RAND Journal of Economics
Vol. 24, No. 2, Summer 1993

Complexity considerations and market behavior

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The article is concerned with market behavior when firms have limited ability to handle effectively the complexity of changing market conditions and strategic interaction. Modelling the managerial bounded rationality by using the concept of strategic complexity as measured by a finite automaton, we show that market behavior can be considerably altered once there is a limit on the complexity of strategies. In particular, we demonstrate that when an incumbent firm operates in several markets, an entry to one market may induce the incumbent to exit from another market (divestiture) in order to "concentrate" on the competition it faces. For different parameters the incumbent may react to such an entry by exit from the same market, creating specialization. We also demonstrate that bounded complexity can serve as an entry barrier, giving an advantage to the established incumbent firm.

1. Introduction

■ Managing corporations is a complex and time-consuming task. Managers need to evaluate changing market conditions, contemplate competitive strategies, decide on new products, production technologies, and new markets, and so forth. The ability to handle such complex situations in an effective way is clearly a key ingredient of successful management. Since corporations are run by managers, one cannot ignore the human factor and the unavoidable boundedness of the ability and rationality of human beings (see Simon (1972, 1978) for the original introduction of bounded rationality into the economic literature). In analyzing the behavior of firms, the economic literature usually assumes that there is no limit to the ability of management to calculate, to remember, to foresee, or to plan. Thus, in evaluating the contributions of this literature one should remember that it relies heavily on the existence of "supermanagers."

A casual observation of the modern business world indicates that firms are indeed aware of their inability to handle complex situations effectively and costlessly. As an example,

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The authors wish to thank Ariel Rubinstein, an anonymous referee, and the participants in seminars at Northwestern University, The Institute for International Economic Studies at the University of Stockholm, Bar-Ilan University, the Hebrew University of Jerusalem, Tel-Aviv University, Trinity College, Dublin, and the International Conference on Game Theory, Florence, Italy in June 1991 for many valuable comments. Kalai's research is partly supported by NSF Grant No. SES-9011790.

consider the recent sale of Fisher-Price by Quaker Oats. As reported by the *Wall Street Journal* (Wednesday, April 25, 1990), "Quaker Oats Co. decided to spin off its troubled Fisher-Price toy business to stockholders allowing management to concentrate on other difficulties at what, for the first time in two decades, is a 'pure' food company." Another example is the recent sale by Zenith Electronics of its computer operations. In its president's words, "In focusing on consumer goods, Zenith returns to its core business and could strengthen its ability to compete."[1] The above statements indicate the advantage that managers might have when they concentrate on one main business, particularly when this business is difficult to manage.[2] This article's main concern is the explanation and ramifications of these business strategies.

The general belief in the existence of managerial diseconomies of scale is the driving force of the segment of the literature claiming that "small is beautiful." The claim that a larger firm is harder to manage can be traced back to Kaldor (1934). In his seminal article, Coase (1937) argued that the optimal size of a firm is determined by the comparative cost of internal transaction versus the cost of market transaction and, more important, that at some point internal transaction becomes more expensive. There are several sources of such organization diseconomies. Calvo and Wellisz (1978), Mirrlees (1976), and Rosen (1982) discuss the cost of monitoring and its effect on effort. Williamson (1967), Geanakoplos and Milgrom (1991), and Guesnerie and Oddou (1988) consider the limited capacity of managers to process information. Rotemberg and Saloner (1991) demonstrate that when contracts are incomplete, firms might benefit from narrow strategies. The narrow strategy enables firms to motivate their employees to search for ways to increase profitability. Consequently, the inefficiency that is due to contract incompleteness implies that the firm has higher profits when it concentrates on one core activity than when it has a broad strategy and carries out several activities. McAfee and McMillan (1990) discuss the suboptimal activities of firms as a result of distorted incentives given by principals, which implies that more layers of hierarchy result in larger distortions. This type of inefficiency can also be regarded as a type of "influence cost," identified by Milgrom (1988) and Milgrom and Roberts (1988, 1989) as one of the main costs of centralization. Such a cost describes workers' incentives to influence their superiors' decisions and the inefficiencies associated with the need to impose mechanisms that offset such behavior.

