $r \in (0, 1)$ will be described in detail below.) Most
importantly, this licensing fee must be paid by the "licensed" firm to the
leading firm if and only if the licensed firm earns at least V in the final-
stage product market competition. Otherwise, the licensed firm, like all
other lagging firms, pays no fee. [12]

When two or more lagging firms successfully develop the innovation, one
is chosen at random to be the licensed firm. The chosen firm is required to
pay V as a licensing fee to the leading firm, provided the licensed firm earns
at least V in final-stage profit. [13] The successful lagging firms that are not
licensed forfeit all rights to their innovation. [14,15]

When the sole forms of compensation for the leading firm are any direct
profit it earns in the product market and the license fees charged to a
licensed lagging firm, it is said that *restricted licensing* (RL) is in place. As
we show in Section 4, there are plausible settings where the ideal incentives
for both the initial sharing of knowledge and subsequent R & D effort by
all firms can be ensured under RL via suitable choice of the licensing fee,
rV. Sometimes, though, strict gains are available when entry into the
R & D race can be priced and controlled directly. Under *unrestricted
licensing* (UL), a uniform entry fee (E) can be imposed on all lagging firms
who wish to enter the second-stage R & D competition. This fee is paid to

[11] If two or more firms disclose the same level of knowledge, one is selected at random to
be the leader and the other firms are designated as lagging firms. With no mass points in the
distribution of $\tilde{n}$, however, the probability of identical disclosures by firms in a first-best
setting is zero.

[12] The practice of making payments contingent upon realized profit is not without
precedent in RJVs. Under the *hojokin* system, the Japanese government commonly provides
funding to RJVs in the form of interest-free loans with the explicit condition that these loans
be repaid from profits that flow from technology produced by the RJV. (See [19, 25]).

[13] Alternatively, the multiple innovating lagging firms could bid for the exclusive right to
employ the innovation as in [20]. An appropriately designed auction will yield V to the
leading firm.

[14] When the licensing fee is V, the licensed lagging firm will technically be indifferent
between competing and not competing in the product market. But the firm will strictly prefer
to compete for any licensing fee strictly less than V. We avoid an uninteresting open set
problem by assuming the authorized firm competes when indifferent between doing so and not
competing.

[15] It is conceivable that multiple lagging firms who develop the innovation simultaneously
will have an incentive to collude and hide their "tie" from the leading firm, so as to pay a
lower licensing fee. However, such collusion would require private communication, clandestine
side-payments, and mutual verification of claims of success. Without this verification, all firms
would have an incentive to claim a tie with a successful lagging firm in order to secure com-
pensation for not disclosing his success to the leading firm. These requirements tend to make
successful collusion unlikely.

the leader and is not contingent upon realized profit. The lagging firms can choose not to engage in R & D if they consider the fee for doing so to be too high. But if they do engage in R & D, they must first pay the fixed charge E to the leading firm. We show in Section 4 that through suitable choice of r and E, the ideal incentives for knowledge sharing and R & D effort can always be ensured under UL if the leader can also dictate the maximum number of firms who compete at the R & D stage. Thus, despite their anticompetitive appearance, entry fees and entry prohibitions may actually promote the social interest. Of course, given the practical difficulties of excluding entry into markets (e.g., legal issues of market definition), UL may not be feasible in some settings where RL is feasible.

There are three key features of these licensing mechanisms that warrant emphasis and explanation. First, the payoffs to individual firms depend upon the number of other firms who have successfully developed the innovation. Because the social value of success by a firm also depends upon whether another firm has succeeded, this feature helps align the private and social incentives to conduct R & D. Second, licensing fees are contingent upon the realization of final-stage profits. [16] This feature makes credible the promise of a successful leading firm to refrain from product-market competition against licensed lagging firms. Thus, dissipative competition is avoided. (See Lemma 2). Third, under UL, lagging firms must pay in advance for the knowledge they receive, and so entry into the R & D race is priced explicitly. [17] As noted, this additional policy instrument can be critical to ensuring adequate incentives for full sharing of knowledge.

Both RL and UL entail an important restriction. Neither scheme allows payments by or treatment of a lagging firm to depend on any cardinal measure of the difference between its knowledge disclosure and those of the leader and/or any other lagging firm. This limited ability to discriminate among firms is natural in settings where the knowledge levels of lagging firms are not verifiable. For example, when imitation lags are short, firms might be able to represent much of the knowledge revealed by others as their own. As other authors have noted (e.g., [2, 6, 24]), it is often difficult in practice to discern the extent to which a lagging researcher is less informed when professional activity entails dissemination of proprietary

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[16] This linking of license fees to realized profit distinguishes the licensing fees we consider from those considered in [21].

[17] The key feature of the entry fee is not that it is paid in advance, but rather that the obligation to pay the fee does not vary with profit earned in the product market. The fee could conceivably be paid after the product market competition has taken place if the overseer can certify in advance that each firm has sufficient wealth to pay the fee even if no profit is earned in the product market. However, such certification may be problematic because the first-best fee will depend on the maximum disclosure of knowledge, and the overseer is not presumed to have any knowledge of the distribution of $\tilde{n}$ (including its upper support).

knowledge. Consequently, feasible licensing fees can only make a crude distinction between the most knowledgeable firm and less knowledgeable ones.

It should also be noted that neither the terms of RL nor those of UL depend upon the initial distribution of knowledge among firms, $f(n)$. Therefore, both licensing mechanisms could be implemented without any information about $f(\cdot)$, which, in practice, might not be readily available. This fact, coupled with the simple lump-sum licensing and entry fees under RL and UL make both schemes relatively straightforward to implement.

Before proceeding to our findings we briefly reiterate the timing and information structure of the model. The first stage begins with the specification of the terms of the overall licensing arrangement. Then, each of the N firms privately observes its knowledge level. Next to complete the first stage, firms make independent, simultaneous public disclosures of knowledge. After observing all disclosures and after the leading firm has been certified (by an impartial overseer), [18] the lagging firms decide whether to enter the second stage R & D contest. Upon entering, the firms simultaneously make independent, irrevocable, and unobservable choices of R & D intensities. The outcome of each firm's R & D process (i.e., success or failure) is then observed publicly. Next, at the start of the third stage, if there are any successful lagging firms, one is selected as the sole lagging firm with the right to employ the superior technology. Next, the firms decide whether to compete in the product market or exit the industry. [19] Finally, after production occurs and profits are realized, the assessed licensing fees are paid. The game is not repeated.

