Problems with the Real Klein-Gordon Field

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1 Comment and 1 Reply can be found at the end.

Textbooks on Quantum Field Theory (QFT) discuss the real Klein-Gordon (RKG) field and claim that a Hamiltonian can be constructed for the free RKG field [1-5]. (The vital role of a QFT Hamiltonian is explained in item 6 below.) The textbooks begin with a Lagrangian density $\mathcal{L}$, derive a Hamiltonian density $\mathcal{H}$ and carry out the 3-dimensional integration.

Contrary to this result, I claim:

1. A real and free RKG field has no Hamiltonian.

   Proof: Assume that a Hamiltonian exists. Then, let us examine the fundamental equation of quantum mechanics:

   $$H\phi = i\hbar \frac{\partial \phi}{\partial t}. \tag{1}$$

   Now $\hbar$, ϕ and t are real. Hence, the Hamiltonian of a free RKG field has pure imaginary eigenvalues. This is an error because an eigenvalue of the Hamiltonian of a free particle represents its energy and should take a real value.

   This point can be stated in a reverse form: textbooks on QFT do not say that there is no Hamiltonian for an RKG particle.

2. Moreover: a bound state of an RKG particle cannot be in an orbital whose angular momentum $l > 0$.

   Proof: The m quantum number satisfies $-l \le m \le l$. Thus, if $l > 0$ then a state $m \ne 0$ exists and the dependence of Y_{lm} on the φ angle is the complex factor $e^{im\varphi}$. This point completes the proof.

Remarks:

3. The RKG field is required for the original form of the Yukawa interaction. This theory claims that a two fermion interaction is mediated by a massive spin-0 particle. The interaction term is $\mathcal{L}_{int} = -g\phi\bar{\psi}\psi$. Now, the Yukawa's wave function ϕ must be real because the Action is real and the same is true for the four coordinates t, x, y, z. Hence, the Lagrangian density must be real. But the Yukawa interaction term is a part of the Lagrangian density and it is proportional to ϕ. Hence, since the coupling constant g and $\bar{\psi}\psi$ are real then one finds that ϕ must be real.

4. The origin of the discrepancy described above [1-5] can be found in a textbook [6], where it is proved that a real wave function has no expression for density. Hence, the spatial integral of the RKG's Hamiltonian density $\mathcal{H}$ has no meaning.

5. The Yukawa theory has been proposed more than 70 years ago. Like the Dirac field $\psi(x^\mu)$, all kinds of the KG field $\phi(x^\mu)$ depend on one set of four space-time coordinates x^μ. Hence, these fields describe a structureless point-like particle. As of today, all kinds of massive point-like particles found in experiment are Dirac particles. In particular, the pion, which is made of a quark-antiquark pair and its radius is not much smaller than that of the proton, is not a Yukawa particle. Here we see that Nature does not respect inconsistent theories and that Dirac was right in his lifelong objection to the KG equation [7].

6. The following lines explain why the Hamiltonian is an indispensable element of RQM and QFT.

The significance of hierarchical relationships between physical theories is explained in pp. 1-6 of [11] and in pp. 85, 86 of [12]. The foundation of the argument can be described as follows. Physical theories take the form of differential equations. These equations can be examined in appropriate limits. Now Relativistic Quantum Mechanics (RQM) is a limit of QFT. The former holds for cases where the number of particles can be regarded as a constant of the motion. Therefore, if examined in this limit, QFT must agree with RQM. By the same token, the classical limit of RQM must agree with classical physics. This matter has been recognized by the founders of quantum mechanics who have proven that the classical limit of quantum mechanics agrees with classical physics.

The following example illustrates the importance of this issue. Let us examine an inelastic scattering experiment. The chronological order of this process is as follows:

a. First, two particles move in external electromagnetic fields. Relativistic classical mechanics and classical electrodynamics describe the process.

b. The two particles are very close to each other. RQM describes the process.

c. The two particles collide and interact. New particles are created. The process is described by QFT.

d. Particle creation ends but particles are still very close to one another. RQM describes the state.

e. The particles depart. Relativistic classical mechanics and classical electrodynamics describe the process.

Now, for the initial and the final phases energy and momentum are well defined quantities and their final state values abide by energy-momentum conservation. It means that the specific values of the energy-momentum of the final state agree with the corresponding quantities of the initial state. Now, the initial and the final states are connected by processes that are described by RQM and QFT. In particular, the process of the new particles creation is described *only* by QFT. Hence, RQM and QFT must "tell" the final state what are the precise initial values of the energy-momentum. It follows that *RQM as well as QFT must use a field function that has a self-consistent Hamiltonian.*

In conclusion, QFT of Dirac particles agrees with RQM of these particles and a self-consistent Hamiltonian exists in this case. By contrast, RQM and QFT of KG particles fail to do that [13]. The experimental evidence described in point 5 above provides a very strong support for the above mentioned Dirac's approach. Indeed, Nature does not like physical theories of particles that have no Hamiltonian.