The main purpose of this article is to study the behavior of a firm when it has bounded capacity to handle complex situations. We are interested in examining the relationships among having bounded ability to handle complexity, the scope of businesses in which a firm operates, and, in particular, the effects of competition on the optimal scope of businesses. We consider two aspects of market complexity: the complexity of operating simultaneously in different markets when the market conditions are uncertain, and the complexity of competing with other firms. These two sources of complexity compete for a firm's limited ability to handle complex situations. Thus when the firm becomes more responsive to changing market conditions, it must use less complex strategies in the games with its competitors. Using this setting, we demonstrate the importance of incorporating bounded rationality into the study of industrial organization and a formal way of doing so.

We will follow the literature (e.g., Abreu and Rubinstein (1988), Aumann (1981), Ben-Porath (1986), Kalai and Stanford (1988), Neyman (1985), Rubinstein (1986), and the survey by Kalai (1990)) and formalize the measure of strategic complexity using the framework of finite automata. Although most of this literature is concerned with repeated games, in discussing the market behavior of firms we do not wish to concentrate solely on

[1] See the statement of Zenith's president in the *Washington Post*, October 3, 1990.

[2] See also Lichtenberg's (1990) empirical work, in which it is pointed out that "the 1960s conglomerate boom may have contributed to the slowdown in U.S. productivity growth that began at or slightly after that time." Lichtenberg pointed out that the slowdown was partly due to the existence of "unmanageable empires."

strategic interaction, excluding the complexity of reacting to changing market conditions.[3] We thus define a model in which the firm faces different demand conditions in every period and needs to react to such exogenous changes at the same time it is dealing with competition. We define the complexity of a strategy as the minimal number of states in an automaton implementing it. We then limit the complexity of the firm's strategies and discuss the implications of such a limit for the scope of the firm's activities and the pattern of competition. In particular, we demonstrate the following possible effects: (i) in the face of competition in one market, the firm might divest from other markets to concentrate on the competition in its main business; (ii) complexity considerations might lead to market segmentation such that a firm facing new competition in one of its markets might decide to exit from it, leaving its competition as a single producer; and (iii) bounded complexity might serve as an entry barrier because the incumbent's bounded complexity might make credible the threat of aggressive response to entry.

We would like to note, however, that putting an upper bound on the complexity of strategies that the firm can use ignores its ability to hire more managers and thereby decentralize decision making and overcome the problem of having bounded complexities. Such a view implies that there is no limit to the ability of an organization to handle complex situations. Indeed, our results do not hold in situations in which complete decentralization of decision is feasible. It is our basic assumption, however, that even when it is possible to hire more decision makers, the ability to handle complex situations is limited.

2. The model

■ **Market conditions.** Let $I = \{1, \ldots, m\}$ be a set of m independent markets. At each market there is a linear demand function $p_i = a_i - q_i$, where p_i and q_i are the price and quantity in market i, respectively. a_i is assumed to be stochastic such that with probability $1/2$ there is a high demand and $p_i = h_i - q_i$ and with probability $1/2$ there is a low demand and $p_i = l_i - q_i$, where $l_i < h_i$ for all $i \in I$. The cost of producing one unit of product i is assumed to be $c_i < h_i$. We divide the set I into two subsets as follows:

$$G = \{ i \in I \mid c_i < l_i \}$$

$$B = I \setminus G.$$

At every period there is a signal σ of an m-tuple of 0 (low) and 1 (high) that specifies the demand condition in each market. We let Σ be the set of all possible signals and σ_i be the ith component of the signal σ. We let σ^0 be the initial market condition given prior to the start of the game.

Consider now a single firm that faces the above market conditions. Let $Q \equiv \mathbb{R}_+^m$ be the set of all possible output combinations. A strategy for the monopolist is a function $A: \Sigma \rightarrow Q$ that specifies an output combination for every possible signal. We further let $\pi_h^i(q_i)$ and $\pi_l^i(q_i)$ be the monopolistic profit function from market i. The monopolist's optimal strategy is:

(i) For $i \in G$, produce $(h_i - c_i)/2$ if $\sigma_i = 1$ and $(l_i - c_i)/2$ if $\sigma_i = 0$.
(ii) For $i \in B$, produce $(h_i - c_i)/2$ if $\sigma_i = 1$ and 0 otherwise.

Such a strategy, however, suggests that the firm adjusts its output vector for every different signal σ and is fully responsive to market conditions. This strategy is optimal as long as being responsive to market conditions does not incur any additional costs. When changing the output level or monitoring market conditions is costly, the above strategy may not be optimal. We let π^i denote the monopolist's expected profit in market i when it plays the above optimal strategy.

[3] Formally, the game we consider is a stochastic game.

