Hilbert on Kinetic Theory and Radiation Theory (1912-1914)

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Introduction and General Background

In 1912 David Hilbert published his first article dealing with physical issues, the foundations of the kinetic theory of gases Over the coming years he would publish additional works on radiation theory and on the general theory of relativity Indeed, Hilbert's interest in physics was neither sporadic nor superficial, it was an organic component of his overall scientific worldview [1] His interest in kinetic theory and radiation theory was only a small, often neglected, part of a more general attitude The present article is a brief account of this portion of Hilbert's scientific work

The strong connection between Hilbert's physics and mathematics is manifest, in particular, in his axiomatic approach Hilbert's axiomatic conception arose in connection with foundational questions of projective geometry, beginning in 1894 [2] But at the same time, he was curious about foundations of mechanics, and knew the recent relevant work by Heinrich Hertz in this domain, this work provided additional impetus to his pursuit of a systematic, axiomatic analysis of geometry From the beginning, Hilbert thought that the method should be applied equally to physical theories

The axiomatic method was never for Hilbert a starting point for research Rather, it was a tool to enhance understanding of existing, elaborate theories This conception is reflected in the following passage, taken from the lecture notes of a course taught in 1905

The edifice of science is not raised like a dwelling, in which the foundations are first firmly laid and only then does one proceed to construct and to enlarge the rooms Science prefers to secure as soon as possible comfortable spaces to wander around, and only subsequently, when signs appear here and there that the loose foundations are not able to sustain the expansion of the rooms, does it seek to support and fortify them This is not a weakness, but rather the right and healthy path of development (Hilbert 1905, 102)[3]

Hilbert was especially concerned about a situation he considered to be typical of the development of physical theories, in which new hypotheses are introduced to explain newly discovered phenomena, without properly checking whether an added hypothesis is consistent with the existing theories The kind of axiomatic analysis he pursued appeared to him as a proper tool to deal with this situation In his well-known correspondence with Gottlob Frege, immediately following the publication of his *Grundlagen der Geometrie*, Hilbert raised this point very explicitly

After a concept has been fixed completely and unequivocally, it is in my view completely illicit and illogical to add an axiom—a mistake made very frequently, especially by physicists By setting up one new axiom after another in the course of their investigations, without confronting them with the assumptions they made earlier, and without showing that they do not contradict a fact that follows from the axioms they set up earlier, physicists often allow sheer nonsense to appear in their investigations One of the main sources of mistakes and mis-

[1]See Corry 1997 1998

[2]See Toepell 1986

[3]Unless otherwise stated translations from the German original are mine Quotations from Hilbert s lecture notes appear here by permission of the library of the Mathematisches Institut Universitat Gottingen I thank the librarian Mr Matheess for his kind cooperation

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understandings in modern physical investigations is precisely the procedure of setting up an axiom, appealing to its truth, and inferring from this that it is compatible with the defined concepts. One of the main purposes of my Festschrift was to avoid this mistake.[4]

By the turn of the century, the kinetic theory of gases had a short, but already particularly convoluted, history, that seemed to furnish an ideal example of the situation described here by Hilbert. In fact, from its inception, the theory gave rise to heated controversies around several issues, such as the so-called reversibility and recurrence paradoxes, the ergodic hypothesis, and the atomistic point of view.[5]

In 1905 Hilbert taught a course in Göttingen, on the axiomatic method and its applications. A considerable portion of the course was dedicated to the axiomatization of physics, and the notes of this course provide the earliest comprehensive account of Hilbert's picture of this subject.[6] The kinetic theory appears in this course as a particular application of the calculus of probabilities, alongside the theory of compensations of errors (*Ausgleichungsrechnung*), and insurance mathematics. Hilbert accepted without reservations the controversial atomistic assumptions underlying the classical approach to this theory as developed by Ludwig Boltzmann. He did stress, however, the problematic use of probabilistic arguments in physical theories. Even if we know the exact position and velocities of the particles of a gas—Hilbert explained—it is impossible in practice to integrate all the differential equations describing the motions of these particles and their interactions. We know nothing of the motion of individual particles, but rather consider only the average magnitudes that constitute the subject of the probabilistic, kinetic theory of gases. In an oblique reference to Boltzmann's replies to the objections raised against his theory, Hilbert stated that the combined use of probabilities and infinitesimal calculus in this context was a very original contribution of mathematics, which may lead to deep and interesting consequences, but which at this stage had in no sense been fully justified.[7]