The equilibrium concept employed is Bayesian–Nash [16], coupled with the criterion of perfection [30] applied to final production decisions. At the start of the second stage of the R & D contest, each firm chooses a success probability to maximize its expected profit, given its beliefs about the knowledge of its competitors and knowing that its competitors will also select profit-maximizing effort levels. At the first stage, firms choose disclosure levels to maximize expected profit, given beliefs about the knowledge levels of competitors and knowing the terms of the licensing scheme, the nature of the subsequent game, and the equilibrium strategies of competitors.

Having specified our basic model in detail, we can contrast our formulation and findings with others in the R & D literature. To begin, our focus

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[18] In practice, the impartial overseer might be a government representative with sophisticated research skills. In Japan, engineers from the government's Ministry of International Trade and Industry played an active role in managing day-to-day activities in the Very Large Scale Integration Consortium (VLSI). (See [19]).

[19] As a normalization, we presume all unsuccessful firms compete in the third-stage product market competition. However, they earn no profit in equilibrium from doing so.

differs from the "standard" models of R & D races (e.g., [7, 22, 28, 29]) in that we are concerned with motivating both the efficient sharing of knowledge and efficient R & D effort levels. Our concern with sharing inputs to the R & D process also distinguishes our work from the standard treatments of cooperative R & D. In [18], for example, the profit-maximizing sharing of research costs and research findings is examined, but there is no explicit consideration of the sharing of knowledge or other inputs to the R & D process. [20] Our focus on the sharing of knowledge and on the maximization of socal welfare, coupled with the different policy instruments we consider, lead us to results which differ from those in [18]. In particular, we find that the ideal social outcome may be obtained from the R & D joint venture even when the product market is characterized by Bertrand competition. This result arises because, as noted above, the successful leading firm will choose not to compete with the licensed lagging firm under RL and UL.

In [20], the decision of a research lab to develop an innovation and to license the innovation to downstream oligopolists is analyzed. In that model, the oligopolists compete in the output market, but they do not use the innovation obtained from the research lab to engage in independent R & D. Thus, the basic innovation needs no further testing or development as it does in (the second stage of) our model. Furthermore, there is no concern in [20] with motivating the sharing of knowledge among members of the research lab. Despite the additional complications in our model, the first-best outcome can be secured in our model when it does not arise in [20] because of differences in feasible licensing mechanisms. In [13], the incentives of a firm to share its superior knowledge with a competitor in the R & D race are considered. However, the licensing fees analyzed in [13] are again different in nature from ours and are not motivated by the efficiency concerns that are central in our analysis.

The importance of examining ways to motivate the sharing of knowledge in RJVs is recognized in [19], but the formal models in [19] do not address this issue. The formal analysis most closely related to the present analysis is our companion paper [1]. In [1], a simple environment is assumed to avoid the nonconcavity in each firm's maximization problem that complicates the present analysis. [21] Our companion paper also focuses

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[20] Similarly, the analysis of RJVs in [26] does not analyze in detail this link between initial sharing of knowledge and subsequent R & D effort levels. In [11], the authors examine whether the ideal incentives can be created for firms to tailor their R & D effort levels to their privately known R & D abilities. No sharing of ability is possible in their model, however.

[21] The nonconcavity arises in our model because the usual "Spencian" monotonicity condition on the incentive payoff structure is not satisfied. In particular, it is not the case that the marginal cost of incremental technological disclosure to a firm is lower the greater is its initial knowledge endowment. This gives rise to an important nonconcavity in the decision problem

on the design of second-best incentive schemes. Our exclusive concern in this paper is whether a first-best outcome can be achieved. We now turn to a formal characterization of a first-best outcome. [22,23]

3. PRELIMINARY RESULTS

In this section, we briefly characterize a first-best outcome in which disclosures $\{d_i\}$ and R & D effort levels $\{P_i\}$ are selected to maximize the expected total surplus less the costs incurred by the firms. We also describe some important characteristics of RL and UL.

DEFINITION. A first-best outcome is the solution to the following problem:

$$\text{Maximize}_{P_., d_., \tilde{N}} \left[1 - \prod_{i=1}^{\tilde{N}} (1 - P_i) \right] V - \sum_{i=1}^{\tilde{N}} C(P_i, \max\{n_i, d\}) \qquad (3.1)$$

subject to: $d \equiv \max\{d_1, ..., d_N\}$, $d_i \leqslant n_i$ $\forall i = 1, ..., N$; and $\tilde{N} \leq N$, where $P_i \equiv$ the second-stage success probability of firm i.

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of the firms, which necessitates a global analysis because the local first-order conditions do not necessarily identify the solution to the problem we consider. This nonconcavity is the source of the potential indeterminacy in sharing rules that attain a first-best outcome. This potential indeterminacy is one feature that differentiates our results from the classic Bayesian implementation result on public goods due to D'Aspremont and Gerard-Varet [8]. As noted above, we find that a first-best outcome may be attained even when transfer payments among firms (i.e., entry fees) are not feasible prior to invention.

[22] As noted, the second-stage development contest in our analysis involves only two outcomes: success or failure. This is in contrast to the models in [7, 22], for example, where the patent race occurs in continuous time. A consequence of the continuous formulation is that "ties" are ruled out with probability one. Efficient development efforts are generally motivated in these models only by reducing the payment to the first innovator below the social value of his innovation. Unfortunately, the magnitude of the optimal "congestion tax" is generally cumbersome to characterize in continuous models. In contrast, the optimal tax on ties in our discrete formulation is readily derived. A more recent analysis in the continuous-time setting is [12]. In contrast to our model, the model in [12] does not permit payments among "winners" at the various stages of the R & D process. These payments are an integral component of our formulation. In other respects, our model has properties similar to those of the continuous-time models. In particular, the aforementioned nonconcavity of the firms' maximization problems arises in both settings. (See [3] for an analysis of this issue in a continuous time setting.)

[23] Other studies, such as [5, 23, 31], analyze conflicts between the attainment of efficiency and different notions of property rights or individual rationality. These studies are concerned with private goods, in settings where production externalities do not arise. Our model may be viewed, in part, as an extension of this line of research to an R & D setting where public goods (i.e., knowledge), endogenous effort choices, and the aforementioned "congestion externalities" all play important roles.

As is evident from (3.1), first-best success probabilities are chosen to maximize the difference between: (1) the product of V and the probability that at least one firm succeeds in the second-stage R & D contest; and (2) the aggregate cost of R & D effort. $\tilde{N}$ is the number of firms that compete in the second-stage R & D contest. We have normalized the identity of the N firms so that the first $\tilde{N}$ enter the second stage. Unless stated otherwise, we will assume that the optimal $\tilde{N}$ is invariant to the maximum realized knowledge endowment $n \in [\underline{n}, \bar{n}]$.