□ **Finite automaton.** Most of the recent literature using the concept of a finite automaton discusses strategic bounded complexity in a repeated-games setting (see Kalai (1990)). Our model differs in that it assumes, besides the strategic interaction, the possibility of having changing market conditions. Thus the definition of an automaton needs to account for complexities induced by nature as well as those induced by strategic interactions.

We define an automaton as a triple $((M, m_0), A, T)$ with the following interpretation. M is the set of states of the automaton, with $m_0 \in M$ being the initial state. $A: M \to Q$ is a behavioral function that prescribes for each state an output combination $q \in Q$. The transition function $T: M \times \Sigma \times Q \to M$ governs the transition of the automaton from one state to another. Thus the input that the automaton receives at every period consists of a signal that describes the new market conditions and the output that was produced at the previous period.[4]

In the single-firm case, an efficient automaton corresponds to a partition of the signal space Σ such that every subset in the partition can be viewed as a state of the automaton and the transition function is degenerate, i.e., does not depend on the current state of the automaton or on the last-period production vector. We choose, however, the more general formulation, described above, since it can be easily extended to the multifirm case in which there is strategic interaction.

Observation 1. Every strategy of the firm can be described by such an automaton.

Proof. See the Appendix.

We now define the complexity of a strategy to be the number of states of the smallest (in the number of states) automaton describing it, or equivalently, as discussed in the Appendix, to be the number of different strategies it induced. We will model bounded rationality by assuming that firms will use strategies not exceeding a certain finite complexity.[5] We will denote by k the bound on the complexity of strategies.

The automaton measure of complexity and Observation 1 can be applied to all extensive-form games. For example, later we replace the monopoly game described above by an oligopoly. Then we assume that the automaton is modified by replacing q by a vector consisting of all firms' production levels in all markets.

Before proceeding it is important to note that the existence of multiple markets does not imply immediately that the strategy used is complex. One can, for example, adopt a strategy in which constant quantities (possibly zero) are produced in all markets. The complexity of such a strategy is 1. In general, however, the complexity of a strategy must equal at least the number of different actions it may prescribe in the play of the game, which implies the following:

Observation 2. The optimal strategy of the monopoly facing the multimarket problem studied above has the complexity of 2^m.

Observation 2 implies that the complexity required to implement the monopolist-optimal strategy is potentially huge, owing to the many possible production combinations. In particular, note that we restrict the demand function to take one of two values. If we change this assumption and let the demand function take many possible values, it will

[4] By including Q, we have chosen the more general notion of automaton, the one allowing for mistakes (the input to the automaton can be actions not consistent with the automaton's earlier prescriptions). In addition to the greater generality, this will enable us to discuss subgame perfection when we switch to the multiplayer model.

[5] An alternative formulation would be to assign a certain cost associated with the number of states in the automaton (see Abreu and Rubinstein (1988)). We choose our formulation for simplicity's sake. We believe, however, that our main results can be obtained with a model of costly states as long as this cost function is convex with the number of states. In this case our formulation is a special case, as we assume zero cost until k states and infinite cost for every state beyond k.

increase the complexity of the monopolist's optimal strategy dramatically. The main question is, of course: What happens if the firm is restricted to use strategies of complexity not exceeding $k < 2^m$?

Remark. We choose to consider automata that process all the aggregate signals, in order to model a manager who has to make decisions about different markets. Decomposing the input signal and letting separate automata deal with different markets is, in our model, equivalent to decentralization. Indeed, one way that firms cope with the complexity problem is to decentralize into divisions, each dealing with a separate market. While decentralization is not the main subject of this article, for our purposes it is important to note that if there is a team of automata that need to be coordinated, then the task of coordination implies that the total size of all the automata is larger than the sums of the states in the separate one-market automata. So to capture this aspect of diseconomies of scale in dealing simultaneously with different markets, we do not allow for decentralization, and we model the human decision problem as one automaton that processes the aggregate market signal.

□ **The optimal automaton for $m = 2$.** Consider a monopolist operating in two markets. Let q_h^i (correspondingly, q_l^i) be the optimal quantities for the ith market for a high (low) demand. Further, let z_i be the optimal constant level of production in market i when the firm does not distinguish between a high and a low signal, i.e., z_i maximizes $(1/2)(h_i - c_i - z_i)z_i + (1/2)(l_i - c_i - z_i)z_i$. Let

$$\gamma_i = (1/2)[\pi_h^i(q_h^i) + \pi_l^i(q_l^i) - \pi_h^i(z_i) - \pi_l^i(z_i)].$$

γ_i is thus the gain from being fully responsive to demand conditions in market i versus producing the optimal constant quantity. Without loss of generality, let $\gamma_1 > \gamma_2$. In describing the monopolist problem, the only complexities the monopolist faces are from changing market conditions. Although we use a finite automaton to model bounded rationality, for the one-person decision-making problem examined in this section, our model implies that the decision maker needs to partition the signal space and that the number of elements in this partition is limited. Thus the basic setting for the monopolist case is similar to Dow (1991), in which a decision maker is searching for the low price and his bounded memory of previous prices he observed is modelled by a limited partition of the set of possible prices.