Between 1898 and 1906, Hilbert lectured several times in Göttingen on mechanics, potential theory, and continuum mechanics. Beginning in 1907, Hilbert's friend and colleague Hermann Minkowski published a series of now famous works on the relativity principle. Hilbert and Minkowski led two seminars on these issues in Göttingen, in 1905 and in 1907, and it is evident that Hilbert was closely involved in Minknowski's current work. In fact, Minkowski's work is best understood against the background of Hilbert's program for the axiomatization of physics.[8]

Between 1903 and 1912, Hilbert's mathematical efforts concentrated on linear integral equations. At the same time, however, after Minkowski's death, Hilbert returned to teach courses on physical issues. He taught statistical mechanics for the first time in the winter semester of 1910–11. In December of 1911 he presented to the Göttingen Mathematical Society (GMG) an overview of his recent investigations on the kinetic theory of gases, which were soon to be published.[9]

Beginning in 1912, Hilbert permanently enrolled an assistant for physics, who was commissioned with the task of keeping him abreast of current developments. Paul P. Ewald had recently finished his dissertation in Munich, and he was the first to hold this position. Hilbert's involvement with physical issues became increasingly broader and deeper, and he devoted much effort to rethinking from a wider perspective the foundations of this discipline. By 1910 Hilbert's approach had become dominated by the view that all physical phenomena could be reduced to mechanics. This view was clearly manifest in the courses he taught and in the works he published on kinetic theory and radiation theory. In 1913, however, although his reductionistic inclinations did not change, he moved from the mechanistic to the electromagnetic point of view. Electromagnetic reductionism dominated his attempts to formulate a unified foundation for all of physics, beginning in 1915.

Hilbert's Lectures on Kinetic Theory and Radiation Theory

In the winter of 1911–12 Hilbert taught a course specifically devoted to the kinetic theory of gases for the first time. In the introduction to the course, he discussed three possible ways of studying different physical theories like hydrodynamics, electricity, etc. First, he mentioned the "phenomenological perspective," often applied to study the mechanics of continua. Under this perspective, the whole of physics is divided into various chapters: thermodynamics, electrodynamics, optics, etc. These can be approached using different assumptions, peculiar to each of them, and deriving from these assumptions different mathematical consequences. The main mathematical tool used in this approach is the theory of partial differential equations.

A much deeper understanding of the physical phenomena involved in each of these domains is reached—Hilbert told his students—when the atomistic theory is invoked. In this case, one attempts to put forward a system of axioms which is valid for the whole of physics, and which can explain all physical phenomena from a single, unified point of view. The mathematical methods called for are obviously quite different from those adopted in the phenomenological perspective. They can be subsumed, in general, under the theory of probabili-

[4] Quoted in Gabriel *et al.* (eds.) 1980, 40.

[5] Two classical detailed accounts of the development of the kinetic theory of gases and the conceptual problems implied by it (particularly during the late nineteenth century) can be consulted: Brush 1976 and Klein 1970 (esp. 95–140).

[6] A detailed account of the contents of this course appears in Corry 1997.

[7] Hilbert 1905, 178–180.

[8] See Corry 1997a.

