Disadvantageous semicollusion

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Standard analysis in industrial organization indicates that firms earn higher profits if they collude rather than compete on prices (or quantities). However, firms choose other strategic variables, such as investment in capacity or R&D, in addition to choosing prices or production levels. Thus the overall evaluation of product market collusion must take into account its effect on the interaction in the other dimensions. This paper demonstrates that collusion in the product market may yield lower overall profits because it intensifies competition in the other dimensions of the interaction.

JEL classification: L10, L13

1. Introduction

Conventional wisdom suggests that firms in oligopolistic markets are better off colluding rather than competing on prices (and quantities).\(^1\) Pricing and output choices, however, are seldom the only decisions that firms must undertake. Firms also invest in production facilities (capacity), R&D, advertising, etc. Thus in principle, firms can collude in some aspects of the interaction and compete in other aspects. We refer to such behavior as semicollusion.

The main message of this paper is that collusion in the product market does not necessarily benefit the firms. When firms compete in some dimensions, the overall evaluation of output market collusion must take into account its effect on the other dimensions of the competition. The paper

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\(^1\)See Tirole (1988, ch. 6) and the surveys by Jacquemin and Slade (1989) and Shapiro (1989) in the Handbook of Industrial Organization.
shows that product market collusion may lead to lower overall profits as it might trigger aggressive competition with respect to the other strategic variables.

The paper analyzes two settings. In the first one, firms invest in cost-reducing R&D in the first stage and then choose production levels in the second stage. In the second example, firms invest in productive capacity in the first stage and then choose prices in the second stage. In both settings we first assume that collusion is not feasible and the firms compete in both stages. We then assume that firms compete in the first stage, but collude in the product market (second stage).

In both settings we show that as a result of collusion in the product market, semicollusion leads to lower production costs or larger capacity than the non-cooperative interaction. Furthermore, this first period 'overinvestment' may be large enough so that the overall semicollusive equilibrium profits are smaller than the non-cooperative equilibrium profits. In such cases the firms are better off not colluding.

Although overall semicollusive equilibrium profits may be smaller than non-cooperative equilibrium profits, it is important to note that for a given choice of variables in the first stage, the firms always are better off colluding in the second stage. Thus, when collusion in the output market yields lower overall profits than competition, the decision not to collude in the second stage must be accompanied by some type of commitment mechanism. Otherwise the firms will wish to renegotiate and the decision not to collude in the second stage will not be credible.

This may shed some light on the results of Asch and Seneca (1976). In an empirical study of American manufacturing firms during the period 1958–1967, they found that on average collusive firms were less profitable than non-collusive firms. Although the direction of causality was not clear from their results, Asch and Seneca believed that the most plausible explanations were that: (1) low profits forced firms into collusive agreements and (2) antitrust prosecutions were most likely to take place when collusion was unsuccessful. They also entertained the possibility that the causality may run in the opposite direction (from collusion to profitability), but dismissed this alternative as unlikely. Our results suggest that this explanation is quite plausible.

Our interest in 'semicollusive environments' in which firms compete in the first stage, but collude in the second stage is motivated by the observation that firms in oligopolistic markets tend to compete with respect to non-price variables such as advertising, capacity or R&D and collude on price. This observation has been made by several authors; see Scherer (1980), Brander and Harris (1984), and Davidson and Deneckere (1990). Cooperation in the output market may be more feasible than cooperation on other variables because output market variables are more easily observed and easier to adjust. A classic example is the cigarette industry of the 1920s and 1930s.
The Big Three (American, Ligget & Myers, and Reynolds) controlled between 70% and 90% of the market during this period and there is evidence that they colluded on price. They did, however, compete intensely on advertising in order to improve the market shares of their premium brands. See Scherer (1980) for more details.

Several papers address semicollusion in which only the second stage is collusive. Fershtman and Muller (1986), Osborne and Pitchik (1987), and Davidson and Deneckere (1990) consider settings similar to our second example and show that semicollusion generally leads to excess capacity in equilibrium. Friedman and Thisse (1993) consider a setting in which two firms choose product locations non-cooperatively in the first stage and then collude on price. They show that the equilibrium is characterized by both firms locating at the market center. These papers do not address the question of whether semicollusion can result in lower profits than the non-cooperative interaction, which is the focus of our paper.