Claim 1. For a monopolist who operates in two markets, the optimal automaton is as follows:

(i) For $k = 1$, there is one state of the automaton. Output is (z_1, z_2) regardless of the signal.

(ii) For $k = 2$, the optimal automaton moves to state 1 whenever the market condition is either $(1, 1)$ or $(1, 0)$. In this state, production is (q_h^1, z_2). The automaton moves to the second state if the market condition is either $(0, 0)$ or $(0, 1)$, and in this case production is (q_l^1, z_2).

(iii) For $k = 3$, an optimal automaton moves to state 1 when the market condition is $(1, 1)$. In this state it produces (q_h^1, q_h^2). It moves to state 2 when the market condition is $(1, 0)$, and it produces (q_h^1, q_l^2). It moves to state 3 whenever the market condition is either $(0, 1)$ or $(0, 0)$. In this state it produces (q_l^1, z_2).

(iv) For $k = 4$, the automaton implements the optimal monopoly strategy as discussed in the previous section.

Proof. See the Appendix.

3. Market scope without competition

■ When a monopolist can use strategies of unlimited complexity, it would operate in all m markets. But when $k < 2^m$, operating in all markets is not necessarily optimal. To illustrate the problem, consider a firm operating in m markets with a strategy of complexity k. Assume that there is an opportunity to enter another market. Given the upper bound on the complexity, k, the firm has three options that illustrate the tradeoff it faces: (i) to choose a strategy that is (partially) responsive to demand conditions in the new market and by so doing reduce its responsiveness to demand conditions in the previous m markets; (ii) to enter the new market by producing a constant quantity; (iii) not to enter the new market.

Since for every $i \in G$, $c_i < l_i$, it is clear that there is a constant quantity that yields positive expected profits. Thus the firm will always enter markets of type G. One cannot extend the above argument to markets of type B.

Let $MB = \{i \in B | [\pi_h^i(z_i) + \pi_l^i(z_i)] \geq 0\}$ and $VB = B \backslash MB$. The monopolist enters markets in MB (moderately bad) as, by definition, producing a constant quantity, z_i, yields positive profits. The entry decisions to a market of type VB (very bad) are more complicated. In markets of type VB, in order to make profits one needs to react to changing market conditions. On the other hand, there are indirect costs associated with being responsive to market conditions. Since the firm can use strategies of bounded complexity, being responsive (even partially) to demand conditions implies that the strategy must be less responsive to demand conditions in some of the other markets, which, of course, reduces profits from these markets.

Claim 2 (market scope). When $m = 2$, $k = 2$, $\gamma_1 > \gamma_2$, and the second market is of type VB, then the firm will not enter the second market.

Proof. When the firm enters both markets, the optimal automaton is specified by Claim 1. For such an automaton the output in the second market is constant and not responsive to demand conditions. Since market 2 is of type VB, the firm loses money in this market and is therefore better off not entering it. *Q.E.D.*

Claim 2 illustrates that bounded complexity considerations can determine the scope of activities of firms even in situations in which the only relationship between the different activities is that they are managed by the same firm.

4. Bounded complexity and market competition

■ Strategic interaction adds another source of complexity to the firm's decision problem. Facing one type of complexity diminishes the firm's ability to handle the other type. This tradeoff between the two types of complexity plays an important role in determining the firm's behavior in oligopolistic markets.

There is a fundamental difficulty in the use of a finite-automaton framework to model market behavior with bounded complexity. The main question is the ability of players to change the automaton they are using. If there is no limit to such ability, then we are back in the world of unbounded complexity, as any strategy of any complexity can be implemented. But to assume that players never change the automaton is too restrictive. Moreover, under such an assumption the use of a finite automaton as a modelling tool will tend more to capture the ability of players to commit themselves rather than modelling their bounded rationality. In this article we assume that once a competitor enters a market, it is possible for the incumbent firm to react by changing the automaton. Thus only when a "major" event such as entry occurs is it possible to change the automaton. Without this assumption it would be possible for the incumbent to commit himself to a certain automaton so as to prevent entry or give the incumbent some advantage in the postentry game.