[9] See the announcement in the *Jahresbericht der Deutschen Mathematiker Vereinigung* (*JDMV*) Vol. 21, p. 58.

ties The most salient examples of this approach are found in the theory of gases and in radiation theory Seen from this point of view, Hilbert stated, the phenomenological perspective appears as a palliative, a primitive stage on the way to real knowledge, which we must however pass through as soon as possible in order to gain entry into the "real sanctuary of theoretical physics" (Hilbert 1911–12, 2) Unfortunately, he said, mathematical analysis is not yet so developed as to enable us to fulfill all the demands of this approach We must therefore do without rigorous logical deductions in this case, and temporarily be satisfied with rather vague mathematical formulas Still, Hilbert said, it is amazing that using this method we nevertheless obtain ever new results that are in close agreement with experience

Yet a third approach, which in Hilbert's view corresponded to the main task of physics, is the study of the molecular theory of matter itself The study of this theory stands above the kinetic theory in its degree of mathematical sophistication and exactitude In the present course, Hilbert intended to concentrate on the kinetic theory, yet he promised to consider the molecular theory of matter in the following semester

Hilbert's next course, during the summer semester of 1912, dealt with the theory of radiation Connecting this topic with the promise issued at the beginning of the preceding semester, Hilbert declared that he now intended to address the "domain of physics properly so-called," based on the atomic theory Hilbert was clearly very much impressed by recent developments in quantum theory The significance of these developments was highlighted at the first Solvay Conference in October 1911[10], echoes of which had most likely reached Hilbert "Never has there been a more propitious and challenging time than now," he said, "to undertake the study of the foundations of physics " What seems to have impressed Hilbert more than anything else were the deep interconnections recently discovered in physics, "of which formerly no one could have even dreamed, namely, that optics is nothing but a chapter of the theory of electricity, that electrodynamics and thermodynamics are one and the same, that also energy possesses inertial properties, that physical methods have been introduced into chemistry as well" (Hilbert 1912c, 2) And above all, the "atomic theory," the "principle of discontinuity," as Hilbert said, "which today is not hypothesis anymore, but rather, like Copernicus's theory, a fact confirmed by experiment " Very much like the unification of apparently distant mathematical domains, which played a leading role throughout his career, the unity of physical laws exerted a strong attraction on Hilbert

Hilbert's Publications on Kinetic Theory

Hilbert's 1912 article on the kinetic theory of gases appeared as the last chapter of his treatise on the theory of linear integral equations (Hilbert 1912), and it was also printed separately in the *Mathematische Annalen* (Hilbert 1912a) In developing his theory of integral equations, Hilbert was working on ideas originally introduced by Henri Poincare, Vito Volterra, and Ivar Fredholm Hilbert treated the equations as limits of systems of an infinite number of linear equations, using infinite determinants to solve them One kind of equations to which Hilbert paid particular attention were those of the form

$$f(s) = \varphi(s) + \int_a^b K(s,t)\varphi(t)dt,$$

Here $f(s)$ and $K(s,t)$ are given and $\varphi(s)$ is an unknown function When $K(s,t)$, the "kernel," is a symmetric function of its arguments, Hilbert proved a series of theorems that greatly helped analyzing and solving the equation, including many important theorems of existence of solutions and convergence of series [11]

Hilbert's research into integral equations turned out to be strongly connected with a central issue of the kinetic theory the Maxwell-Boltzmann transport equation James Clerk Maxwell, who was the first to formulate this equation, had been able to find only a partial solution of it, valid only for a very special case In 1872 Boltzmann reformulated Maxwell's equation in terms of a single integro-differential equation, in which the unknown function represents the velocity distribution of the given gas The only exact solution Boltzmann was able to find was valid for the same particular case that Maxwell had treated in his own model [12]

By 1912, some progress had been made on the solution of the Maxwell-Boltzmann equation The laws obtained from the partial knowledge concerning those solutions, describing the macroscopic movement and thermal processes in gases, seemed to be qualitatively correct However, the mathematical methods used in the derivations seemed *ad hoc* and unconvincing It was quite usual to depend on average magnitudes, and thus the calculated values of the coefficients of heat conduction and friction appeared unreliable A more accurate estimation of these values remained a main concern of the theory, and the techniques developed by Hilbert offered the means to deal with it [13]