Two papers illustrate that semicollusion in which only the second stage is collusive can increase consumer surplus relative to the non-cooperative interaction. Matsui (1989) considers an environment in which firms choose capacity in the first period and choose quantities in the second period. He shows consumer surplus may increase when firms are permitted to collude in the second stage. Sevy (1992) considers an environment in which firms invest in R&D in the first period and choose prices in the second period. He shows that consumer surplus may increase when the firm with the lower production cost is granted a monopoly in the second stage. The enticement of monopoly profits restores the incentives to invest in R&D when spillovers are high, and the increase in R&D is large enough to offset the effect of monopoly prices. In both of our settings, consumer surplus is lower under semicollusion: the monopoly effect of increased prices and reduced quantities more than offsets the increased investment in capacity and R&D.

In a two-stage game there are two possible types of semicollusion: (1) firms collude in the first stage, but compete in the second stage, and (2) firms compete in the first stage, but collude in the second stage. Our analysis emphasizes that the market outcome is not only sensitive to which variables the firms choose collusively, but also depends on the timing of collusive arrangements in the market.

When collusion occurs in the first stage, firms cannot be worse off under semicollusion than in the non-cooperative interaction. An example of this type of semicollusion is D'Aspremont and Jacquemin (1987) who consider a two-stage game in which R&D occurs in the first stage and production takes place in the second stage. They compare competition in both stages, and cooperation in the first (R&D) stage and competition in the second

\footnote{Benoit and Krishna (1987) allow firms to collude on \textit{both} capacity and price. They also show that collusive equilibria are characterized by excess capacity.}
(production) stage and show that when there are significant spillovers in R&D, cooperative R&D is unambiguously welfare improving, since it results in lower equilibrium prices and larger equilibrium profits. Kamien et al. (1992) consider a setting similar to that of D'Aspremont and Jacquemin and show that a regime in which firms cooperate in R&D and fully internalize spillovers is Pareto superior to a regime in which firms choose research levels cooperatively but do not internalize spillovers. Katz (1986) considers a four-stage game in which firms respectively decide whether to participate in an R&D agreement, then choose R&D sharing rules, next invest in R&D, and finally compete in the product market. He shows that industrywide cooperation is welfare enhancing when there are significant spillovers in R&D and when the degree of product market competition is relatively low.

2. Sharing rules

According to the folk theorem, as long as the discount rate is close to one, any division of the collusive profits among the firms can be supported. Thus, in the semicollusive regime, the firms need to bargain on how to divide the 'pie' or adopt some focal rule. If the firms have made different choices in the first stage of the semicollusive regime, they will be asymmetric in the second stage; clearly the choice of capacity or R&D in the first period will affect their relative bargaining positions. In both settings, we assume that the firms adopt a focal rule so that the interaction in the first stage determines both the pie to be divided in the second stage and the way it will be divided.

In our first setting, firms might have different unit costs when they enter the production stage. In such a case, firms' preferred prices differ. Thus, the first question to address is: Which collusive technology is employed? Do firms coordinate quantities or prices, are there side payments, etc.? This problem has been addressed by Schmalensee (1987), who considered three types of collusive technologies: market sharing, market division, and proportional reduction. We adopt the Market Division (MD) collusive technology, in which each actual or potential consumer is assigned to a single firm, because as Schmalensee (1987, p. 356) notes, it corresponds to 'a frequently-observed pattern of cartel behaviour: the firms can divide the market'.

Under market division, firms will charge different prices whenever costs differ. Such a division can only exist, therefore, when there is some mechanism to prevent arbitrage. Scherer (1980) notes that in the power switching equipment industry in the late 1950s, General Electric, Allais-Chambers and other manufacturers 'coordinated their bidding so that each firm was the low bidder in enough transactions to gain its predetermined

3See Schmalensee (1987) and Harrington (1989) for a discussion of oligopoly collusion when firms are asymmetric.

4For more on the market division technology, see Stigler (1964) and Blair and Kaserman (1985).
share of the market'. A similar mechanism was used to divide the market in the high voltage switchgear field.

