CONSERVATION LAWS AND THE ELECTRIC AHARONOV–BOHM EFFECT

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The original derivation of the electric Aharonov–Bohm effect is analysed. It is shown that the operation of a simple device based upon the principles used in that derivation is incompatible with the law of energy conservation. It is concluded that the original proof of the effect is incorrect.

The Aharonov–Bohm (AB) effects [1,2] are considered to be a controversial issue for a long time [3]. The ambivalent approach to these effects has been expressed by Weisskopf who remarked on the first AB article: "the first reaction to this work is that it is wrong; the second is that it is obvious" [14].

The AB effects predict the phase shift of a split electronic beam moving in a nonsimply connected field-free region. The electric AB effect uses an electric potential associated with an inaccessible electric field. In the magnetic effect, a magnetic field plays the role of the inaccessible field. The present work discusses the derivation of the electric AB effect. The magnetic AB effect is completely beyond the scope of the present work and no claim is made concerning the relevance of the following conclusions to it.

Unlike the magnetic AB effect, the electric one has never been tested experimentally [‡1]. Hence, this effect lacks a substantial support required for the establishment of a physical prediction. Thus, arguments in favour or against it are based solely on analyses of thought experiments.

The following experiment is designed for the measurement of the electric AB effect [1,2] (see fig. 1). An electronic beam is chopped into rather short packets at A. Each of these packets is split coherently into two subpackets at B. An electron prepared in this way is called hereafter the moving electron. The cylinder consists of two layers made of insulating materials. The outer rigid layer is covered uniformly with positive charges and the inner flexible one is covered analogously with the same amount of negative charges. When the subpacket P_I approaches the cylinder, the difference between the radii of the two layers is infinitesimal and the electric potential vanishes everywhere. Later, when P_I is well inside the long cylinder and P_O is far from its two ends, a special device releases a fixed amount of energy that pushes adiabatically the inner layer in a radial direction in a manner that conserves it cylindrical shape (see fig. 2). The inner flexible layer shrinks until it reaches a minimal finite radius. At this position it is held fixed for a while after which it expands adiabatically back to its original size. During this period, the inner subpacket is still inside the cylinder and the two subpackets are very far from the two cylindrical ends. Thus, each of them continues to move in a field-free region. Later, the subpackets interfere on the screen

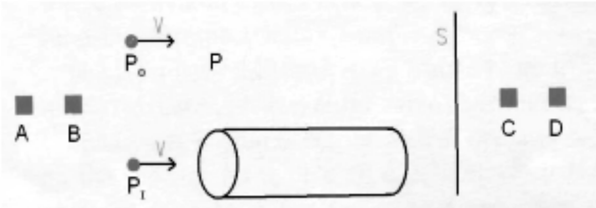

Fig. 1. Two coherent electronic subpackets, P_I and P_O, move with velocity V from left to right. P_I moves through a very long hollow cylinder and P_O moves at its outer side. Later, the subpackets interfere on the screen S. (The role of the devices A, B, C and D is explained in the text.)

[‡1] This claim is made by Bocchieri and Loinger [5] and is implicitly admitted by Aharonov et al. [6], who responded to other arguments of that article.

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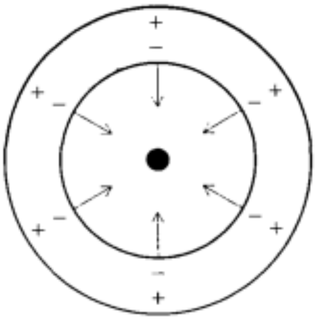

Fig. 2. A cross section of the two cylindrical layers. The outer rigid layer is motionless and the inner flexible one shrinks. The black circle denotes the inner subpacket of the moving electron.

S. This interference pattern is compared with the one obtained from a null experiment where the cylindrical layers are completely motionless. (The devices C and D are required for another experiment.)

In ref. [1] the analysis of this experiment is incomplete because it ignores the dynamics of the potential source. This aspect of ref. [1] is acknowledged in ref. [2] which aims to correct this point. Therefore, the derivation of ref. [2] is discussed in the following lines. This derivation uses the following hamiltonian:

$$H=H_e+H_S+V, \qquad (1)$$

where H_e denotes the single particle hamiltonian of the moving electron, H_S stands for the hamiltonian of the cylindrical source by itself and V designates the interaction between the moving electron and the cylindrical charges (see eq. (11) of ref. [2]). The analysis of ref. [2] proceeds to show that H_e+V can *approximately* be written as a function of the coordinates of the moving electron (see page 1518):

$$H_e+V=H_e(x)+V(x,y_i(t)), \qquad (2)$$

where x denotes the coordinates of the moving electron and $y_i(t)$ are parameters denoting the classical orbits of the cylindrical charges.

It is also claimed in ref. [2] that the wavefunction of the entire system can, *under certain approximations*, be written as a product

$$\Psi=\phi(y_1,...,y_n)\psi(x), \qquad (3)$$

where y_i denotes the coordinates of the ith cylindrical charge (see eq. (24) and the text following it). Substituting (2) and (3) into the Schrödinger equation of the hamiltonian (1), it is deduced that the rates of phase accumulation on the two subpackets are different. Indeed, the Schrödinger equations for the two parts are

$$i\hbar\frac{\partial\Psi_I}{\partial t}=H\Psi_I$$

$$\equiv[H_S+(H_e+V)]\phi(y_1,...,y_n)\psi_I(x), \qquad (4)$$

$$i\hbar\frac{\partial\Psi_O}{\partial t}=H\Psi_O$$

$$\equiv[H_S+(H_e+V)]\phi(y_1,...,y_n)\psi_O(x), \qquad (5)$$

where $\psi_I(x)$ and $\psi_O(x)$ denote the inner and outer subpackets, respectively.

The AB phase shift is derived from (4) and (5). In the two cases, H_S makes the same contribution to the phases of Ψ_I and of Ψ_O because it operates only on the coordinates y_i of ϕ. The moving electron is confined to a field-free region and conserves its kinetic energy. Hence, also H_e makes the same contribution to the phases. However, ψ_O is finite only outside the cylinder and the electric potential vanishes at its location during all times. On the other hand, ψ_I moves through the cylindrical inner part where the potential V is finite during a part of the time. Thus, a nonvanishing phase shift is obtained and the electric AB effect is deduced.

The following counter-example shows that the approximations used on page 1518 of ref. [2] are unjustified. To this end, let us carry out a similar experiment. In the new experiment the screen S is removed and the inner layer is held fixed in its contracted shape until the moving electron leaves and reaches C. Thus, the two experiments are identical until, in the first one, the inner subpacket starts its expansion. At C, which is very far from the cylinder, the two subpackets, P_O and P_I, have different kinetic energies because P_O conserves its original kinetic energy whereas the energy of P_I increases due to the process of being ejected by the cylindrical negative potential. Device C discriminates the subpackets according to their kinetic energy. A subpacket with the original kinetic energy is diverted in an external magnetic field and comes back to join the beam entering device A. A more energetic subpacket is directed to D where its additional kinetic energy is absorbed. Now this subpacket recovers it original kinetic energy and is also diverted back to be merged in the beam entering A. Meanwhile, when the subpackets are very far from the cylinder, the flexible layer returns adiabatically to its original size.

This process is incompatible with the law of energy conservation. Consider the size of the discrepancy, $-eV$, and the difference between the rates of phase accumulations in the first experiment, as claimed in the derivation of the electric AB effect. These quantities are equal while the inner layer is held fixed in its contracted form. It follows that if energy conservation is restored then the electric AB effect disappears. Hence, it is proved that the original derivation of the electric AB effect is wrong.

References

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