Shortly after the publication of his article on the kinetic theory, Hilbert organized a seminar on this topic, together with his former student Erich Hecke The seminar was also attended by the Gottingen docents Max Born, Paul Hertz, Theodor von Kármán, and Erwin Madelung The issues discussed included the following the ergodic hypothesis and its consequences, theories of Brownian motion, the electron theory of metals in analogy to Hilbert's theory of gases, Hilbert's theory of gases, temperature split by the walls, the theory of dilute gases using Hilbert's theory, the theory of chemical equilibrium, including a report on the related work of Sackur, dilute so-

[10]Kormos Barkan 1993

[11]See Hellinger 1935 Toeplitz 1922

[12]See Brush 1976 432–446

[13]See Born 1922 587–589

lutions[14] The names of the younger colleagues that participated in the seminar indicate that these deep physical issues could not have been discussed only superficially Especially indicative of Hilbert's surprisingly broad spectrum of interests is the reference to the work of Otto Sackur Sackur was a physical chemist from Breslau, whose work dealt mainly with the laws of chemical equilibrium in ideal gases and on Nernst's law of heat He also wrote a widely used textbook on thermochemistry and thermodynamics (Sackur 1912) His experimental work was of considerable significance, and generally his work was far from the kind of mathematical physics which is usually associated with Hilbert and the Gottingen school[15]

Hilbert evidently considered his investigations to be more than just a major contribution to the development of the kinetic theory as such As with his more purely mathematical works, Hilbert was always after the larger picture, searching for the underlying connections among apparently distant fields On many occasions he stressed the connections of his work on the kinetic theory with other physical domains, and in particular with radiation theory, as in the following passage

In my treatise on the "Foundations of the kinetic theory of gases," I have showed, using the theory of linear integral equations, that starting alone from the Maxwell-Boltzmann fundamental formula—the so-called collision formula—it is possible to construct the kinetic theory of gases systematically This construction is such that it only requires a consistent implementation of the methods of certain mathematical operations prescribed in advance, in order to obtain the proof of the second law of thermodynamics, of Boltzmann's expression for the entropy of a gas, of the equations of motion that take into account both internal friction and heat conduction, and of the theory of diffusion of several gases Likewise, by further developing the theory, we obtain the precise conditions under which the law of equipartition of energies over the intermolecular parameter is valid A new law is also obtained, concerning the motion of compound molecules, according to which the continuity equation of hydrodynamics has a much more general meaning than the usual one

Meanwhile, there is a second physical domain whose principles have not yet been investigated at all from the mathematical point of view, and for the establishment of whose foundations—as I have recently discovered—the same mathematical tools provided by integral equations are absolutely necessary I mean by this the elementary theory of radiation, understanding by it the phenomenological aspect of the theory, which at the most immediate level concerns the phenomena of emission and absorption, and on top of which stand Kirchhoff's laws concerning the relations between emission and absorption (Hilbert 1912b, 217–218)