In our second setting, in which firms compete on capacity in the first stage, we adopt a 'relative capacity' sharing rule in the cooperative regime, i.e. sales are proportional to plant size when there is excess capacity. We find it attractive because there is a large body of anecdotal evidence that cartels use such a rule in setting output quotas. Scherer (1980) notes that in Germany in the 1920s and 1930s, cartels allocated market shares according to production capacity. There is evidence that cartel members added capacity in an effort to increase their market shares. In the Rhineland–Westphalian coal cartel, for example, capacity exceeded demand by 25%.

In the 1950s and 1960s, the Japanese government allowed firms in many industries to form cartels which controlled prices and quantities. These industries often were characterized by excess capacity; the explanation cited for this excess capacity was that in the cartels, market shares were allocated according to the amount of capital equipment each firm possessed. See Matsui (1989) and the references cited therein.

OPEC has used production or supply quotas in an attempt to control oil prices. While no explicit formula has been employed to allocate quotas among members, analysis by Gault et al. (1990) concludes that the allocations have been systematic and that the principal factor in determining production levels is each individual member's oil production capacity. Although the relative capacity sharing rule may not be efficient for the cartel, it is clearly in the interest of Saudi Arabia, whose productive capacity exceeds the combined productive capacity of the next three largest OPEC members, to insist that the cartel allocate productive quotas according to this rule.

3. An R&D model

Consider a duopolistic industry in which firms invest in cost-reducing R&D in the first stage, while production takes place in the second stage. We employ a variant of the D'Aspremont and Jacquemin (1987) model. Specifically, we assume that market (inverse) demand is linear and given by \( P(Q) = 1 - Q \), where \( Q = q_1 + q_2 \) is the total quantity produced. Each firm has a cost of production \( C_i(q_i, x_i) = (A - x_i)q_i \), where \( x_i \leq A \) is the amount of investment that the firm undertakes in the first stage. The cost of investing \( x_i \) is given by \( \lambda x_i^2/2 \). We now compare two scenarios: in the first one, the firms compete non-cooperatively in both stages of the game, while in the second scenario the investment stage is non-cooperative, but there is collusion in the product market.

3.1. Competition in both stages

Suppose that the firms compete in both stages. We calculate the subgame perfect equilibrium in the standard way. Given $x_1$ and $x_2$, the Cournot–Nash equilibrium in the second stage is given by

$$\frac{1 - A + 2x_i - x_j}{3}, \quad i = 1, 2, j \neq i.$$ 

(1)

Given the second-stage strategies $q_i^*(x_i, x_j)$, the second-period profits are $R_i^*(x_i, x_j) = (1 - A + 2x_i - x_j)^2 / 9$. Thus the profit functions of the R&D game in the first stage are

$$\pi_i^*(x_i, x_j) = R_i^*(x_i, x_j) - \frac{\lambda x_i^2}{2}, \quad i = 1, 2, j \neq i.$$ 

(2)

We assume that $A \leq 1/2$, which guarantees an interior solution for every $(x_1, x_2)$. It is easy to show that, given (2), the unique equilibrium levels of R&D in the first stage are

$$x_i^* = \frac{4(1 - A)}{9\lambda - 4}, \quad i = 1, 2.$$ 

(3)

3.2. Semicollusion: Cooperation in the product market

We now consider the case in which firms collude in the second stage. We assume for convenience that in the collusive stage, firms can sign a binding agreement. Alternatively, one can imagine that given the choice of $(x_1, x_2)$, the second stage is a repeated oligopoly game in which the collusive outcome is supported by some non-cooperative strategies. In this setting we adopt the market division (MD) collusive technology. Recall that in such a case, each actual or potential consumer is assigned to a single firm. Given a share $s_i$ of the market, the demand function of firm $i$ is given by

$$q_i = s_i d(P_i) = s_i (1 - P_i), \quad i = 1, 2, \text{ where } s_1 + s_2 = 1.$$ 

Note that under the MD technology, $s_i$ is not the market share of firm $i$, but its contingent market share as defined by Shubik (1959). Given $s_i$, firm $i$ maximizes its second-stage profit by producing

$$q_i(x_i, x_j) = (1 - A + x_i) s_i(x_i, x_j) / 2, \quad i = 1, 2; j \neq i.$$ 

(4)

Thus the second-stage profits of firm $i$ net of investment $x_i$ are

$$R_i^c(x_i, x_j) = \frac{(1 - A + x_i)^2 s_i}{4}, \quad i = 1, 2; j \neq i.$$ 

(5)