□ **Competition in a single market.** Consider a single duopolistic market in which demand is as specified in Section 2 and both firms have the same cost function. Let x_h and x_l be the Cournot equilibrium output for the high and low demand respectively, and let $\tilde{\pi}$ be the Cournot equilibrium payoffs. Our symmetry assumption implies that both firms realize the same profits.

Observation 3. The Cournot equilibrium can be implemented with strategies of complexity 2 (for each firm).

Proof. Consider the following simple automaton: There are two states, M_1 and M_2, with the initial state M_1 if the market is high initially and M_2 otherwise. The behavioral function is $A(M_1) = x_h$ and $A(M_2) = x_l$. The transition function is $T(M_i, 1, q_1, q_2) = M_1$ and $T(M_i, 0, q_1, q_2) = M_2$ for every q_1, q_2, and M_i. Clearly, if for both firms $k = 2$ and one firm uses the above automaton, the best response of the other firm is to use the same automaton. *Q.E.D.*

Now consider other equilibria of the repeated play of the above dupolistic game, and let the two firms maximize discounted profits. Let r_h and r_l be the collusive output level for the high and low demand respectively, and let $\hat{\pi}^i$ be the expected collusive payoffs from market i.[6] As is well documented in the literature, when the discount factor is sufficiently close to one, the collusive outcome can be supported as a noncooperative (subgame-perfect) Nash equilibrium. For example, one can use the well-known grim-trigger strategies, such that firms cooperate until one defects and then both switch to the Cournot-Nash equilibrium forever.

Observation 4. The grim-trigger strategy equilibrium can be implemented as an equilibrium with strategies of complexity 4.

Proof. Can be proved by a straightforward construction.

One can also verify that it is possible sometimes to economize on the punishment phase of the grim-trigger strategies and obtain the collusive outcome by using strategies of complexity 3. The need to have states assigned to the punishment phase leads to the following observation:

Observation 5 (collusion is complex). In a single-market duopoly, supporting the collusive outcome requires the use of strategy of complexity $k > 2$, i.e., above the complexity of the Cournot-Nash equilibrium strategies.

Proof. When an automaton of only two states is used to support the collusive outcome it must be that $A(M_1) = r_h$ and $A(M_2) = r_l$. Clearly, a pair of such strategies is not an equilibrium. *Q.E.D.*

□ **Multimarket competition.** Let us move to a multimarket setup. Although we assume that markets are independent with respect to demand and cost conditions, complexity considerations may introduce interdependence among markets, as we have already seen in the monopoly case. In particular, the introduction of competition in one market may lead to a different behavior in the other. To demonstrate this, consider the following example: An incumbent firm operates in two markets, $k = 2$ and $\gamma_1 > \gamma_2$. As Claim 1 suggests, the optimal automaton is to be fully responsive in market 1 and produce a constant quantity in market 2. Assume now that a new firm enters market 1. As a response to such an entry, the incumbent firm may decide to exit from one of the two markets. If it decides to stay in both, it needs to determine whether to continue being responsive in the first market and

[6] Although there might be several collusive levels, let us choose the one yielding the highest possible profits among the symmetric outcomes.

whether producing a constant quantity in the second market is still its optimal strategy. It is possible that as a result of an entry to market 1, it becomes optimal for the firm to be responsive to demand conditions in market 2, in which there is no competition, and to produce a constant quantity in market 1.

Competition and divestiture policy of firms. Consider now a firm that operates in two markets. Letting $k = 4$ implies that the firm operates in both markets and is fully responsive to demand conditions. We further assume that $\gamma_1 > \gamma_2$ and that the second market is of type *VB*.

Assume now that a new firm enters the first market. The two firms can now compete and produce the Cournot equilibrium quantities, or they can collude. Note, however, that as indicated by Observation 5, collusion is complex, i.e., in order to support a collusive outcome the firms need to use strategies of complexity exceeding the complexity of the Cournot-Nash equilibrium.