One must always approach this kind of pompous declaration coming from Hilbert with a modicum of critical spirit But even if the self-evaluation of his works turns out to be exaggerated under closer scrutiny, one can be sure that Hilbert's ideas on the kinetic theory positively influenced significant work developed by several of his students First were two doctoral dissertations written under his supervision on related issues, by Hans Bolza and by Bernhard Baule[16] Second, other young Gottingen scientists, like Max Born, Theodor von Kármán, and Erich Hecke, who had attended Hilbert's seminar, published in this field under its influence[17] But perhaps of a much greater impact was the work of the Swedish physicist David Enskog, who attended Hilbert's lectures of 1911–12[18] Building on ideas contained in Hilbert's article, Enskog developed what has come to constitute, together with the work of Sydney Chapman, the standard approach to the whole issue of transport phenomena in gases[19] Although a detailed analysis of Hilbert's influence on Enskog is yet to be written, there can be little doubt that it indeed goes back to the 1911–12 lectures Last is the possible influence of Hilbert on the publication of Paul and Tatyana Ehrenfest's famous article on the conceptual foundations of statistical mechanics (Ehrenfest 1959 [1912]) Paul Ehrenfest studied in Gottingen between 1901 and 1903, and returned there in 1906 for one year, before moving with his wife Tatyana, who was also a Gottingen-trained mathematician, to St Petersburg The idea of writing this article arose following a seminar talk in Gottingen, to which Paul Ehrenfest was invited by Felix Klein[20] The Ehrenfests' style of theory clarification, as manifest in this article, is strikingly reminiscent of Hilbert's lectures in many respects, and strongly suggests a direct influence

Hilbert's Publications on Radiation Theory

Hilbert's published papers on radiation theory are mainly concerned with axiomatic derivation of Kirchhoff's laws of emission and absorption Gustav Kirchhoff had established the laws governing the energetic relations of radiation in a state of thermodynamic equilibrium According to these laws, in the case of purely thermal radiation the relation between the emission and absorption capacities of matter is a universal function of the temperature and the wavelength, independent of the nature and the other characteristics of the body in question In his work on the theory of radiation, Planck substituted for Kirchhoff's concepts of emission and absorption capacity the

[14]References to this seminar appear in Lorey 1916 129 Lorey took this information from the German students journal *Semesterberichte des Mathematischen Vereins*

[15]See Sackur s obituary in *Physikalische Zeitschrift* Vol 16 1915 113–115

[16]The latter one was published as Baule 1914

[17]Cf for instance Bolza Born & van Karman 1913 Hecke 1918 Hecke 1922

[18]See Mehra 1973 178

[19]See Brush 1976 449–468

[20]See Klein 1970 81–83

coefficients of emission and absorption, ϵ and α, respectively, defined for an element of volume. Planck showed that Kirchhoff's law can be formulated as follows: the ratio $q^2\epsilon/\alpha$ (q being the speed of light propagation in the body) is independent of the substance of the body involved, and is a universal function of the temperature and the frequency of radiation.[21]

In his first article on radiation theory (Hilbert 1912b), Hilbert attempted to provide the foundations of this theory, while avoiding the kinds of simplifications usually introduced by physicists (e.g., that the body is homogeneous, simply limited, etc.). Hilbert assumed that the three parameters ϵ, α, and q are given by some arbitrary functions of their spatial location, and showed that the requirement of energy equilibrium for each color leads to a separate, homogeneous integral equation of the second type for ϵ, whose unique solution is $\epsilon = (\alpha/q^2)\, K$ (where K is a constant).

Although Hilbert declared that his foundational study of radiation theory was axiomatic, it was only in an article published the following year (Hilbert 1913), and especially in his second talk on the topic before the Göttingen Scientific Society (Hilbert 1913a), that he articulated the axioms that lay at the basis of his theory and studied their interrelations more systematically. In the footnotes and references appearing in his articles, Hilbert mentioned a considerable number of works in the field by Planck, Ernst Pringsheim, W. Behrens, Rudolf Ladenburg, Max Born, and S. Bougoslawski. It would appear, however, that Hilbert read some of those works, if at all, only after a number of objections to his first article were raised by Pringsheim, which led to a somewhat heated debate between the two. This debate is illustrative of the typical way in which a physicist could have reacted to Hilbert's approach to physical issues, and of how Hilbert's treatment, rather than presenting the systematic and finished structure characteristic of the *Grundlagen*, was piecewise, *ad hoc*, and sometimes confused or unilluminating.