$^6$Second-order conditions require $\lambda > 8/9$. This also insures that $x_i^* > 0$. In order to insure that $x_i^* < A$, we need $9\lambda A > 4$. Equilibrium profits are non-negative if $\lambda \geq 8/9$. 
Once MD is chosen as the collusive technology, we need to specify how firms choose the division \((s_1, s_2)\) in the second stage. One can model this stage as a bargaining game between the firms by letting \((x_1, x_2)\) affect both the threat point and the feasible set of the game. However, to keep the analysis simple, we choose not to model the bargaining game. Instead, we assume that the original non-cooperative equilibrium is a natural focal point, and that the allocation of consumers in the cooperative solution is such that firms receive equal percentage gains over the profits that would be earned in the Cournot equilibrium.\(^7\) Thus, given investment levels \(x_1\) and \(x_2\) in the first stage, the shares are determined so that:

\[
\frac{R_i^c(x_1, x_2)}{R_i^c(x_1, x_2)} = \frac{R_i^s(x_1, x_2)}{R_i^s(x_1, x_2)}.
\]

Setting \(s_2(x_1, x_2) = 1 - s_1(x_1, x_2)\) and substituting the expressions for \(R_i^c\) and \(R_i^s\) into (6) yields

\[
s_i(x_1, x_2) = \frac{(1 - A + x_2)^2(1 - A + 2x_1 - x_2)^2}{(1 - A + x_1)^2(1 - A + 2x_2 - x_1)^2}.
\]

In the first stage of the game each firm chooses its R&D investment level \(x_i\) to maximize the following profit function:

\[
\pi_i^c(x_i, x_j) = \frac{(1 - A + x_i)^2 s_i(x_i, x_j)}{4} - \lambda x_i^2/2, \quad i = 1, 2, j \neq i,
\]

where \(s_i(x_i, x_j)\) is determined by (7). Maximizing (8) with respect to \(x_i\) yields the following best response functions:\(^8\)

\[
(1 - A + x_i) \frac{s_i(x_i, x_2)}{2} + \frac{(1 - A + x_i)^2}{4} \frac{\partial s_i(x_i, x_2)}{\partial x_i} - \lambda x_i = 0, \quad i = 1, 2.
\]

The first term in (9) gives the increase in profits due to lower production costs. The second term is the increase in profits due to a larger market share derived from lower production costs,\(^9\) while the last term is the marginal cost of investing in R&D. A simultaneous solution of the two best response

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\(^7\) Clearly this sharing rule is individually rational, i.e. for every \((x_1, x_2)\), following this rule yields payoffs larger than non-cooperative payoffs.

\(^8\) We verified that second-order conditions hold for \(\lambda > 9/8\) by computational analysis of the payoff function (8). This analysis is available from the authors upon request.

\(^9\) It can be shown that \(\partial s_i(x_1, x_2)/\partial x_i > 0\) for all \((x_1, x_2)\).
functions (9) yields the unique equilibrium levels of R&D undertaken in the first stage:

\[ x_i^c = \frac{(1 - A)}{2\lambda - 1}, \quad i = 1, 2. \]  

(10)

3.3. Comparison of semicollusion and competition

Comparing the two regimes yields the following two propositions:

**Proposition 1.** The equilibrium level of investment in R&D is higher when firms cooperate in the second stage, i.e. \( x_i^c > x_i^n \).

**Proof.** Compare (3) and (10). Q.E.D.

**Proposition 2.** When investment costs are relatively small, semicollusion will result in lower equilibrium profits than the non-cooperative interaction.

**Proof.** Substituting the non-cooperative equilibrium level of investment (3) into (2), the non-cooperative equilibrium profits are

\[ \pi_i^n = \frac{\lambda(1 - A)^2(9\lambda - 8)}{(9\lambda - 4)^2}, \quad i = 1, 2. \]  

(11)

Substituting the semicollusive equilibrium level of investment (10) into (8), the semicollusive equilibrium profits are

\[ \pi_i^c = \frac{\lambda(1 - A)^2(\lambda - 1)}{2(2\lambda - 1)^2}, \quad i = 1, 2. \]  

(12)

Comparing the above two equations yields that for \( \lambda < 1.4 \), semicollusive equilibrium profits are smaller than non-cooperative equilibrium profits. Q.E.D.