7. On top of the foregoing arguments, relativistic considerations prove that an expression for energy must be a 0-component of a 4-vector that has the dimensions of energy. The operator $i\hbar\frac{\partial}{\partial t}$ satisfies these requirements. Now, a free *massive* particle is motionless at its rest frame. It follows that at this frame a real function that describes a massive particle must be time-independent. Hence, the time-derivative of such a function vanishes and no energy can be defined for a massive particle whose wave function is real. This is the underlying reason for using complex wave functions in a quantum description of a massive particle. The real QFT theory fails to do that.

References:

[1] N. N. Bogoliubov and D. V. Shirkov, *Introduction to the Theory of Quantize Fields* (Interscience, New York, 1959). (See p. 28, subsection 3.1).

[2] S. S. Schweber *An Introduction to Relativistic Quantum Field Theory* (Harper and Row, New York, 1964). (See the discussion beginning at the middle of p. 187).

[3] J. D. Bjorken and S.D. Drell, *Relativistic Quantum Fields* (McGraw-Hill, New York, 1965). (See p. 26).

[4] G. Sterman, *An Introduction to Quantum Field Theory* (University Press, Cambridge, 1993). (See p. 14 after eq. (1.51).)

[5] M. E. Peskin and D. V. Schroeder *An Introduction to Quantum Field Theory* (Addison-Wesley, Reading Mass., 1995) (See Section 2.3).

[6] V. B. Berestetskii, E. M. Lifshitz and L. P. Pitaevskii, *Quantum Electrodynamics* (Pergamon, Oxford, 1982). (See pp. 42, 43).

[7] P. A. M. Dirac, *Mathematical Foundations of Quantum Theory*, Ed. A. R. Marlow (Academic, New York, 1978). (See pp. 3,4).

[8] E. Wigner, Ann. Math. **40** 149 (1939).

[9] S. S. Schweber, *An Introduction to Relativistic Quantum Field Theory*, (Harper & Row, New York, 1964). (See pp. 44-53.)

[10] S. Sternberg, *Group Theory and Physics* (Cambridge University Press, Cambridge, 1994). (See pp. 143-150.)

[11] F. Rohrlich, Classical Charged Particle, (Addison-Wesley, Reading MA, 1965).

[12] A. Einstein, Albert Einstein in His Own Words (Portland House, New York, 2000).

[13] For a proof of the inconsistency of the complex KG equation with the Hamiltonian, see E. Comay *Progress in Physics* **4**, 91 (2009) and references therein.

———————— Comment #1 ————————

This comment is taken from an email containing remarks made by a Weizmann Institute Prof. which is described as a person having an excellent understanding of QFT.

The claim about real Klein-Gordon fields is completely wrong, it seems that (like many people from the old generation) he misunderstands quantum field theory and the differences it has from quantum mechanics.

...point him to ref. [5] of his note on Klein-Gordon, for detailed discussions both of the consistency of Klein-Gordon theory ..."

————————- End of Comment #1 ————————

———————— Reply #1 ————————-

It is very easy to see that the first paragraph of Comment #1 contains no scientifically acceptable argument that supports its conclusions. The second paragraph is not much better. Indeed, the main discussion presented in item 1 makes a fair usage of [5] and proves that if the books [1-5] are right then a contradiction follows.

Assuming that the Comment's writer does not deliberately want to establish his QFT on a clear contradiction, one is obliged to deduce that in his opinion QFT is constructed in a form which *bears no relationship* to RQM and that this point is not known to "many people from the old generation".

In order to clarify this matter I've added items 6,7 to the main text. Item 6 explains the hierarchical relationships between QFT and RQM. Furthermore, this item proves that every form of QFT must be related to RQM and that if QFT and its corresponding RQM have no expression for energy (namely, a Hamiltonian) then both are just wrong.

The discussion presented in item 6 above proves that understanding the meaning of hierarchical relationships between theories is a necessary condition for making a correct distinction between a true and a false theory. Unfortunately, it turns out that this very important notion is not an indispensable topic of contemporary physicist education. I hope that item 6 as well as references [11,12] will help people to close this gap.

Item 7 describes another decisive argument that refutes the physical meaning of using real wave functions.

Item 5 of the main text points out that in spite of more than 70 years of very hard experimental work, not even a single example of a genuine elementary pointlike KG particle has been found. Thus, the following question arises: for how long can people adhere to an erroneous physical theory that fails time and again in every experimental test?