Pringsheim objected to the general approach adopted by Hilbert, and to many of the details of his arguments. Pringsheim also stressed the significant differences between Hilbert's successive articles, in spite of the latter's insistence that there were none. It is noteworthy that also in his later work in general relativity, Hilbert published several versions and claimed that they were essentially identical—a claim that is not confirmed by a detailed examination of the various versions.[22] At any rate, Pringsheim claimed that in focusing on the inadequacies of all earlier proofs of Kirchhoff's theorem Hilbert was assuming, as grounds for his own proof, a fact that Kirchhoff and all other physicists had considered to be in urgent need of proof, namely, that the radiation at each wavelength separately is in equilibrium and no interchange of energy takes place between different spectral regions. In fact, Pringsheim claimed, a main task of Kirchhoff's work was precisely to prove this assertion.[23] Hilbert had to admit the validity of these objections, and his successive articles were attempts to reorganize his thoughts trying to take care of Pringsheim's criticism. Hilbert claimed throughout the articles, however, that the main reason for applying the axiomatic method to this particular physical theory was precisely the need to introduce order into the entanglement of physical assumptions and mathematical derivations that, in Hilbert's opinion, affected it.

In order to prove the impossibility of deriving the Kirchhoff-Planck equations starting from the assumption of equilibrium of total energy for all wavelengths, Hilbert had set the values of q and α equal to 1, independently of the value of the wavelength λ. Pringsheim considered this step inadequate, because no actual body in nature has as its absorption coefficient $\alpha = 1$, and at the same time no dispersion whatsoever (i.e., $q = 1$, the velocity of light in vacuum). A second objection of Pringsheim's was that Hilbert had not taken into account the effects of dispersion and reflection. Hilbert's last article was written as an attempt to extend his proof to take these into consideration (Hilbert 1914).

Hilbert also claimed to have provided a definitive proof of the internal consistency of his system of axioms and of its consistency with the laws of optics. Hilbert was going here much farther than he had gone in the axiomatic analysis of any other physical domain. In the past, when he discussed axiomatic systems for individual disciplines, he never accompanied his discussion with a detailed analysis of the kind he had performed for geometry, though he often declared he had. This time he at least included some detailed arguments concerning the consistency of his system, although they are far from a complete proof. As in his earlier papers, Hilbert's analysis of the logical interrelations among the basic concepts and the principles of the theory, and of their relations to other physical domains, certainly provided a degree of clarity unlike that of his predecessors. Still, his declarations about the strict logical character of his axiomatic analysis—and especially about its similarity with what he had formerly done in geometry—overstate what he actually did in the article.

Hilbert's articles on radiation theory attracted scant attention from physicists. Max Born attributed this neglect to new works appearing soon after, dealing with deeper problems of radiation theory (especially the law of spectral energy distribution of the black body), which became far more important than the issues dealt with in Hilbert's articles. These new works, Born claimed, uncovered many interesting connections with the foundations of physics, which then led to a turning point in our understanding of radiation.[24]

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[21] For Planck's work see Kuhn 1978.

[22] See Corry, Renn and Stachel 1997.

[23] Pringsheim published his objections in Pringsheim 1913, 1913a.

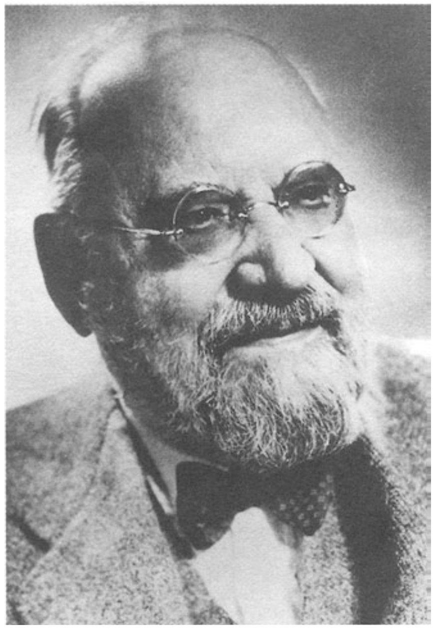
[24] See Born 1922, 592–593.