**Corollary 1.** Semicollusion results in higher equilibrium prices than the non-cooperative interaction.

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\(^{10}\)See appendix A. Note that \( \lambda > 9/8 \) insures that \( x_i^f > 0 \). In order to insure that \( x_i^f < A \), we need \( 7\lambda A > 1 \). Thus the binding constraints for both the cooperative and non-cooperative settings are \( \lambda > 9/8 \) and \( \lambda A > 1/2 \geq A \).
Proof. Substituting (3) into (1), and then substituting (1) into the expression for market (inverse) demand yields the equilibrium price under the non-cooperative interaction. Similarly, substituting (10) and (7) into (4), and then substituting (4) into the expression for market (inverse) demand yields the equilibrium price under semicollusion. Comparing these prices establishes the result. Q.E.D.

From Proposition 1, firms in semicollusive markets invest more in R&D. Clearly the equilibrium level of investment in R&D depends on the cost parameter $\lambda$. Proposition 2 indicates that when $\lambda$ is relatively small, the equilibrium level of investment in R&D is sufficiently large to more than offset the short-run gains from collusion in the output market. Regardless of the affect on firm profitability, Corollary 1 shows that despite the higher level investment in R&D in the semicollusive setting, prices are still lower in the non-cooperative interaction.

4. Capacity model

In this section we consider a duopolistic industry in which firms choose production capacity in the first stage and prices in the second stage. There are no production costs, but the cost of installing capacity $k_i$ for firm $i$ is given by $C(k_i) = \gamma k_i$. There are rigid capacity constraints, such that firm $i$ can produce any quantity $q_i \leq k_i$ costlessly in the second stage, but it cannot produce more than $k_i$.

In the second stage, firms simultaneously and independently choose prices. For simplicity, we assume that when both firms choose the same price, total quantity demanded is $D(p) = 1 - p$. When firms charge different prices, consumers purchase first from the cheaper supplier. Since firms cannot sell beyond their capacities, it is possible that the firm charging the higher price will have positive demand. For convenience, we follow Kreps and Scheinkman (1983) and assume efficient rationing so that if $p_2 > p_1$, firm 2 sells $q_2 = \min\{k_2, \max(0, 1 - p_2 - k_1)\}$. When firms set the same price $p$, each firm sells $q_i = \min\{k_i, (1 - p)/2 + \max(0, (1 - p)/2 - k_j)\}$.

4.1. Competition in both stages

Kreps and Scheinkman (1983) showed that under the above setting, the equilibrium of the game has the same output and profits as the Cournot equilibrium of the induced quantity game in which firms determine output rather than prices, and production cost is $\gamma q_i$. That is, the equilibrium capacity (and production) levels are

$$k_i^o = q_i^o = \frac{(1 - \gamma)}{3}$$

(13)
and the non-cooperative equilibrium profits are
\[ \pi_i^c = \frac{(1 - \gamma)^2}{9}. \] (14)

For convenience, we assume that capacity costs are sufficiently small so that
the Cournot equilibrium price of the induced quantity game, \( p = \frac{(1 + 2\gamma)}{3} \), is
below the second-stage monopoly price \( (p^m = \frac{1}{2}) \). This assumption implies
that \( \gamma < \frac{1}{4} \).

4.2. Semicollusion: Cooperation in the second stage

Assume now that the firms choose their capacities non-cooperatively, but
collude in the second stage. We further assume that firms adopt the market-
sharing collusive technology [see Schmalensee (1987)]. Using this technology,
total industry profits are maximized as follows: for \( k_1 + k_2 \leq \frac{1}{2} \), both firms
produce at capacity and when \( k_1 + k_2 > \frac{1}{2} \), firms set the price equal to \( \frac{1}{2} \)
(the monopoly price) and produce \( q_1 + q_2 = \frac{1}{2} \).

