Inherent Contradictions of the Majorana Neutrino Theory

Consider the Lagrangian density of a chargeless Dirac particle

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi, \tag{1}$$

and its Dirac equation

$$(i\gamma^\mu\partial_\mu - m)\psi = 0. \tag{2}$$

The definition of the Dirac γ^μ matrices is not unique. Majorana has contrived a set of pure imaginary γ matrices [1]. In his representation, the Dirac equation (2) is mathematically real. The physical meaning of the Majorana equation is a neutrino that is identical to its own antineutrino.

The de Broglie hypothesis concerning the wave nature of a free massive particle is a fundamental principle of quantum theories. This hypothesis says that the following relation holds between the particle's wave-length and its linear momentum

$$\lambda = 2\pi\hbar/p. \tag{3}$$

The undulating properties of the particle's wave function can be written as a linear combination of these expressions (see [2], p. 18)

$$\sin(\boldsymbol{k}\cdot\boldsymbol{x} - \omega t), \ \ \cos(\boldsymbol{k}\cdot\boldsymbol{x} - \omega t), \ \ \exp\pm(\boldsymbol{k}\cdot\boldsymbol{x} - \omega t). \tag{4}$$

Here is an application of this principle. A mathematically real function can be written as a linear combination of the first and the second functions of (4). Hence, a mathematically real wave function of a free massive particle moving along the x-direction takes the form

$$\psi(t, x) = A\sin(kx - \omega t - \delta), \tag{5}$$

where A and δ are real constants.

Here is an analogous argument: Like the Dirac equation, the Majorana equation (2) is of the first order. Hence, the Majorana mass of this equation multiplies $\sin(kx - \omega t - \delta)$, whereas its derivative term multiplies $\cos(kx - \omega t - \delta)$. It means that the Majorana function must vanish. This is a contradiction.

An important fingerprint of the existence of a Majorana neutrino is the neutrinoless double beta decay. The failure of the experimental search for the neutrinoless double beta decay results from the theoretical inconsistency of the Majorana theory.

References

[1] P. B. Pal, *Am. J. Phys.*, **79**, 485 (2011).

[2] L. I. Schiff, *Quantum Mechanics* (McGraw-Hill, New York, 1955).