Lemma 1 characterizes a first-best outcome. In the statement of the lemma, $S^h(n)$ refers to the expected social surplus when the first h firms participate in the second-stage R & D contest, each with knowledge level n, and when each firm undertakes the first-best level of R & D effort.

DEFINITION. $S^h(n) = \left\{ 1 - \prod_{i=1}^{h} [1 - P_i^{h*}(n)] \right\} V - \sum_{i=1}^{h} C(P_i^{h*}(n), n)$, where

$$P_i^{h*}(n) \equiv \underset{P}{\operatorname{argmax}} \left\{ PV \prod_{\substack{j=1 \\ j \neq i}}^{h} [1 - P_j^{h*}(n)] - C(P, n) \right\} \qquad \forall i = 1, ..., h. \qquad (3.2)$$

LEMMA 1. *In a first-best outcome*:

(i) $d = n \equiv \text{maximum}\{n_1, ..., n_N\}$;

(ii) $S^{\tilde{N}} = \operatorname{argmax}_{h \leq N} S^h(n)$; *and*

(iii) $P_i = P^{\tilde{N}*}(n) \equiv \operatorname{argmax}_P \{ P[1 - P^{\tilde{N}*}(n)]^{\tilde{N}-1} V - C(P, n) \}$ (3.3)
$\forall i = 1, ..., \tilde{N}$.

The conclusions of the lemma are quite intuitive. All knowledge is disclosed and shared (i.e., $d = n$) in order to maximize the capabilities of all firms. The number of firms $(\tilde{N})$ that participate in the second-stage R & D contest is the number that maximizes the total expected surplus. Furthermore, success probabilities (P_i) are chosen to maximize their expected net contribution to social surplus.[24]

An important feature of a first-best outcome is reported in Corollary 1.

COROLLARY 1. *In a first-best outcome, the expected marginal contribution of each firm that participates in the second-stage R & D contest is strictly positive, i.e.*,

$$M^{\tilde{N}}(n) \equiv P^{\tilde{N}*}(n)[1 - P^{\tilde{N}*}(n)]^{\tilde{N}-1} V - C(P^{\tilde{N}*}(n), n) > 0. \qquad (3.4)$$

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[24] Recall that the symmetry reflected in (3.3) follows from the assumption that $[1 - P] C_P(P, n)$ is strictly monotonic in P for all n.

Corollary 1 follows directly from the presumed fixed cost, C_0, of achieving any success probability $P > 0$. The corollary suggests that the ability to control participation in the second-stage R & D contest will be important if a first-best outcome is to be ensured, since the private profit incentives of individual firms may not be perfectly aligned with the social incentives.[25]

Our concern throughout the analysis is whether a first-best outcome can be ensured under RL and/or UL. To investigate this issue, it is useful to first examine the incentives RL and UL provide at the second-stage R & D contest and the third-stage product market competition. Lemmas 2 and 3 address these incentives.

LEMMA 2.   *Under both RL and UL, if a lagging firm succeeds at the R & D stage, the leader will not engage in product market competition with the licensed lagging firm.*

The proof of Lemma 2 is immediate. If the leader competes with the licensed lagging firm, the profit of both firms is driven to zero by Bertrand competition. On the other hand, the leader can ensure itself a strictly positive payoff, rV, by not competing, since the licensed firm must pay the specified license fee when its profits from the product–market competition are sufficiently large. Thus, the licensing rules in RL and UL prevent dissipative final-stage competition between the leading and lagging firms. In doing so, RL and UL can provide ideal incentives for second-stage R & D effort, as indicated in Lemma 3.

LEMMA 3.   *Suppose full disclosure $(d = n = \max\{n_1, ..., n_N\})$ and the first-best participation level $(\tilde{N})$ are ensured. Then first-best R & D effort levels will arise as the unique symmetric Nash equilibrium of the second-stage R & D contest under RL and UL when a sole lagging innovator is assessed a licensing fee of $P^{\tilde{N}*}(n) V$ (i.e., when $r = P^{\tilde{N}*}(n)$).*

Lemma 3, which is closely related to the findings in [29], helps explain the appeal of the simple licensing rules in RL and UL. With the licensing fee set at V when two or more lagging firms succeed at the R & D stage, Lemma 2 ensures a lagging firm anticipates a profit of $[1 - r]V$ if and only if it succeeds alone or in conjunction with the leading firm. Otherwise, its

profit is zero. Thus, the expected profit of a lagging firm that chooses success probability P when all other firms choose $P^{\tilde{N}*}(n)$ is

$$P[1-r] V[1-P^{\tilde{N}*}(n)]^{\tilde{N}-2} - C(P,n)$$

$$= P[1-P^{\tilde{N}*}(n)]^{\tilde{N}-1} V - C(P,n) \qquad \text{when} \quad r = P^{\tilde{N}*}(n). \quad (3.5)$$

Similarly, under RL and UL, the second-stage expected profit of the leading firm does not vary with its success probability unless all of the lagging firms fail. Thus, with $r = P^{\tilde{N}*}(n)$ under RL and UL, the calculus of the leading firm, too, is reflected in (3.5). Therefore, since social surplus is realized only when a firm succeeds alone (i.e., when the other $\tilde{N}-1$ firms fail, which occurs in equilibrium with probability $[1 - P^{\tilde{N}*}(n)]^{\tilde{N}-1}$), the private and social incentives for R & D effort coincide under both RL and UL when the licensing fee for a sole lagging innovator is set at $P^{\tilde{N}*}(n) V$. Since our exclusive concern is determining when a first-best outcome is ensured under RL and UL, henceforth any reference to these licensing schemes will presume the licensing fee, rV, is set equal to the product of V and the first-best success probability given first-best disclosure and participation levels.

Finally, before proceeding to our main conclusions, it is useful to derive expressions for the net expected payoffs of the firms under RL and UL. In Lemma 4, $\pi^R(n, d | N)$ represents the expected profit under RL of the firm (say firm N) with knowledge level n who discloses $d \leq n$ to its $N-1$ first- and second-stage competitors, given that these competitors fully reveal their knowledge endowments. In Lemma 5, $\pi^U(n, d | N)$ represents the corresponding expected profit measure under UL when the $N-1$ lagging firms who enter the second-stage R & D competition pay the specified entry fee, E.

LEMMA 4.