When \( k_1 + k_2 > \frac{1}{2} \), capacity exceeds total output; we therefore need a
collusive solution concept that determines the allocation of market shares,
taking into account the possible asymmetry in capacity. We assume that
when \( k_1 + k_2 > \frac{1}{2} \) and total output is \( \frac{1}{2} \), the market shares are allocated
according to the firms' relative capacities. In other words, each firm \( i \) is
allocated \( \frac{k_i}{(k_i + k_j)} \) of the monopoly quantity. First note that this rule is
feasible since it implies that \( q_i \leq k_i \). In appendix B we prove that this sharing
rule is individually rational so that for every \( (k_1, k_2) \), following this rule yields
higher payoffs than the non-cooperative payoffs.

We now turn to the first stage of the game in which capacity is
determined. Given the above collusive technology, the profit function of firm
\( i \) in the first stage is
\[ \pi_i(k_i, k_j) = \begin{cases} k_i(1 - k_i - k_j) - \gamma k_i, & \text{if } k_i + k_j \leq \frac{1}{2}, \\ \frac{k_i}{4(k_i + k_j)} - \gamma k_i, & \text{if } k_i + k_j \geq \frac{1}{2}. \end{cases} \] (15)

We can state the following lemma.

**Lemma 1.** \( k_i^* = k_j^* = \frac{1}{16\gamma} \) is the equilibrium capacity level of the game.

**Proof.** First note that under our assumptions, there is no equilibrium with
\( k_1 + k_2 < \frac{1}{2} \). Given the payoff function (15), any equilibrium \( (\hat{k}_1, \hat{k}_2) \), with
\( \hat{k}_1 + \hat{k}_2 < \frac{1}{2} \) is also a Cournot equilibrium of the induced quantity game, but
the equilibrium price of this game will exceed \( \frac{1}{2} \), which contradicts our
assumption that \( \gamma < \frac{1}{4} \). Assume now that firm \( j \) has capacity \( k_j^* \). Using (15),
it can easily be verified that if firm \( i \) chooses a capacity \( \hat{k}_i \) such that
If $k_i + k_j^* \geq 1/2$, then its best capacity choice is $k_i^*$. But firm $i$ can decide to choose its capacity $k_i$ such that $k_i + k_j^* < 1/2$. Its profit function is then $\bar{k}_i(1 - \bar{k}_i(1/16\gamma) - \gamma)$. However, in the range for which there is no excess capacity $0 \leq \bar{k}_i \leq 1/2 - (1/16\gamma)$, the above profit function is maximized at $\bar{k}_i = 1/2 - (1/16\gamma)$, which implies that $k_i + k_j^* = 1/2$. Since the two pieces of the profit function in (15) have the same value for $k_i + k_j = 1/2$, and since for all $k_i + k_j \geq 1/2$ we obtain that $k_j^*$ is the best response to $k_i^*$, we can conclude that $(k_i^*, k_j^*)$ is an equilibrium of the game. Uniqueness is guaranteed since there is no other $(k_i, k_j)$, $k_i + k_j \geq 1/2$, which satisfy the best response property. Q.E.D.

We can now state the following corollary.

**Corollary 2.** (Excess capacity). The semicollusive equilibrium is characterized by excess capacity.

**Proof.** The equilibrium output of each firm is $q_i^* = 1/4$, while the equilibrium capacity choice $k_i^* = 1/16\gamma > q_i^*$. Q.E.D.

The firms invest in this excess capacity not for future production, but rather as an investment to enhance their bargaining power in the 'negotiations' concerning the division of the collusive profits, since larger capacity leads to a greater share of the collusive market.

4.3. Comparisons: Semicollusion vs. competition

The analysis of the preceding two subsections leads to the following proposition.

**Proposition 3.** (Disadvantageous semicollusion). Semicollusion yields lower equilibrium profits than when firms compete non-cooperatively in both stages.

**Proof.** Equilibrium profits in the semicollusive case are

$$\pi_i^* = 1/8 - \gamma k_i^* = 1/16,$$

while the equilibrium profits in the non-cooperative case are given by (14). Using the assumption that $\gamma < 1/4$, a comparison of (14) and (16), shows that equilibrium profits are always higher under the non-cooperative setting. Q.E.D.

Recall that the equilibrium semicollusive price, $p^m = 1/2$, exceeds the equilibrium price under the non-cooperative interaction, $p = (1 + 2\gamma)/3$, since, by assumption, $\gamma < 1/4$. Thus, we can state the following corollary.
Corollary 3. Semicollusion results in higher equilibrium prices than the non-cooperative interaction.