Concluding Remarks

From the end of 1913 onwards, Hilbert's attention focused on the structure of matter, more particularly, on the Lorentz-covariant, electromagnetic theory of matter developed by Gustav Mie. This was to become the basis for his work on a unified foundation for physics, which included treatment of the field equations of gravitation in the general theory of relativity (Hilbert 1916, 1917). Hilbert's interest in Mie's theory implied a significant change in his basic physical conceptions, from a wholehearted support of an extreme mechanical reductionistic approach throughout his early career, to a similar support of an extreme electromagnetic reductionism. Typically, Hilbert never mentioned this change, and of course he did not explain what caused it. One may infer, with support from the historical evidence, that he increasingly realized the deep difficulties involved in the mathematical treatment of physical phenomena based on the atomistic hypotheses. Certainly, no one was better qualified than Hilbert to assess the degree of these difficulties.

Pringsheim's reaction to Hilbert's articles on radiation is only one example of the lack of enthusiasm aroused by Hilbert's many incursions into physics. Einstein, for instance, criticized Hilbert's approach to the general theory of relativity as being "childish in the sense of a child that recognizes no malice in the external world."[25] Hermann Weyl considered that Hilbert's work in physics was of rather limited value, especially when compared to his work in pure mathematics. Weyl thought that a valuable contribution to physics required a different kind of skills than those in which Hilbert excelled.[26]

Max Born was one physicist who expressed a more consistent enthusiasm for Hilbert's physics. He seems to have truly appreciated the nature of Hilbert's program for axiomatizing physical theories and the potential contribution that program could make. Born explained why, in his opinion, Pringsheim had misunderstood Hilbert and why his reproaches were unjustified.



Gustav Mie. Courtesy of Klaus Mie, Kiel.

The physicist sets out to explore how things are in nature, experiment and theory are thus for him only a means to attain an aim. Conscious of the infinite complexities of the phenomena with which he is confronted in every experiment, he resists the idea of considering a theory as something definitive. He therefore abhors the word "Axiom", which in its usual usage evokes the idea of definitive truth. The physicist is thus acting in accordance with his healthy instinct, that dogmatism is the worst enemy of natural science. The mathematician, on the contrary, has no business with factual phenomena, but rather with logical interrelations. In Hilbert's language the axiomatic treatment of a discipline implies in no sense a definitive formulation of specific axioms as eternal truths, but rather the following methodological demand: specify the assumptions at the beginning of your deliberation, stop for a moment and investigate whether or not these assumptions are partly superfluous or contradict each other. (Born 1922, 591)

In fact Hilbert never performed for a physical theory exactly the same kind of axiomatic analysis he had done for geometry. His derivations of the basic laws of the various disciplines from the axioms were rather sketchy, at best. Many times he simply declared that such a derivation was possible. Among his published works, his last article on radiation theory contains—perhaps under the pressure of criticism—his most detailed attempt to prove independence and consistency of a system of axioms for a physical theory. But what is clear in every case is that Hilbert always considered that an axiomatization along the lines he suggested was plausible and could eventually be fully performed following the standards established in the *Grundlagen*.

Whether or not physicists should have looked more closely at Hilbert's ideas than they actually did, and whether or not Hilbert's program for the axiomatization of physics had any influence on subsequent developments in this discipline, it is nevertheless important to stress that a full picture of Hilbert's own conception of mathematics must include his views on physical issues and on the relationship between mathematics and physics. More specifically, a proper understanding of Hilbert's conception of the role of the axioms in physical theories—a conception condensed in the above quoted passage of Born—helps us understand his conception of the role of axioms in mathematical theories as well. The picture that arises from such an understanding is obviously very far away from the widespread image of Hilbert as the champion of a formalistic conception of the nature of mathematics.

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[25] In a letter of November 23, 1916. Quoted in Seelig 1954, 200.

[26] See Sigurdsson 1994, 363.

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