$\pi^R(n, d | N)$

$$= \{V[P^N(n,d)[1 - P^{N*}(d)]^{N-1} + T(d|N)] - C(P^N(n,d),n)\} G^{N-1}(d)$$

$$+ \int_d^n \{VP^N(n,m)[1 - P^{N*}(m)]^{N-1} - C(P^N(n,m),n)\} dG^{N-1}(m)$$

$$+ \int_n^{\bar{n}} \{VP^{N*}(m)[1 - P^{N*}(m)]^{N-1} - C(P^{N*}(m),m)\} dG^{N-1}(m), \quad (3.6)$$

where

$$T(d|N) \equiv 1 - [1 - P^{N*}(d)]^{N-1} [1 + (N-1) P^{N*}(d)], \qquad (3.7)$$

$$G^{N-1}(x) \equiv \text{Prob}(\max\{n_1, ..., n_{N-1}\} \leq x), \qquad (3.8)$$

$P^{N*}(y)$ *solves*: $V[1 - P^{N*}(y)]^{N-1} - C_P(P^{N*}(y), y) = 0,$ *and*   (3.9)

$P^{N*}(x, y)$ *solves*: $V[1 - P^{N*}(y)]^{N-1} - C_P(P^N(x, y), x) = 0,$ *for* $x > y.$

(3.10)

The first line in (3.6) reflects the firm's expected profit when it turns out to be the leader. This expected profit, net of R & D costs, is composed of: (1) the leader's reward (V) when it succeeds alone (which happens with probability $P^N(\cdot)[1 - P^{N*}(\cdot)]^{N-1}$); and (2) its reward ($P^{N*}(\cdot) V$) when exactly one of the lagging firms succeeds (which occurs with probability $[N - 1] P^{N*}(\cdot)[1 - P^{N*}(\cdot)]^{N-2}$), and its reward ($V$) when two or more of the lagging firms succeed (which occurs with probability $1 - [1 - P^{N*}(\cdot)]^{N-1} - [N - 1] P^{N*}(\cdot)[1 - P^{N*}(\cdot)]^{N-2}$). The last two lines in (3.6) reflect the profit of a lagging firm. Recall that the lagging firm receives $[1 - P^{N*}(\cdot)]^V$ when it succeeds and all of the other lagging firms fail. Otherwise it receives 0.

LEMMA 5.   *With entry fee $E(d)$,*

$$\pi^U(n, d \mid N) = \pi^R(n, d \mid N) + [N - 1] E(d) G^{N-1}(d) - \int_d^{\bar{n}} E(\xi) \, dG^{N-1}(\xi).$$

(3.11)

Thus, the expected profit of the firm under UL is exactly its profit under RL plus expected revenues from entry fees when it is the leader less the expected entry fee it must pay when its disclosure, d, is not the highest.[26]

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[26] It can be inferred from (3.6) and (3.8) that neither $\pi^R(n, d \mid N)$ nor $\pi^U(n, d \mid N)$ is necessarily monotonic in d. This is the case because even though the marginal expected return from disclosure for a firm may be positive for "high" disclosures of knowledge, it may be negative for smaller disclosures. In particular, the marginal expected payoff from disclosure at a given level of d is decreasing in n because $P^N(n, d)$, defined in (3.10), is increasing in n and decreasing in d. Consequently, the fact that the necessary condition for optimal disclosure is satisfied at $d = n$ does not guarantee that full disclosure is globally optimal. This nonconcavity in the firm's problem implies that a global analysis is required. It is also the case that the marginal incentives for disclosure are not everywhere greater under UL than under RL (or vice versa). More precisely, it can be shown that

$$\text{sign} \left[\frac{\partial \pi^U(d, n \mid N)}{\partial d} - \frac{\partial \pi^R(d, n \mid N)}{\partial d} \right]_{d=n} = \text{sign} \left[\frac{dM^N(n)}{dn} \right]$$

$$> 0 \quad \text{if} \quad P^{N*}(n) \leqslant 1/N \text{ and } (d/dn)[C(P^{N*}(n), n)] < 0;$$

$$< 0 \quad \text{if} \quad P^{N*}(n) \geqslant 1/N \text{ and } (d/dn)[C(P^{N*}(n), n)] > 0.$$

Note that $dM^N(n)/dn < 0$ can occur because of the negative externality of ties that firms impose on each other's second-stage payoffs, even though $dS^N(n)/dn > 0$ always.

4. FINDINGS

We state our main conclusions formally in this section. In Proposition 1, we report that judicious use of UL will always ensure a first-best outcome. Propositions 2 and 3 present conditions which are sufficient to ensure RL also leads to a first-best outcome. Proposition 4 reports that the difference in expected payoff for a firm under UL versus RL becomes negligible as the first-best number of firms becomes infinitely large. Because of the restrictions it embodies, however, RL will not always implement a first-best outcome. Proposition 5 concludes that if knowledge is sufficiently rare and valuable, RL may not induce full disclosure of knowledge. On the other hand, Proposition 6 reports that RL may result in too much disclosure and excessive participation in the second-stage R & D contest.

PROPOSITION 1. *Suppose the leading firm can dictate the number* $(\tilde{N} \leq N)$ *of firms that participate in the second-stage R & D contest. Also suppose* $E(d) = M^{\tilde{N}}(d)$ *as defined in (3.4), where d is the level of knowledge disclosed by the leading firm. Then a first-best outcome is ensured under UL.*

By construction, both RL and UL induce first-best R & D effort levels when all knowledge is disclosed and the first-best number of second-stage competitors is ensured. (Recall Lemma 3.) Under the conditions of Proposition 1, the leading firm receives the full social value of the knowledge it reveals, because the lagging firms are charged an entry fee that drives to zero their expected profit from participating in the second-stage R & D contest. Furthermore, since the leading firm's objective coincides with the social objective (recall (3.1)) under UL, the leading firm will choose the number of second-stage participants to maximize the expected social surplus.[27]

From this point on, it is assumed that under UL, $E(d)$ is set at the level identified in Proposition 1. It remains to determine whether a first-best outcome can be ensured even when lagging firms cannot be charged such an entry fee for the right to engage in second-stage R & D. Proposition 2 and Corollary 2 address this question.

PROPOSITION 2. *Full disclosure of private knowledge by all firms is guaranteed under RL if* $P^{N*}(\underline{n}) \geq 1/N$, *where N is the number of participating firms.*

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[27] It should be noted that the result in Proposition 1 is related to the finding in [8], although we incorporate the additional features of: (1) productive spillovers from disclosed knowledge; (2) an endogenous number of firms.

Intuitively, if the equilibrium probability of success is sufficiently large even with the minimal level of knowledge, RL can provide sufficient incentive for disclosure, and additional entry fees are not essential. Hence, when disclosure is in the social interest, RL ensures a first-best outcome.