5. Concluding remarks

In discussing two-stage games in which each stage is either collusive or non-cooperative, there are in principle four different possibilities: (1) non-cooperative competition in both stages; (2) semicollusion in which only the first stage is collusive; (3) semicollusion in which only the second stage is collusive; and (4) sequential collusion in which both stages are collusive.

Clearly in cases (2) and (4) the firms are better off than under the non-cooperative interaction. The objective of this paper was to establish the possibility that firms can earn lower profits under semicollusion than under the non-cooperative setting when the timing is such that collusion occurs in the later stages of the game: case (3).

Finally, we showed that even when semicollusion is disadvantageous to the firms, consumers do not necessarily benefit. Hence, regulators still need to be diligent in the enforcement of antitrust laws.

Appendix A: Equilibrium semicollusive level of investment

In this appendix we derive the equilibrium semicollusive level of investment [eq. (10)]. Differentiating (7) with respect to $x_1$ yields

$$
\frac{\partial s_1(x_1, x_2)}{\partial x_1} = \frac{4(1-A+x_2)^2(1-A+2x_1-x_2)^2}{D} - (1-A+x_2)^2(1-A-x_2+2x_1)^2 \frac{\partial D}{\partial x_1},
$$

where

$$
D = (1-A+x_2)^2(1-A+2x_1-x_2)^2 + (1-A+x_1)^2(1-A+2x_2-x_1)^2.
$$

Furthermore,

$$
\frac{\partial D}{\partial x_1} = 2(1-A+x_1)(1-A+2x_2-x_1)^2 - 2(1-A+x_1)^2(1-A-x_1+2x_2)
$$

$$
+ 4(1-A+x_2)^2(1-A-x_2+2x_1).
$$

As we are looking for a symmetric equilibrium, setting $x_1 = x_2 = x^*$ yields
Substituting the above two expressions into (9) and solving for $x^*_i$ yields eq. (10).

**Appendix B: Individual rationality of the relative capacity sharing rule**

To show that the sharing rule for the capacity game is individually rational, we must show that for any $(k_1, k_2)$ short-run profits are higher under cooperation than under competition. Kreps and Scheinkman (1983) prove that for each $(k_1, k_2)$, the associated subgame has unique equilibrium expected revenues which depend on which of the three regions $(k_1, k_2)$ fall into. In our setting these three regions are: (1) $k_1 < (1 - k_2)/2$ and $k_2 < (1 - k_1)/2$; (2) $k_1 \geq k_2$ and $k_1 > (1 - k_2)/2$; and (3) $k_2 \geq k_1$ and $k_2 > (1 - k_1)/2$.

We now show that the sharing rule we employ is individually rational in all three regions. Region I: If $k_1 < (1 - k_2)/2$ and $k_2 < (1 - k_1)/2$, the equilibrium revenues for firm $i$ are $k_i(1 - [k_i + k_j])$, which are less than or equal to revenues earned under collusion, $k_i/4(k_i + k_j)$, if $(k_i + k_j)(1 - [k_i + k_j]) \leq 1/4$, which is true for all $(k_i, k_j)$. Region II: If $k_1 \geq k_2$ and $k_1 > (1 - k_2)/2$ it can easily be shown that the non-cooperative equilibrium revenues for firm 1 are $(1 - k_2)^2/4$. Since $k_1 \geq k_2$ and since the revenues earned under collusion, $k_1/4(k_1 + k_2)$ are increasing in $k_1$, collusive revenues are larger than $(1 - k_2)/4(1 + k_2)$. Thus collusive revenues are (weakly) larger than non-cooperative revenues if $(1 - k_2)(1 + k_2) \leq 1$, which is true.

It also can be easily shown that equilibrium noncooperative revenues for firm 2 are $k_2(1 - k_2)^2/4k_1$. Since cooperative revenues for firm 2 are $k_2/4(k_1 + k_2)$, non-cooperative revenues are weakly less than cooperative revenues for firm 2 if $(1 - k_2)^2/4$ is less than or equal to $k_1/4(k_1 + k_2)$, which is the same condition that insures that cooperation is individually rational for firm 1.

Region III: The final region we must consider is $k_2 \geq k_1$ and $k_2 > (1 - k_1)/2$. Since this region and region II are symmetric, the result follows immediately.

**References**


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