Chapter 15

Concluding Remarks

This chapter briefly describes some important points that have been discussed in this book. This presentation aims to help readers see the full picture of a coherent structure of particle physics theories. The arguments emphasize the central role of the variational principle and the Dirac theory of elementary massive particles in the theoretical framework of particle physics.

15.1 The Merits of the Variational Principle

At present (August 2021), mainstream physicists regard the SM as an assembly of correct theories. This book disagrees with many SM elements. However, it is important to point out that two fundamental SM elements are agreed on:

1. This book agrees with the SM on the definition of the domain of physical phenomena that should be examined. The present Wikipedia entry states [170]: "The Standard Model of particle physics is the theory describing three of the four known fundamental forces (the electromagnetic, weak, and strong interactions, and not including the gravitational force) in the universe..." The present book is dedicated to physical phenomena that belong to this domain.

2. Textbooks on QFT aim to describe details of the SM (see e.g. [14, 20, 46]). These textbooks use the variational principle as the basis for a QFT of a given elementary particle. Each specific interaction that applies to a given elementary

particle has a Lagrangian density. This Lagrangian density is regarded as the theoretical cornerstone of the interacting particle. This book agrees with this approach but not with the detailed form of the SM Lagrangian density of the presently known elementary particles and their interactions.

Physics is a mature science, and every theory should abide by relevant principles. These principles impose constraints, and any theory that violates even one of these constraints should be rejected. However, it is a good idea to reexamine the validity of the constraints every once in a while.

A description of the constraints that this book adopts were listed in section 3.9 that begins on p. 39. Each constraint is regarded here as a crucial element of a quantum theory of an elementary particle. Other valid constraints are derived from them. They are restated here in a different form:

Req.I The theory should be derivable from a Lagrangian density.

Req.II The theory should abide by SR.

Solution: The Lagrangian density should be a Lorentz scalar.

Req.III Specific terms of the Lagrangian density represent each of the three relevant interactions.

Req.IV A partial differential equation is a crucial element of a theory of an elementary particle. This equation is the Euler-Lagrange equation, derived from the Lagrangian density of the particle's theory.

Req.V Solutions of the theory's equations of motion must describe the states and the time evolution of the relevant elementary quantum particle.

Req.VI A theory of a massive quantum particle should provide a coherent expression for the wave equation, and the limit of its free particle solution should agree with the de Broglie principle.

Solution: The Euler-Lagrange equation of the variational principle should take the form of a wave equation. The action is the phase, and its free-particle limit should agree with the de Broglie principle. The action should be a mathematically real dimensionless Lorentz scalar. For this reason, the Lagrangian density must be a mathematically real Lorentz scalar whose dimension is $[L^{-4}]$.

Req.VII The appropriate limit of quantities of a higher rank theory should be compatible with corresponding quantities of a lower rank theory. Hence, the appropriate limit of quantities of a QFT of a massive particle should agree with corresponding quantities of QM (see [20], p. 49).

Req.VIII An elementary particle is point-like.

Solution: The theory's Lagrangian density should depend on quantum functions with the form $\psi(x)$, where x denotes a set of the four space-time coordinates.

Req.IX The theory must conserve energy, momentum, and angular momentum.

Solution: The Lagrangian density should not depend explicitly on the space-time coordinates. In this case, the Noether theorem yields the theory's energy-momentum tensor.

Req.X A theory of a massive quantum particle should provide a coherent expression for density.

Solution: The Noether theorem provides this kind of expression. A coherent expression for density should not depend on derivatives of the quantum functions with respect to the space-time coordinates.

Req.XI The interaction term of the Lagrangian density of a charged particle should be proportional to its electric charge e.

Req.XII Maxwell equations are independent of the 4-potential A_μ. In VE, Maxwellian electrodynamics uses A_μ as the coordinate of the electromagnetic Lagrangian density (see [3], section 30). Therefore, the Euler-Lagrange equations prove that no term of the Lagrangian density should have A_μ whose power is greater than unity.

Req.XIII A theory of a quantum particle should abide by Wigner's classification of physical particles: There are two sets of physically meaningful particles – massive particles that have positive mass and spin, and massless particles that have positive energy and two degrees of helicity [16].

This book adheres to the idea that elementary massive particles are spin-1/2 particles that are described by the Dirac equation. (For details, see the next section. Experiments have shown that this set of particles comprises the following:

- Three flavors of neutrinos: ν_e, ν_μ, ν_τ.
- Three flavors of charged leptons: e, μ, τ.
- Six flavors of quarks: u, d, s, c, b, t.

Each of these elementary particles has an antiparticle.

It is proved in this book that the SM theoretical description of the particles called $W^{\pm}$, Z, and Higgs bosons contains erroneous elements. Hence, these particles are not elementary point-like particles, but instead, mesons of the top quark.

Some textbooks put forward the problem of whether the variational principle is a mandatory theoretical element of QFT. For example, Weinberg says: "If we discovered a quantum field theory that led to a physically satisfactory S-matrix, would it bother us if it could not be derived by the canonical quantization of some Lagrangian?" (see [20], p. 292). However, in this same textbook Weinberg examines a Lagrangian density of the form

$$\mathcal{L}[\psi(x), \psi(x)_{,\mu}] \tag{15.1}$$

and its associated Lagrangian, and states (see p. 300): "All field theories used in current theories of elementary particles have Lagrangians of this form." This book adopts the form of (15.1).