COROLLARY 2. *Suppose $P^{\tilde{N}*}(\underline{n}) \geqslant 1/\tilde{N}$ and the optimal participation level, $\tilde{N}$, is implemented. Then RL ensures a first-best outcome.*

A conclusion analogous to Proposition 2, but one that is based entirely on the primitives in the model, can be derived when the optimal number of firms (N) is large. When N is large, the probability that two or more lagging firms succeed simultaneously becomes large, provided the R & D effort level of each firm is not too small. Therefore, since the leading firm receives the entire surplus in the event of such "ties" under RL, sufficient incentive will be provided to motivate full disclosure of knowledge. Furthermore, sufficiently high R & D effort levels are motivated from all firms when the marginal cost of increasing the second-stage success probability above zero is small, even for the lowest level of knowledge. These observations are made precise in Proposition 3 and Corollary 3.

PROPOSITION 3. *When the number of participating firms, N, is sufficiently large, RL guarantees full disclosure of private knowledge if $C_P(0, \underline{n}) < V/e$, where e is the base of the natural logarithm.*

COROLLARY 3. *Suppose the first-best participation level, $\tilde{N}$, is implemented.[28] Then for $\tilde{N}$ sufficiently large, RL ensures a first-best outcome if $C_P(0, \underline{n}) < V/e$.*

Proposition 3 suggests a relationship between the expected profit of firms under RL and UL as the number of firms becomes large. As Proposition 4 reports, the difference between these two payoffs may become negligible for the leading firm when N is sufficiently large.

PROPOSITION 4. *Suppose: (i) $C_P(0, n) = 0 \; \forall n \in [\underline{n}, \bar{n}]$; or (ii) N, the number of second-stage participants, is equal to that in a first-best outcome, $\tilde{N}$. Then,*

$$\lim_{N \to \infty} \{ \pi^U(n, n \mid N) - \pi^R(n, n \mid N) \} = 0.$$

Under condition (i) in Proposition 4, the probability that exactly one firm will succeed in the second-stage R & D contest approaches zero as

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[28] Note that participation by $\tilde{N}$ firms will result under free entry into the second-stage R & D contest if $M^{\tilde{N}+1}(n) \leq 0$.

$N \to \infty$. Consequently, the expected profit of a lagging firm becomes negligible and $[(N-1) M^N(n)]$ tends to zero. Furthermore, the fact that $\lim_{\tilde{N} \to \infty} [\tilde{N} - 1] M^{\tilde{N}}(n) = 0$ holds generally when the first-best number of researchers is large. Consequently, with the leading firm being nearly certain that it will secure the entire surplus (V) because a second-stage "tie" among lagging firms is so likely, RL can generate nearly the same incentive for disclosure of knowledge that UL can.

More generally, however, the inability to impose second-stage entry fees will hinder the performance of RL. In particular, when high knowledge is both rare and valuable and the optimal number of second-stage participants is small, the reward for disclosure under RL will be too meager. This intuition is made precise most readily by introducing a mass point at the lowest level of knowledge, n.

PROPOSITION 5. *Suppose* $\tilde{N} = 2$, $P^{2*}(n) = \varepsilon$, *and* $F(n) = 1 - \delta$. *Then for* $\varepsilon > 0$ *sufficiently small and* $\delta > 0$ *sufficiently small, RL will not induce full disclosure of knowledge.*

Proposition 5 considers a setting where it is very likely that a typical firm will have the lowest level of knowledge, n. Furthermore, effort is sufficiently costly with low levels of knowledge that the first-best success probability is very small. Therefore, by disclosing a level of knowledge only slightly above n, the more knowledgeable firm can be almost certain that it will be established as the leader and that its competitors will not succeed at the second stage. In this setting, a firm with a high level of knowledge cannot be induced to fully disclose this knowledge to competitors because the reward for disclosure under RL is too small, relative to the expected profit from concealing superior knowledge and then employing it in an attempt to succeed alone.

The inability of RL to ensure full disclosure in situations where full disclosure is desired is not its only flaw. RL may also elicit to much disclosure. This possibility is demonstrated in the proof of Proposition 6 using an example where the optimal $\tilde{N}$ varies with n.

PROPOSITION 6. *There exist settings where RL induces full disclosure of private knowledge, even though less than full disclosure is in the social interest.*

The fact that too much disclosure may be elicited under RL stems from a different type of divergence between private and social incentives. Recall that the leading firm receives a licensing fee equal to the entire social surplus (V) whenever two or more lagging firms succeed at the R & D stage. Thus, the leader gains from such ties, even though the social value of a second success is zero. Consequently, when the primary effect of disclosure

is to enhance the probability of a second-stage tie (and thus create "excessive" R & D competition), RL can even induce full disclosure when no disclosure is in the social interest, as the proof of Proposition 6 illustrates. [29]

5. Conclusions and Extensions

We have examined the ability of two simple licensing mechanisms to motivate efficient sharing of knowledge and subsequent independent R & D activity. Unrestricted licensing (UL), in which lagging firms are charged a fixed fee for the knowledge they acquire from the leading firm, was shown to ensure a first-best outcome whenever the leader can control entry into the R & D contest. Restricted licensing (RL), where entry fees are not feasible, can also ensure a first-best outcome in some settings (e.g., where a successful innovation is sufficiently likely), but will fail to do so in other settings (e.g., where superior knowledge is sufficiently rare and valuable). Under both UL and RL, the leading firm receives licensing fees from profitable lagging firms that succeed at the R & D stage. Thus, licensing fees which require lagging firms to share realized profit with the leading firm will be sufficient to ensure efficient sharing of knowledge and R & D effort in some settings, but not in others.

One implication of our findings is that social gains may arise if: (1) some firms who successfully develop an innovation are prohibited from employing the innovation; (2) some firms can be charged for the right to engage in independent R & D; and (3) some firms are prohibited by others from engaging in R & D. Of course, great care must be taken in interpreting these conclusions for three reasons. First, these policies are optimal in our model only under certain circumstances, and the obvious antitrust concerns these policies raise in other circumstances are real and important. [30] Second, the conclusions arise in a particular economic model with some special features. It remains to determine how robust these insights are to variations in the model. [31] Third, it may be difficult in practice to control entry into a market, particularly since precise market definitions are not easily formulated.

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[29] Note that this inefficiency could be reduced if the licensing scheme specified a *maximum* level of disclosure for which compensation would be paid.