Claim 3. Consider an entry to the first market. (i) When $\hat{\pi}^1 > \tilde{\pi}^1 + \pi^2$ and $\tilde{\pi}^1 > 0$, the incumbent's optimal response is to exit from the second market. (ii) When $\tilde{\pi}^1 \leq 0$ and $\hat{\pi}^1 < \pi^2$, the incumbent's optimal response is to exit from the first market.[7]

Proof. Facing competition in market 1, the incumbent has three options: he can exit from market 1 and remain in market 2, he can exit from market 2 and cooperate in market 1, or he can stay in both markets and play the Cournot strategies in market 1. Given that $k = 4$ and collusion is complex, the option of staying in both markets—being responsive to market conditions in market 2 and yet supporting the collusive outcome in market 1—is not available to him, since it requires strategies of complexity exceeding 4. The conditions $\hat{\pi}^1 > \tilde{\pi}^1 + \pi^2$ and $\tilde{\pi}^1 > 0$ imply that supporting the collusive outcome in the first market is the incumbent's best strategy. In such a case the incumbent cannot be responsive to demand conditions in market 2, and since this market is of type *VB*, getting out of it is part of the optimal strategy. This completes the proof of (i).

When $\hat{\pi}^1 < \pi^2$ and $\tilde{\pi}^1 < 0$, supporting the collusive outcome in market 1 is not optimal, as it requires getting out of market 2 and losing π^2. Staying in market 2 while colluding in market 1 implies that the firm produces a constant quantity in market 2, and since this market is of type *VB*, the firm will realize losses from such a policy. *Q.E.D.*

Claim 3 demonstrates two possible scenarios. Part (i) demonstrates that limited complexity may lead to divestiture, while part (ii) demonstrates that it can result in specialization. Divestiture occurs when the competitor enters the market that contributes significantly to the firm's total profits. In such a case, the firm decides to exit from markets that are not its main business and to concentrate on its main business, when by "concentrating" we mean using a strategy with high complexity. Specialization occurs when, as a result of entry into one of the markets, the firm decides to leave this market because the strategic interaction is too complicated and thus costly. The outcome of this behavior is a complete specialization such that in both markets there is a monopoly.

Note also the importance of the independence assumption in Claim 3. If markets 1 and 2 are related such that the conditional probability p (the demand is high in market 2 | the demand is high in market 1) > ½, the optimal behavior might be different. The firm can use the correlation to reduce the complexity of its strategy. For example, when the above conditional probability is 1, such that the two markets are perfectly correlated, one can produce the optimal quantity with strategies of complexity 2. Thus our result of divestiture will not hold when the markets are sufficiently correlated. The firm can collude (use

[7] One of the conditions of (ii) is that the expected Cournot equilibrium profits, $\tilde{\pi}^1$, is negative. This can happen, for example, when there are sufficiently large fixed costs.

a strategy of higher complexity) in the first market and still produce the optimal quantities in the second market. This claim supports our intuition that the divestiture will occur in a conglomerate firm in which the businesses are not related and not in a firm producing in related markets.

Bounded complexity and entry deterrence. Intuitively, one may think that having bounded complexity is always disadvantageous for the firm, as it limits its ability to use complex strategies. But as often happens in strategic interaction, one can sometimes benefit from a handicap, i.e., it is possible that having bounded complexity will enhance profits (see also Gilboa and Samet (1989)).

Consider a firm, I, operating in two markets such that $\gamma_1 > \gamma_2$. There is a firm E that considers entering the second market. In the postentry game the incumbent may choose to cooperate and get the profits $\hat{\pi}_I^2$, or to use strategies of lower complexity and get the Cournot-Nash equilibrium profits $\tilde{\pi}_I^2 < \hat{\pi}_I^2$. The entrant equilibrium profits are $\tilde{\pi}_E^2 < 0$ if the incumbent chooses not to cooperate and $\hat{\pi}_E^2 > 0$ if the incumbent cooperates. Thus entry is attractive only when the incumbent cooperates. We assume that the entrant does not have any complexity constraints, and thus it is the incumbent who decides upon the type of the postentry game, i.e., cooperation or fighting. Note, however, that since $\tilde{\pi}_E^2 < 0$, an entrant with a complexity constraint will not enter the market without the ability to support the collusive outcome.

The above setup can be regarded as the last-period problem in the chain store paradox (Selten, 1978). Indeed, without a limit on the complexity of strategies, subgame-perfection arguments imply that once entry occurs, the incumbent will cooperate and thus entry is profitable. This result holds since supporting cooperation is not costly. This well-known result does not hold, however, if there is a bound on the complexity of strategies of the incumbent firm.

Claim 4. (i) When k is sufficiently large, the incumbent will react cooperatively and the entrant will enter. (ii) When $k \leq 4$ and $\hat{\pi}_I^2 - \tilde{\pi}_I^2 < \gamma_1$, the incumbent will react aggressively to an entry and thus the entrant will not enter.