The following arguments explain why this book evades the problem of the possibility of constructing a coherent physical theory that cannot be derived from the variational principle:

- The variational principle has a magnificent mathematical tool called the Noether theorem. This theorem instructs people on how to build a Lagrangian density that abides by many of the requirements Req.IV – Req.XIII.
- This book aims to solve the problems of *existing particles*. As stated above, it explains why all elementary massive particles are spin-1/2 Dirac particles.
- It is well known that the Dirac equation can be derived from a Lagrangian density. Therefore, there is no indispensable need for a theory that takes a different form.
- The long list of requirements Req.IV – Req.XIII explains why the construction of a coherent theory of an elementary particle that *cannot* be derived from a Lagrangian density is a very difficult assignment.

- This book describes a novel application of the Noether theorem. It can be applied to a given Lagrangian density and discover erroneous theoretical elements (see chapters 5, 6). Evidently, error removal is a vital element of any scientific work.

15.2 Theory of an Interacting Dirac Particle

As explained in the previous section, this book argues that the Lagrangian density is the cornerstone of a quantum theory that describes the state and the time evolution of a quantum particle. Let us see the complete Lagrangian density of a spin-1/2 Dirac particle and its interactions:

$$\mathcal{L}_{full} = \bar{\psi}[\gamma^\mu i\partial_\mu - m - e\gamma^\mu A_{(e)\mu} - g\gamma^\mu A_{(m)\mu} - \mathrm{d}\sigma_{\mu\nu}\mathcal{F}^{\mu\nu}_{(w)}]\psi. \quad (15.2)$$

(The self term of the electromagnetic fields $-\frac{1}{16\pi}F_{\mu\nu}F^{\mu\nu}$ is omitted from (15.2)). Each term of this Lagrangian density is a Lorentz scalar with a dimension of $[L^{-4}]$. The first term inside the square brackets is the ordinary kinetic term, the second is the mass term, the third is the QED interaction term, the fourth is the strong interaction term, and the last is the weak interaction term. Here $\mathcal{F}^{\mu\nu}_{(w)}$ denotes the field of weak interactions. For a neutrino, e=g=0, and only the terms 1, 2, and 5 of (15.2) hold. For a charged lepton, $g = 0$, and the fourth term does not hold. All terms of (15.2) apply to quarks.

Each interaction term of (15.2) comprises three factors. The first factor denotes the strength of the interaction between the Dirac particle and the external field. The second factor comprises the γ^μ 4-vector or the antisymmetric product $\sigma_{\mu\nu}$ of two γ_μ. Finally, the last factor is a tensorial expression of the external field. The γ^μ matrices are a crucial mathematical asset of the Dirac theory. They are numerical dimensionless 4-vectors, and they (or their antisymmetric product) may contract with a tensorial expression of an external field and produce the required Lorentz scalar *without changing the dimension of the term.* In particular, every interaction term does not contain the derivative operator ∂_μ. Furthermore, the Noether expression for the 4-current (see 3.14 on p. 24) depends on the derivative of the quantum function ψ_μ. Hence, like in the well-known electromagnetic interaction of a Dirac particle, the absence of the derivative operator from the interaction terms of

(15.2) is a very important aspect of the Dirac theory: *it means that the interaction term and the Noether theorem for the 4-current do not modify the original expression for density of the Dirac particle.*

The literature calls the electromagnetic interaction of (15.2) the *minimal interaction* (see, e.g., [2], p. 11). It was shown above that every interaction term of (15.2) has an analogous structure. It is just one term that is a product of three factors. For this reason, one may argue that every interaction term of (15.2) is a *minimal interaction.* Another argument that justifies this conclusion goes as follows: Each of the three relevant interactions has specific physical properties. Hence, the associated interaction term of the Lagrangian density should have a unique form. Therefore, each of the three relevant interactions should have at least one distinct interaction term. Hence, remembering that the metric is used for the gravitation interaction, (15.2) is a *minimal interaction* because just one term is used for every interaction. A justification for this assertion relies on a comparison with the excessive number of terms used by the SM for its description of the strong and weak interactions.

Indeed, the SM strong interaction sector is called QCD. This theory is certainly more complicated than that of (15.2) because it adds another degree of freedom, called color. Color is an extension of the charge concept and together with its field they belong to the non-commutative SU(3) group. QCD also relies on a hypothesis that forbids a colored structure of its quark to exist as a free particle. As proved in chapter 10, QCD has been constructed on an erroneous basis, and it is inconsistent with many kinds of experimental data.

The case of the SM weak interaction sector, called the electroweak theory, is even worse than QCD. Its Lagrangian density has more than 30 terms!!! (See subsection 11.6.8). Chapter 11 pointed out many problematic electroweak issues. For example, its description of the electrically charged particles called $W^{\pm}$ violates Maxwellian electrodynamics.

Finally, the general criterion called Occam's razor certainly favors the Lagrangian density (15.2) over the multitude of terms of the SM version of these interactions. Moreover, as stated in the Occam's razor section (see section 2.2, p. 9): "if one theory relies on six assumptions while the other theory relies on more than thirty assumptions, then the Occam's razor criterion is decisive!"

Conclusion: The Occam's razor principle strongly supports the theories of this book (15.2) and denies the SM in general and its electroweak theory in particular.