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The influence of David Hilbert and Hermann Minkowski on Einstein's views over the interrelation between physics and mathematics

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In the early years of his scientific career, Albert Einstein considered mathematics to be a mere tool in the service of physical intuition. In later years, he came to consider mathematics as the very source of scientific creativity. A main motive behind this change was the influence of two prominent German mathematicians: David Hilbert and Hermann Minkowski.

Recent historical research has increasingly stressed the centrality of two different, though interrelated, factors that must be recognized when trying to understand in its proper context the scientific career of Albert Einstein. First are the deep changes that affected some of his most basic scientific conceptions along the years. Second is the crucial role played in the evolution of his ideas by intensive interaction with colleagues, friends and critics.

Among the farthest-reaching changes that affected Einstein's conceptions we find those that concern the interrelation between physics and mathematics: from an early view of physical intuition as guiding his research and of mathematics as no more than an empty tool, to an identification of the latter with the very source and essence of knowledge[1]. The grounds for this noticeable transition are as diverse as they are convoluted, and even a cursory examination of all of them cannot be undertaken here. In this article I point out just one meaningful manifestation of how Einstein's interaction with other scientists affected his views, and in particular, concerning the interrelation between physics and mathematics. The scientists in question are David Hilbert and Hermann Minkowski, two of the most prominent and influential mathematicians of their time.

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Einstein's changing views on mathematics and physics

In June 1933 Einstein delivered the Herbert Spencer lecture at Oxford. In describing 'The Method of Theoretical Physics' he declared[2]:

> If, then, it is true that the axiomatic basis of theoretical physics cannot be extracted from experience but must be freely invented, can we ever hope to find the right way? ... I answer without hesitation that there is, in my opinion, a right way, and that we are capable of finding it. Our experience hitherto justifies us in believing that nature is the realization of the simplest conceivable mathematical ideas. I am convinced that we can discover by means of pure mathematical constructions the concepts and the laws connecting them with each other, which furnish the key to the understanding of natural phenomena ... Experience remains, of course, the sole criterion of the physical utility of a mathematical construction. But the creative principle resides in mathematics.

Such a pronouncement was no mere lip-service for Einstein, but rather an actual description of his current approach to physical research. Since 1922 (and over the rest of his scientific life) he directed his best efforts to formulate a unified field theory that would provide a foundation for all of physics. Einstein's quest – that, as it happened, yield practically no result of enduring value and estranged him from the main stream of the physical community – was dominated by the belief expressed in the above quotation, namely, the belief in the creative powers of mathematics and in the primacy of mathematics in the research of nature.

There can be no doubt, however, that before 1920 Einstein himself would have strongly opposed that belief. Einstein's student years and early career was characterized by a cavalier approach to mathematics: he saw mathematics as a tool in the service of physical ideas and sought to command only as much mathematical knowledge as needed for his immediate purposes. He distrusted mathematical sophistication as such and repeatedly manifested his distaste for 'formal approaches' and 'pure speculation' as opposed to 'real physics'. Among the many recorded instances where he did this, an often quoted (and relatively late) one appears in an interchange with Felix Klein in 1917 concerning Klein's mathematical treatment of the equations of general relativity (GTR). Thus Einstein[3] wrote to Klein:

> It seems to me that you strongly overrate the value of formal points of view. These may be valuable when an already *found truth* needs to be formulated in a final form, but they fail almost always as heuristic aids.

Quite certainly, a major factor behind this fundamental change in Einstein's conception was the difficult struggle he maintained between 1912 and 1915 in the quest after a relativistic theory of gravitation[4]. This quest was characterized by a complex interaction between physical and mathematical considerations that appeared as mutually interwoven within each other and that Einstein found difficult to separate[5]. The unexpected application of Riemannian geometry and of the so-called 'absolute differential calculus' of Gregorio Ricci-Curbastro and Tullio Levi-Civita to the solution of this quintessential physical problem obviously impressed Einstein very much. Still, it would take some years until he came fully under the spell of the idea that pure mathematical can dominate the development of physical theory. This idea, by contrast, had traditionally been one of the main philosophical underpinnings of the scientific work conducted at that time in Göttingen under the leadership of Hilbert and Minkowski.

Hilbert and Minkowski in Göttingen

David Hilbert was the most influential mathematician of the beginning of the century, and perhaps the last universalist. He was brought to Göttingen in 1895 by Felix Klein, who had undertaken to establish there a world center of mathematics and science. Hilbert's studies and early mathematical career between 1880 and 1895 took place in his native city of Königsberg. Königsberg had a small university, with a very respectable tradition of research and education in mathematics and physics (especially rational mechanics) that had been established during the first half of the 19th century by Carl Gustav Jacobi and Franz Ernst Neumann. During his first years as a student, Hilbert attended the lectures of the distinguished mathematician Heinrich Weber, whose interests covered an astonishing variety of issues ranging from the theory of polynomial equations, to elliptic functions, to mathematical physics.

