FURTHER COMMENTS ON THE ORIGINAL DERIVATION OF THE ELECTRIC AHARONOV–BOHM EFFECT

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The original derivation of the electric Aharonov–Bohm effect is analysed. The underlying reasons for the inherent incompatibility of this derivation with the law of energy conservation is pointed out.

It has recently been shown that the original derivation of the electric Aharonov–Bohm (AB) effect [1,2] is incompatible with the law of energy conservation [3]. The objective of the present Letter is to reveal the underlying physical error which is inherent in that derivation of the effect. A consequence of this analysis is that the counter-example presented in ref. [3] is not accidental.

Classical physics teaches us that energy conservation of a system is associated with the time independence of the hamiltonian function [4]. A corresponding relation holds in quantum mechanics. In particular, it is shown that the wavefunction of a closed system is an eigenfunction of the hamiltonian (ref. [5], p. 28). In quantum mechanics Heisenberg's uncertainty relation

$$\Delta E \, \Delta t \sim \hbar \tag{1}$$

holds (ref. [5], pp. 150–153). Thus, conservation of energy is approached if the entire system can be left uninterrupted for a sufficiently long time.

Equiped with these theoretical foundations of quantum mechanics, let us examine the original derivation of the electric AB effect [2]. This effect measures the interference pattern of a charge moving in a nonsimply connected field-free region of space where the electric potential varies in time. Henceforth, this charge is called "the moving electron". It is assumed that other charges of the system are not electrons so that no antisymmetrization is required for the moving electron. This assumption simplifies notation but does not affect the results of the following analysis. It is also assumed that particles move slowly and the nonrelativistic limit holds.

In the experiment, the moving electron is chopped into rather short wave packets, each of which is split later into two coherent subpackets. The subpacket P_I runs through a very long hollow cylinder while P_0 runs at the cylindrical outer region (see figs. 1 and 2 of ref. [3]). The cylinder consists of two layers made of insulating materials which are covered uniformly with the same amount of positive and negative charges, respectively. The initial difference between the radii of the cylindrical layers is infinitesimal. When P_I is well inside the cylindrical inner region, a special device releases a fixed amount of energy which pushes the cylindrical flexible layer and reduces its radius. Thus, P_I as well as P_0 move in a field-free region of space. However, while P_0 travels in a region where the electric potential vanishes identically, P_I "sees" a nonzero electric potential. Hereafter, the cylinder and its charges are sometimes called "the source".

The hamiltonian of the system is

$$H = H_s + H_e + V, \tag{2}$$

where H_s operates on the coordinates of the source, H_e is the single-particle hamiltonian of the moving electron and V denotes the interaction energy of the moving electron with the source (see eq. (11) of ref.

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[2]). The authors of ref. [2] aim to derive the effect when the moving electron as well as the source are treated quantum mechanically (see section 3 of their work). They claim to prove that the wavefunction of the entire system can be written in the form of a single product

$$\Psi = \phi(y_1, ..., y_n, t)\psi(x, t), \tag{3}$$

where the single-particle wavefunction of the moving electron factors out (see top of page 1518 and also the text after (19)). In (3), $y_1, ..., y_n$ denote the coordinates of the charges at the source and x is that of the moving electron. Assuming that their approximations holds, the authors of ref. 2 use the multiplicative wavefunction (3) to derive the electric AB effect.

In the following lines, the multiplicative wavefunction (3) is examined. To this end, let us write it in the form

$$\Psi = \phi(y_1, ..., y_n, t)[\psi_1(x, t) + \psi_0(x, t)], \tag{4}$$

where ψ_1 and ψ_0 denote the inner and the outer subpackets, respectively. Operating on (4) with the hamiltonian (2), one finds

$$\begin{aligned} H\Psi &= (H_s + H_e + V)\Psi \\ &= (H_s\phi)\psi_1 + (H_s\phi)\psi_0 + \phi(H_e\psi_1) + \phi(H_e\psi_0) \\ &\quad + V\phi\psi_1 + V\phi\psi_0. \end{aligned} \tag{5}$$

At the location of ψ_1, the potential V is nonzero during the relevant period whereas it vanishes identically for ψ_0. Hence, the last term of (5) can be eliminated. The moving electron travels in a field-free region of space which is macroscopically far from the cylindrical charges. Therefore, due to the validity of the classical limit, one finds that the third and the fourth terms of (5) make the same contribution to the required eigenvalue. This conclusion obviously holds for the first two terms of (5) where H_s operates on the same function ϕ. Therefore, it is proved that (4) is *not an eigenfunction of the hamiltonian* (2) and that the size of the discrepancy is V.

Another point is the time duration of the energy imbalance. This period is measured between t_0, when P_I is far from the cylinder and t_1, which denotes an instant when P_I is inside the cylinder and "sees" the potential V. This time difference can be as long as one likes if the moving electron is slow enough. Hence, $\Delta E\,\Delta t$ can be made bigger than any preassigned value. It follows that the wavefunction used by AB violates the basic requirements of Landau and Lifshitz (ref. [5], pp. 28, 150–153). This analysis proves that the incompatibility of this wavefunction with the law of energy conservation is inherent in the original derivation of the AB effect [2]. Hence, the example of ref. [3] is not a trick but an illustration of this substantial inconsistency.

References

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