[30] It should be emphasized that the antitrust concerns here are not with the usual problems of collusion in the product market, but with *ex ante* agreements concerning R & D activities and the use of developed technologies. In practice, it may be difficult for antitrust authorities to detect and prove that a firm is not employing a technology it has developed.

[31] One might reasonably interpret our conclusions as providing support for the "rule of reason" treatment afforded by RJVs under the National Cooperative Research Act of 1984.

A number of extensions of our model remain to be considered. First, a richer space of outcomes for the R & D process should be explored. The success versus failure dichotomy in our model is illustrative, but special. Second, alternative forms of product–market competition should be considered (as in [18]). With less intense final-stage competition among members of the RJV, it may be less difficult to motivate the sharing of knowledge. Third, actual research need not be carried out independently by the members of the RJV. If firms can coordinate R & D activities, they may be more willing to share basic knowledge. Fourth, the R & D race might proceed in a different fashion. For example, firms may be able to alter their R & D efforts over time after observing the progress of their competitors (as in [13]). Fifth, alternative characterizations of innate knowledge should be analyzed. In many instances, members of an RJV may possess complementary knowledge. In this case, firms might be less reluctant to share their knowledge if they are certain to receive useful complementary knowledge in return. However, the differences may only be a matter of degree. Perhaps of greater practical importance in the case of complementary knowledge is that it may be more difficult to specify the appropriate compensation for revelation of knowledge, because it may not even be straightforward to identify the firm that revealed the "most" knowledge. [32]

Sixth, it would be interesting to examine the governance rules for an RJV that would arise endogenously from bargaining among firms with asymmetric information. Seventh, repeated interactions among firms in the RJV should be considered. Repeated play introduces a broader range of possible policy instruments, including the ability to base licensing fees on the entire history of a firm's contributions to the joint venture. Furthermore, a firm may be more willing to provide knowledge to competitors today if it anticipates they will reciprocate in the future. Eighth, it would be interesting to incorporate the possibility that research within an RJV might proceed in different directions, as it does in practice. It may be easier to motivate a firm to reveal knowledge when the reward for doing so includes the right to choose a direction for future research in the RJV. [33] Finally, the task of motivating individual researchers within an RJV merits careful attention. [34]

Of course, a complete investigation of RJVs must provide a characteriza-

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[32] In the case of perfectly complementary knowledge, where firm i's knowledge is of no value without the knowledge of firm j and vice versa, recursive implementation of the licensing mechanism in [20] may prove valuable in motivating the sharing of knowledge. With less extreme forms of complementary knowledge, complex public goods problems beyond those in [8] will generally arise because the sharing of knowledge will alter firms' cost functions.

[33] See the interesting description in [19] of how the Japanese VLSI Consortium was structured to promote a variety of research programs.

[34] A useful description of the incentive structures employed by MCC is presented in [27].

tion of the optimal second-best rules when a first-best outcome is not obtainable. Two qualitative departures from the ideal are possible. First, too much or too little disclosure may be induced. Second, deviations from the optimal second-stage R & D effort levels may be motivated. In the simplified setting of [1], full disclosure is always motivated, but inefficient R & D effort levels are induced when a first-best outcome is not feasible.[35]

APPENDIX

Proof of Lemma 1.   Only the symmetry reflected in (iii) needs discussion. It is apparent that a first-best outcome requires

$$V \prod_{\substack{j=1 \\ j \neq i}}^{\tilde{N}} [1 - P_j^{\tilde{N}*}(n)] - C_p(P_i^{\tilde{N}*}(n), n) = 0 \qquad \text{for} \quad i = 1, ..., \tilde{N}.$$

Therefore, for $l, \kappa \in \{1, ..., \tilde{N}\}$, $l \neq \kappa$,

$$V \prod_{\substack{j=1 \\ j \neq l, \kappa}}^{\tilde{N}} [1 - P_j^{\tilde{N}*}(n)] = C_p(P_l^{\tilde{N}*}(n), n)/[1 - P_\kappa^{\tilde{N}*}(n)], \qquad \text{(A1)}$$

and

$$V \prod_{\substack{j=1 \\ j \neq \kappa, l}}^{\tilde{N}} [1 - P_j^{\tilde{N}*}(n)] = C_p(P_\kappa^{\tilde{N}*}(n), n)/[1 - P_l^{\tilde{N}*}(n)]. \qquad \text{(A2)}$$

(A1) and (A2) imply:

$$C_p(P_\kappa^{\tilde{N}*}(n), n)[1 - P_\kappa^{\tilde{N}*}(n)] = C_p(P_l^{\tilde{N}*}(n), n)[1 - P_l^{\tilde{N}*}(n)]. \qquad \text{(A3)}$$

Hence, since $C_p(P, n)[1 - P]$ is strictly monotonic in P by assumption, (A3) implies $P_l^{\tilde{N}*}(n) = P_\kappa^{\tilde{N}*}(n)$.   ∎

Proof of Proposition 1.   Let us focus on a particular firm i and assume that all other firms truthfully reveal their level of knowledge, i.e., $d_j = n_j$ for all $j \neq i$. Let $n_{-i} = \text{Max}\{n_j | j \neq i\}$. Then,

(a) if $n_i < n_{-i}$, firm i's expected net payoff is always zero, independent of its disclosure;

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[35] In [1], we assume binary support for the underlying knowledge level. This simplification is introduced to avoid the nonconcavity in the firms' problem that complicates the present analysis.

(b) if $n_i > n_{-i}$, firm i's payoff is maximized at $d_i = n_i$, since this report ensures it the full social surplus, which is maximized at $d_i = n_i$ given the optimal number of firms. ∎

Proof of Proposition 2. In the ensuing calculations, the reference to N is dropped, but should be understood. From (3.6),

$$\frac{\partial \pi^R(n, d)}{\partial d} = \{V[1 - P^*(d)]^{N-1} - C_p(P(n, d), n)\} \, G^{N-1}(d) \frac{\partial}{\partial d} P(n, d)$$

$$- V \left\{ \left[P(n, d)[N-1][1 - P^*(d)]^{N-2} \frac{dP^*(d)}{dd} + \frac{\partial T(d)}{\partial d} \right] \right.$$

$$\left. \times G^{N-1}(d) + T(d) \, g^{N-1}(d) \right\}, \tag{A4}$$

where $g^{N-1}(x) \equiv dG^{N-1}(x)/dx$.