Proof. When k is sufficiently large, the bounded complexity is not a binding constraint and subgame perfection implies (i). When $k = 4$, the incumbent has two options: The first is to cooperate in the second market and to produce a constant quantity in the first market. The second is not to react cooperatively and to be responsive to market conditions in the first market. Note that the option of reacting cooperatively in the second market while remaining responsive to market conditions in the first market is not available, as it requires strategies of complexity exceeding 4. Since $\hat{\pi}_I^2 - \tilde{\pi}_I^2 < \gamma_1$, the second possibility yields higher payoffs, the incumbent will not react cooperatively, and thus the entrant will not enter. *Q.E.D.*

The bounded complexity serves here as a credibility device to the threat of noncooperation in the postentry game. Given the bound on complexity, a cooperative behavior becomes costly. If the incumbent reacts cooperatively to an entry it will have to be less responsive to market conditions in the first market, which reduces its profits by γ_1.

5. Concluding remarks

■ The literature on transaction cost economics emphasizes the need to revise the analysis of markets by taking into account that engaging in a contract, changing production level, or generally changing strategies is not without cost. The major claim of this article is that the economic analysis of markets needs also to account for the limited ability of management to handle effectively the complexity of changing market conditions and strategic interaction with competitors. Modelling the managerial limited rationality by using the concept of strategic complexity as measured by automata, we show that the outcome of market behavior and conduct can be considerably altered once there is a limit on the complexity of strategies. We believe that such an analysis can explain market behaviors that the classical industrial organization literature cannot. In this article we discussed only two aspects of market complexity. There are, however, many other aspects of complex market situations unaccounted for here, for example, entry and exit decisions, R&D decisions, contracting complexities, and so on. We believe that a positive approach to industrial organization ought to account for the effects of such complexities on managerial decision making and market behavior.

Appendix

■ Proofs of Observation 1 and Claim 1 follow.

Proof of Observation 1. We use a generalized version of the construction in Kalai and Stanford (1988). Given a game with an initial condition σ^0, we define the set of histories of length zero as $H^0 = \{e\}$, with e denoting the empty history. The set of histories of length l, H^l, consists of all vectors $q^1\sigma^1, \ldots, q^l\sigma^l$, with $\sigma^t \in \Sigma$ and $q^t \in Q$ for $t = 0, 1, \ldots, l$. We let $H = \cup_{t=0}^{\infty} H^t$ denote the set of all (finite-length) histories. Now we formally define a strategy to be a function $f: H \rightarrow Q$.

Given a history $h \in H$ it is useful to discuss entities defined on the subgame induced by h. h is of the form $h = q^1\sigma^1, \ldots, q^l\sigma^l$, and we define the game induced by h, G_h, to be the one with the initial condition σ^l. The set of induced histories is $H_h^0 = \{e\}$, and H_h^r is the set of all vectors $\bar{q}^1\bar{\sigma}^1, \ldots, \bar{q}^r\bar{\sigma}^r$. Starting with a strategy f and a history $h \in H$, we define the strategy induced by f and h on the game G_h to be the function $f_h: H_h \rightarrow Q$ such that for every t and every $\bar{h} \in H_h^t$, $f_h(\bar{h}) = f(h\bar{h})$ with $h\bar{h}$ denoting the concatenation of the two histories, i.e., $h\bar{h} = q^1\sigma^1, \ldots, q^l\sigma^l, \bar{q}^1\bar{\sigma}^1, \ldots, \bar{q}^t\bar{\sigma}^t$ (we make the convention that $eh = he = h$). We use $f_H \equiv \{f_h: h \in H\}$ to denote the set of all strategies induced by f. Notice that even if $G_h \neq G_{\bar{h}}$ (when $\sigma^l \neq \sigma^{\bar{l}}$), the set of strategies of the two games coincide. Thus the set f_H induces all the induced strategies from all different induced games and thus it may be small even if the number of induced games is larger. Suppose, for example, that a constant strategy is used with $q^t = q^*$ for all t. Then f_H includes only one element (the constant strategy).

Now for a given strategy we will exhibit an automaton implementing it. Its states will correspond to the different strategies it induces, i.e., $M = f_H$, with the initial state corresponding to f itself, i.e., $m_0 = f$. The behavior function assigns to each state the initial action taken by the corresponding induced strategy, i.e., $A(m) = f_h(e)$. The transition function is defined by

$$T(f_h, \sigma, q) = f_{\bar{h}} \qquad \text{with} \qquad \bar{h} = q^1\sigma^1, \ldots, q^l\sigma^l q\sigma.$$

It is easy to check that the automaton just described is well defined and that it implements f.

