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Maxwell equations versus the longitudinal magnetic field of the photon

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Abstract

A new theory ascribing an additional longitudinal magnetic field to a circularly polarized electromagnetic wave is discussed. An analysis of the fields of a source of circularly moving charges proves that the new theory is inconsistent with Maxwell equation $\nabla \cdot \boldsymbol{B} = 0$.

1. Introduction

The classical theory of radiation is known for more than 100 years. Solving Maxwell equations for an appropriate system of charges, one finds radiation fields associated with charge acceleration. This effect is characterized by transverse electric and magnetic fields [1, 2]. The transverse fields span the two degrees of freedom of polarization of radiation fields. This property of classical fields corresponds to photon polarization as derived in quantum mechanics [3]. Attempts to modify electrodynamics have been carried out for several years in this journal and elsewhere [4–10]. The modified theory claims that, beside the transverse fields, circularly polarized waves are endowed with a longitudinal magnetic field, namely, a field which is parallel to the direction of the wave propagation. (This theory is called hereafter modified electrodynamics.) Furthermore, the attack on ordinary electrodynamics continues and it is claimed that "the basic electrodynamical notion that there can be only two degrees of field polarization in three-dimensional space is therefore geometrical nonsense" (see Ref. [7, bottom of p. 1520]). Objections to the modified electrodynamics have already been published [11–14] and have been followed by responses [15, 16]. Further articles advocating the modified electrodynamics continue to be published [17–20]. The purpose of the present work is to add arguments to the debate by showing that the modified electrodynamics is inconsistent with Maxwell equation $\nabla \cdot \boldsymbol{B} = 0$.

The present work uses units where the speed of light c is unity. $\boldsymbol{i}, \boldsymbol{j}$ and $\boldsymbol{k}$ denote unit vectors in the x, y and z-directions, respectively.

Essential elements of the modified electrodynamics are presented in the second section. Fields of a system which consists of two circularly moving charges are discussed in the third section. Concluding remarks are the contents of the last section.

2. The modified electrodynamics

The modified electrodynamics does not alter the well known transverse fields of radiation as derived

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in ordinary electrodynamics. Here one can use a presentation of fields of a circularly polarized wave which is written by means of complex quantities [1, 2]. Define the following expression (see Ref. [8, p. 1674])

$$\boldsymbol{B}^{(1)} = \frac{B^{(0)}}{\sqrt{2}}(\mathrm{i}\boldsymbol{i} + \boldsymbol{j})\mathrm{e}^{\mathrm{i}\phi}, \tag{1}$$

where $\phi = \omega t - \boldsymbol{k}\cdot\boldsymbol{r}$ and $B^{(0)}$ is a real quantity. The complex conjugate of Eq. (1) is

$$\boldsymbol{B}^{(2)} = \frac{B^{(0)}}{\sqrt{2}}(-\mathrm{i}\boldsymbol{i} + \boldsymbol{j})\mathrm{e}^{-\mathrm{i}\phi}. \tag{2}$$

A third quantity used by the modified electrodynamics is

$$\boldsymbol{B}^{(3)} = B^{(0)}\boldsymbol{k}. \tag{3}$$

These quantities satisfy the following relation

$$\boldsymbol{B}^{(l)} \times \boldsymbol{B}^{(m)} = \mathrm{i}B^{(0)}\boldsymbol{B}^{(n)*}, \tag{4}$$

where $\{l, m, n\}$ is a cyclic permutation of $\{1, 2, 3\}$ and the asterisk denotes complex conjugation.

The main point of the modified electrodynamics is that a circularly polarized wave has a longitudinal magnetic field $\boldsymbol{B}^{(3)}$ whose amplitude is the same as that of the transverse magnetic field. Consequences of this assumption are discussed in the next section.

3. A test of the modified electrodynamics

The modified electrodynamics is tested here by means of the following simple device. Consider a disk made of an insulating material whose permitivity and permeability are the same as those of vacuum. Let R denote the disk's radius. A positive charge Q (denoted by a filled circle) and a negative one $-Q$ (denoted by an open circle) are attached to two antipodal points of the disk (see Fig. 1). The disk's plane coincides with the (x, y) plane and its center O is at the origin of coordinates. This disk rotates around the z-axis with a constant angular velocity ω. This system is well known and its dipole radiation fields can be found in the literature (see Ref. [1, pp. 173–176]). Here one finds that the radiation in a general direction is polarized elliptically and that $\cos\theta$ measures the circularity. It follows that the radiation in a small neighbourhood at the z-axis is circularly polarized. Therefore, it is suitable for an examination of the modified electrodynamics. Thus, a point on the z-axis at the wave zone is examined.