From (3.10), the expression in the first { }-brackets in (A4) vanishes. Also from (3.7),

$$\frac{\partial T(d)}{\partial d} = [N-1] \{ [1 - P^*(d)]^{N-2} [1 + [N-1] P^*(d)]$$

$$- [1 - P^*(d)]^{N-1} \} \frac{dP^*(d)}{dd}$$

$$= [N-1][1 - P^*(d)]^{N-2} NP^*(d) \frac{dP^*(d)}{dd}. \tag{A5}$$

Therefore,

$$\frac{\partial \pi^R(n, d)}{\partial d} = V[N-1][1 - P^*(d)]^{N-2} \frac{dP^*(d)}{dd}$$

$$\times \{ NP^*(d) - P(n, d) \} \, G^{N-1}(d) + VT(d) \, g^{N-1}(d). \tag{A6}$$

The expression in (A6) is strictly positive if

$$NP^*(d) > P(n, d),$$

which is true if $P^*(\underline{n}) > 1/N$. Therefore, $\pi^R(n, n) > \pi^R(n, d)$ for all $d \in [\underline{n}, n)$. ∎

For the asymptotic results, we need a preliminary characterization.

LEMMA A1. *Suppose* $P^{N*}(n)$ *satisfies* (3.9) $\forall N$. *Then* $\lim_{N \to \infty} P^{N*}(n) = 0$ $\forall n$.

Proof. Suppose $\lim_{N \to \infty} P^{N*}(n) = \delta(n) > 0$. Then $\lim_{N \to \infty} V\{1 - P^{N*}(n)\}^{N-1} = \lim_{N \to \infty} V\{1 - \delta(n)\}^{N-1} = 0$. Equation (3.9) implies that $\lim_{N \to \infty} V\{1 - P^{N*}(n)\}^{N-1} = \lim_{N \to \infty} C_p(P^{N*}(n), n) = C_p(\delta(n), n) > 0$, by assumption. Hence, the proof is complete by contradiction. ∎

Proof of Proposition 3. Since we will focus only on the level of knowledge n, let $P^{N*} \equiv P^{N*}(n)$ and $C_p(P) \equiv C_p(P, n)$. Dividing both sides of (3.9) by V, taking the natural log, and rearranging provides

$$-\log(1 - P^{N*}) = [\log(V/C_p(P^{N*}))]/(N-1). \tag{A7}$$

Notice that if $C_p(0) < V/e$, then $\log(V/C_p(0)) > 1$. Let $\delta > 0$ be defined by

$$\log(V/C_p(0)) = 1 + \delta. \tag{A8}$$

Now choose $K > 0$ sufficiently large to ensure

$$[1 - \delta/K][1 + \delta - \delta/K] > 1. \tag{A9}$$

From Lemma A1 and L'Hopital's rule,

$$\lim_{N \to \infty} \frac{P^{N*}}{-\log(1 - P^{N*})} = 1. \tag{A10}$$

From Lemma A1, we also know

$$\lim_{N \to \infty} C_p(P^{N*}) = C_p(0). \tag{A11}$$

From (A10) and (A11), there exists an N_0 such that for $N > N_0$,

$$P^{N*} > [1 - \delta/K][-\log(1 - P^{N*})] \tag{A12}$$

and

$$\log(V/C_p(P^{N*})) > \log(V/C_p(0)) - \delta/K. \tag{A13}$$

Therefore, for all $N > N_0$,

$$
\begin{aligned}
P^{N*} &> [1 - \delta/K][\log(V/C_p(P^{N*}))]/[N-1] \\
&> [1 - \delta/K][\log(V/C_p(0)) - \delta/K]/[N-1] \\
&= [1 - \delta/K][[1 + \delta - \delta/K]/[N-1] \\
&> 1/N.
\end{aligned}
\tag{A14}
$$

The first inequality follows from (A7) and (A12), the second from (A13), and the last from (A9). The equality follows from (A8). Then, using Proposition 2, the proof is complete. ∎

Proof of Proposition 4.   From Lemma A1, $\lim_{N \to \infty} M^N(d) = 0$ for all d, where $M^N(d)$ is defined by (3.4). Therefore, from (3.11) and from the fact that $E(d) = M^N(d)$,

$$\lim_{N \to \infty} \left\{ \pi^U(n, n) - \pi^R(n, n) \right\}$$

$$\propto \lim_{N \to \infty} \left\{ (N-1) M^N(n) \right\} \tag{A15}$$

$$= \lim_{N \to \infty} \left[(N-1) V P^{N*}(n)[1 - P^{N*}(n)]^{N-1} - (N-1) C(P^{N*}(n)) \right]. \tag{A16}$$

Multiplying both sides of (3.9) by $(N-1) P^{N*}(n)$ yields

$$[N-1] V P^{N*}(n)[1 - P^{N*}(n)]^{N-1} = [N-1] P^{N*}(n) C_P(P^{N*}(n)). \tag{A17}$$

However, since $C_P > 0$ and $C_{PP} > 0$, (A17) implies

$$[N-1] V P^{N*}(n)[1 - P^{N*}(n)]^{N-1} \geqslant [N-1] C(P^{N*}(n)) \; \forall N. \tag{A18}$$

Therefore, in order to prove part (i) of the proposition, it suffices to show that

$$\lim_{N \to \infty} \left\{ (N-1) V P^{N*}(n)[1 - P^{N*}(n)]^{N-1} \right\} = 0 \qquad \text{if} \quad C_P(0, n) = 0.$$

Let $P^{N*} \equiv P^{N*}(n)$ and $C_P(P) \equiv C_P(P, n)$. By (A7) and (A10) we know that

$$\lim_{N \to \infty} \frac{P^{N*}}{\log(V/C_p(P^{N*}))/[N-1]} = 1. \tag{A19}$$

Therefore, from (A17) and (A19), and since $\lim_{N \to \infty} \{ [\log(V/C_P(P^{N*}))] C_P(P^{N*}) \}$ exists,

$$\lim_{N \to \infty} \left\{ (N-1) V P^{N*}[1 - P^{N*}]^{N-1} \right\} = \lim_{N \to \infty} \left\{ [\log(V/C_p(P^{N*}))] C_p(P^{N*}) \right\}$$

$$= 0 \qquad \text{if} \quad C_p(0) = 0. \tag{A20}$$

The last equality follows from Lemma A1, using L'Hopital's rule. This completes the proof of part (i) of the proposition.