It is worth noting that the above construction used a number of states equal to the cardinality of the set of strategies induced by f. This shows that the number of states needed to implement f does not exceed the number of strategies induced by f. It is easy to see that the converse is also true, i.e., the number of states needed to implement a strategy equals at least the number of different strategies it induces. Thus, we can conclude that the number of states needed to implement a strategy equals the number of strategies it produces. *Q.E.D.*

Proof of Claim 1. $k = 1$ is trivial. For $k = 2$ let us consider all possible partitions. Since $\gamma_1 > \gamma_2$, it is evident that having the partition $\{(1, 1), (0, 1)\}$, $\{(1, 0), (0, 0)\}$ is not optimal, as it represents a full partition in the second market while producing a constant level in the first market. For the partition $\{(1, 1), (0, 0)\}$, $\{(1, 0), (0, 1)\}$, the optimal production levels are (z_1, z_2) for all states of the automaton, and thus the monopolist does not exploit his ability to be partially responsive to market conditions.

Consider now the partition $\Sigma_1 = \{(1, 1)\}$ and $\Sigma_2 = \{(1, 0), (0, 0), (0, 1)\}$. For such a partition the optimal production levels are described in Figure A1, where f_i is the optimal output when the probability of having a low demand is ⅔. We will now show that transforming the quantities produced in Figure A1 to the ones specified in Claim 1(ii) and depicted in Figure A2 yields higher profits for the firm. We will first demonstrate that the quantities in Figure A3 (which cannot be implemented by a strategy of complexity) yield higher payoffs than those in Figure A1. We will then demonstrate that the quantities in Figure A2 yield higher payoffs than those in Figure A3.

Changing from Figure A1 to Figure A3 yields higher payoffs as we first change the quantity produced in the second market at state (h, l) to the optimal quantity q_l^2. A similar change occurs in the first market in the state (l, h). Then notice that, conditional on having a low demand in the second market, the constant that maximizes profits in the first market is z_1. Thus changing f_1 in the states (h, l) and (l, l) to z_1 increases payoffs. In a similar manner, changing f_2 in states (l, h) and (l, l) to z_2 yields higher profits.

Changing from Figure A3 to Figure A2 yields the following changes in payoffs:

$$[-\pi_h^2(q_n^2) + \pi_h^2(z_2) + \pi_h^1(q_h^1) - \pi_h^1(z_1) - \pi_l^2(q_l^2) + \pi_l^2(z_2) + \pi_l^1(q_l^1) - \pi_l^1(z_1)]/4, \tag{A1}$$

which, after rearranging and using our assumption that $\gamma_1 > \gamma_2$, yields that (A1) is positive and thus the firm has higher payoffs with figure A2 than with Figure A1. The other partitions can be analyzed similarly.

FIGURE A1

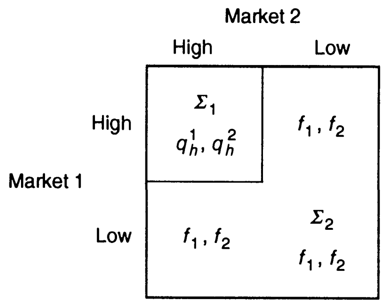


FIGURE A2

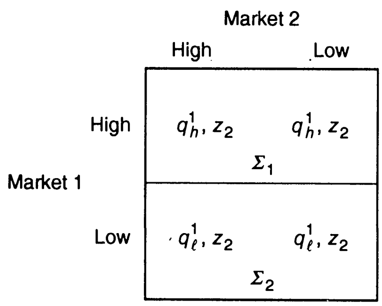
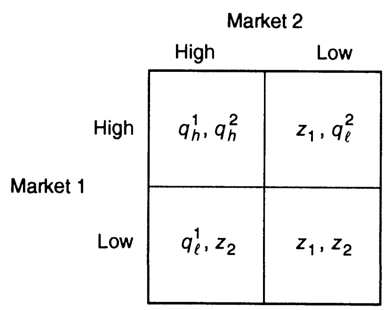


FIGURE A3



For $k = 3$, the only other partition is the one that perfectly uses the information regarding market 2 and only partially uses the information regarding market 1. Using our assumption that $\gamma_1 > \gamma_2$ and following the above procedure would indicate that such a partition yields lower payoffs. The case of $k = 4$ is trivial. *Q.E.D.*

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