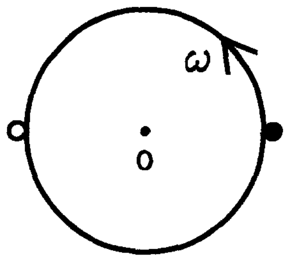

Fig. 1. A disk rotates in the (x, y) plane around its center O (see text).

Let $\boldsymbol{B}_\mathrm{T}$ denote the transverse magnetic field of dipole radiation. The form of this field at a point on the z-axis is

$$\boldsymbol{B}_\mathrm{T} = A_\mathrm{T}(\boldsymbol{i}\cos\omega t + \boldsymbol{j}\sin\omega t)/r. \tag{5}$$

Here the coefficient A_T is independent of the radial distance r and of the time t

$$A_\mathrm{T} = 2Q\omega^2 R. \tag{6}$$

The magnetic field (5) satisfies the Maxwell equation

$$\nabla\cdot\boldsymbol{B}_\mathrm{T} = 0. \tag{7}$$

Now assume that the modified electrodynamics holds. It follows that we have also a longitudinal magnetic field $\boldsymbol{B}^{(3)}$ in the radial direction (namely in the z-direction). Relations (1)–(4) of the modified electrodynamics yield

$$|\boldsymbol{B}^{(1)}| = |\boldsymbol{B}^{(3)}|, \tag{8}$$

and $\boldsymbol{B}_\mathrm{T}$ is related to the real part of $\boldsymbol{B}^{(1)}$. Taking the absolute value of expression (5), one finds

$$|\boldsymbol{B}_\mathrm{T}| = A_\mathrm{T}/r. \tag{9}$$

It follows that the radial dependence of $|\boldsymbol{B}^{(3)}|$ is like that of expression (9). Thus, we have

$$\boldsymbol{B}^{(3)} = \frac{A_\mathrm{L}}{r}\boldsymbol{n}, \tag{10}$$

where $\boldsymbol{n}$ is a unit vector in the radial direction and the coefficient $A_{\mathrm{L}} = A_{\mathrm{T}}$ does not depend on the radial distance r and on the time t.

Let us examine the validity of Maxwell's equations within the framework of the modified electrodynamics

$$\nabla \cdot \boldsymbol{B} = \nabla \cdot (\boldsymbol{B}_{\mathrm{T}} + \boldsymbol{B}^{(3)}) = A_{\mathrm{L}}/r^2 \neq 0. \tag{11}$$

Here Eqs. (7) and (10) are used and the calculation is carried out in spherical polar coordinates. This outcome proves that the modified electrodynamics does not satisfy Maxwell equations.

4. Concluding remarks

Advocates of the modified electrodynamics claim that this theory satisfies Maxwell equations. This claim is followed by an examination of circularly polarized *plane* wave (see, e.g. Ref. [8, p. 1674]). Here the magnetic field $\boldsymbol{B}^{(3)}$ is assumed to be uniform. Thus, its divergence vanishes. However, an argument of this kind, which is just an *example*, cannot be considered as a *proof*. Moreover, a plane wave is a *limit* of actual physical phenomena. The present work examines an example of spherical waves and shows that in this case, the modified electrodynamics violates Maxwell equation $\nabla \cdot \boldsymbol{B} = 0$. As is well known, one counterexample suffices for disproving a theory.

Maxwell equations are real and linear. This property enables the addition of pure imaginary quantities to real solutions, provided one is careful about retaining the linearity of mathematical operations [1]. Obviously, this requirement is not conserved by the cross product (4) of the modified electrodynamics. Therefore, one cannot be sure whether or not the field $\boldsymbol{B}^{(3)}$ satisfies Maxwell equations. The specific examination carried out above proves that, indeed $\boldsymbol{B}^{(3)}$ of the modified electrodynamics is inconsistent with Maxwell equations. This outcome casts serious doubts on the validity of the new theory.

The modified electrodynamics claims to explain phenomena found in interaction of a circularly polarized laser beam with matter. It turns out that this explanation is inconsistent with a very well established theory, namely, Maxwell equations, which nobody thinks to be incorrect. Therefore, one should take a different course and explain the laser beam interaction with matter in a way which is consistent with Maxwellian electrodynamics.

It is interesting to note that a recent experiment has disproved Evans' theory concerning the optical Faraday effect [21]. The discussion presented here indicated that this experiment can also be considered as a refutation of a theory which is incompatible with Maxwellian electrodynamics.

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