To prove part (ii), let $S^N \equiv V[1 - (1 - P^{N*})^N] - NC(P^{N*})$ be the social surplus given the (socially optimal) probability of success P^{N*}. Then, using the envelope theorem,

$$\frac{dS^N}{dN} = V[1 - P^{N*}]^N \log(1/[1 - P^{N*}]) - C(P^{N*})$$

$$= V\{[1 - P^{N*}]^N \log(1/[1 - P^{N*}]) - P^{N*}[1 - P^{N*}]^{N-1}\}$$

$$+ VP^{N*}[1 - P^{N*}]^{N-1} - C(P^{N*}). \tag{A21}$$

Therefore,

$$[N-1]\frac{dS^N}{dN} = [N-1] VP^{N*}[1 - P^{N*}]^{N-1}$$

$$\times \left\{\frac{\log(1/[1 - P^{N*}])}{P^{N*}/[1 - P^{N*}]} - 1\right\} + [N-1] M^N, \tag{A22}$$

where $M^N \equiv VP^{N*}[1 - P^{N*}]^{N-1} - C(P^{N*})$.

Now, for a given V and a given cost function (and ignoring integer problems):

$$dS^N/dN = 0 \quad \text{at the socially optimal number of firms.} \tag{A23}$$

Therefore, if $\{N\}_1^\infty$ is a sequence of the socially optimal number of firms for a subsequence of cost functions for which the left-hand side of Eq. (A22) is identically zero, then

$$\lim_{N \to \infty} [N-1]\frac{dS^N}{dN} = 0. \tag{A24}$$

In the right-hand side of (A22) it is easy to see that

$$\lim_{N \to \infty} \frac{\log(1/[1 - P^{N*}])}{P^{N*}/[1 - P^{N*}]} = \lim_{x \to 1} \frac{\log x}{x - 1} = 1 \tag{A25}$$

(by L'Hopital's rule), where $x \equiv 1/(1 - P^{N*})$ and using the fact that $\lim_{N \to \infty} P^{N*} = 0$. Therefore, by (A22), (A23), (A24), and (A25), we know that $\lim_{N \to \infty}[N - 1] M^N = 0$. Therefore, by (A15), the proof is complete. ∎

Proof of Proposition 5. If $N = 2$ and $F(\underline{n}) \approx 1$, then for $n > d > \underline{n}$:

$$\pi^R(n, d) \approx VP^2(n, d) - C(P^2(n, d), n). \tag{A26}$$

Notice that $P^2(n, d)$ and $P^{2*}(d)$ are continuous functions of d on the interval $[\underline{n}, n]$. Therefore if a firm with knowledge $n > \underline{n}$ discloses $\underline{n} + \gamma$ and if $\gamma > 0$ is sufficiently small, its payoff will be

$$\pi^R(n, \underline{n} + \gamma) \approx VP^2(n, \underline{n}) - C(P^2(n, \underline{n}), n). \tag{A27}$$

If, on the other hand, the firm fully discloses n, its payoff will be

$$\pi^R(n, n) \approx VP^{2*}(n) - C(P^{2*}(n), n). \tag{A28}$$

Therefore, if $P^{2*}(\underline{n}) = \varepsilon > 0$, then for ε and γ sufficiently small,

$$\begin{aligned}
\pi^R(n, n) - \pi^R(n, \underline{n} + \gamma) &= VP^{2*}(n) - C(P^{2*}(n), n) \\
&\quad - [VP^2(n, \underline{n}) - C(P^2(n, \underline{n}), n)] \\
&< 0.
\end{aligned} \tag{A29}$$

The inequality in (A29) follows from the fact that if ε is sufficiently small, then by (3.10), $P^2(n, \underline{n})$ satisfies

$$V - C_P(P^2(n, \underline{n}), n) = 0,$$

which implies that

$$P^2(n, \underline{n}) = \underset{P}{\operatorname{argmax}} \{VP - C(P, n)\}. \quad \blacksquare$$

Proof of Proposition 6. An example will suffice as proof. Suppose $V = 1$, $N = 2$, $\underline{n} = 0$, $\bar{n} = 1$, $F(0) \approx 1$, and $C(P, n) = \max\{0, [P^2/2 - 1/8]\}$ $[1 - n]\}$. Then $C_P(P, n) = P[1 - n]$, $P^{2*}(d) = 1/[2 - d]$, and $P(n, d) = [1 - d]/[(2 - d)(1 - n)]$ in the relevant range.

Now observe that

$$P^{2*} = 0.5, \qquad\qquad C(P^{2*}(0), 0) = 0,$$

$$P^{2*}(0.25) = 0.57, \qquad C(P^{2*}(0.25), 0.25) = 0.0287,$$

$$P^2(0.25, 0) = 0.666, \qquad C(P^2(0.25, 0), 0.25) = 0.0729.$$

Since $P^{2*}(0) = 0.5$, we know from Proposition 2 that full disclosure of knowledge is motivated under RL. We shall show, however, that with $N = 2$, the social surplus is higher if the firm with knowledge level $n = 0.25$ discloses none rather than all of its knowledge.

With full disclosure, the social surplus is

$$S^2(n_1 = n_2 = 0.25) = 1 - \{1 - [P^{2*}(0.25)]\}^2 - 2C(P^{2*}(0.25), 0.25) = 0.7577.$$

With no disclosure, the corresponding social surplus is:

$$S^2(n_1 = 0.25, \ n_2 = 0) = P^2(0.25, 0) + [1 - P^2(0.25, 0)] \ P^{2*}(0)$$
$$- C(P^2(0.25, 0), 0.25) - C(P^{2*}(0), 0)$$
$$= 0.7596.$$

Thus, the social surplus absent any disclosure is higher than the social surplus under full disclosure.

To see the nature of the critical externality, notice that with $F(\underline{n}) = 1$, firm 1 will disclose its knowledge $n > \underline{n}$ fully, if and only if

$$0 < \{P^{2*}(n) - C(P^{2*}(n), n)\}$$
$$- \{P(n, \underline{n})[1 - P^{2*}(\underline{n})] + [P^{2*}(\underline{n})]^2 - C(P(n, \underline{n}), n)\}.$$

On the other hand, no disclosure $(d_1 = \underline{n})$ is socially optimal if

$$0 > \{2P^{2*}(n) - [P^{2*}(n)]^2 - 2C(P^{2*}(n), n)\}$$
$$- \{P(n, \underline{n}) + P^{2*}(\underline{n}) - P(n, \underline{n}) \ P^{2*}(\underline{n}) - C(P(n, \underline{n}), n) - C(P^{2*}(\underline{n}), \underline{n})\}.$$

These two conditions imply

$$0 > M^2(n) - M^2(\underline{n}) = \{P^{2*}(n)[1 - P^{2*}(n)] - C(P^{2*}(n), n)\}$$
$$- \{P^{2*}(\underline{n})[1 - P^{2*}(\underline{n})] - C(P^{2*}(\underline{n}), \underline{n})\}. \quad \blacksquare$$

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