The deepest influence on shaping Hilbert's intellectual horizon in Königsberg came from his exceptional relationship with two other young mathematicians: Adolf Hurwitz, first Hilbert's teacher and later his colleague, and the younger Hermann Minkowski. As a student, Minkowski spent three semesters in Bonn before receiving his doctorate in Königsberg in 1885. He returned to Bonn as a *Privatdozent* and remained there until 1894, when he moved to Zürich. Among his students there was Albert Einstein. Not until 1902 did he join Hilbert in Göttingen, following Klein's success in persuading the Prussian educational authorities to create an unprecedented third chair of mathematics especially for him.

Hilbert's and Minkowski's main interests lay in pure mathematics, but they by no means remained confined to it. Attention to current developments in physics was never foreign to them. When Hilbert elaborated his axiomatic approach, whose first full-fledged manifestation was the famous *Grundlagen der Geometrie* published in 1899, Hilbert had in mind physical theory no less than he had geometry. Hilbert's call in 1900 to the axiomatization of physics, as part of his list of 23 problems for the new century, reflected a very concrete program that he had in mind. Minkowski's enthusiasm for physics had been stimulated very much by his contact with Heinrich Hertz in Bonn, and it was certainly a major source of encouragement for Hilbert's own interest in these issues[6].

A main philosophical notion that underlay Hilbert and Minkowski's involvement in physics, alongside an impressive group of colleagues and students in Göttingen, was the belief in a 'pre-established harmony between mathematics and physics'. This notion had deep roots in German intellectual circles and now in Göttingen it became object of an interesting revival[7]. We find explicit references to it in many of Hilbert and Minkowski's lectures, and we can see how it helped providing philosophical and ideological coherence to much of what was being done in Göttingen. It also lent support, in a natural way, to an idea that seems to have been common among local mathematicians, namely, that 'physics is too difficult to be left to the physicists'. It also encouraged the view that mathematical sophistication was the key to unveiling the secret of nature.

On first sight it is noticeable that both Minkowski and Hilbert were actively involved in the development of both the special and the general theories of relativity respectively. But seen against the background of the central place conceded in Göttingen mathematical circles to the investigation of physical theories, this fact appears to us as anything but a coincidence.

Minkowski and the special theory of relativity

Immediately after his arrival in Göttingen, Minkowski became deeply involved in all the scientific activities of Hilbert, including his current interests in the axiomatization of physics[8]. In 1905 Hilbert and Minkowski, together with other Göttingen professors, organized an advanced seminar that studied recent progress in the theories of the electron[9]. In 1907, the two conducted a joint seminar on the equations of electrodynamics. Beginning in 1907 and until his death in 1909, Minkowski devoted all his efforts to the study of the equations of electrodynamics and the postulate of relativity.

Minkowski reformulated Einstein's special theory of relativity (STR) in terms of a four-dimensional space-time manifold. His formulation provided an extremely elegant and simplified derivation of the theory by focusing on the invariance under linear, orthogonal transformations of the magnitude

$$x^2 + y^2 + z^2 - (ct)^2$$

where c represents the speed of light. Minkowski's presentation was in many senses programmatic rather than systematic. It stressed the possibility of deriving the central conclusions of the theory starting from mathematical principles alone, and without recourse to any experiments. Such an achievement in a theory whose essence was a deep and unexpected unification of mechanics and electrodynamics could not but reinforce the sense of a 'preestablished harmony between mathematics and physics'.

The importance of Minkowski's formulation was quickly grasped by physicists such as Max von Laue and Arnold Sommerfeld. Sommerfeld was a former student of Klein's and his own views were akin to those of the Göttingen circle. He published two articles in 1910 that elaborated in a systematic fashion the ideas introduced by Minkowski and served as a standard point of reference for physicists over the coming years[10]. Laue published in 1911 the first introductory textbook on STR[11] that because his use of Minkowski's formulation presented the theory in a level of clarity and sophistication that surpassed by far Einstein's original one.

Einstein's own reaction to Minkowski's work was quite different. Indeed, it was typical of his early approach to the interrelation between physics and mathematics. Einstein knew Minkowski from his days as a student in Zürich and had mixed feelings about him: he had stayed away from Minkowski's advanced courses on mathematics and at the same time had appreciated very much the high quality of his lectures on analytical mechanics. Einstein considered Minkowski's reformulation of his theory to be no more than 'superfluous erudition'[12] and complained that 'since the mathematics pounced on the relativity theory I no longer understand myself'[13]. He even expressed similar feelings towards Laue's textbook, boasting that he could 'hardly understand' it[14].

But the more Einstein became involved in his efforts towards the GTR, the more he was forced to recognize the intrinsic merits of Minkowski's approach. In fact, the invariant line element of Minkowski's space-time, when taken infinitesimally,

$$ds^2 = dx^2 + dy^2 + dz^2 - (cdt)^2,$$

turned out to be fundamental for representing the 'curved' space-time of GTR and for establishing the connection between the structure of space-time and the gravitational potentials. Around 1912 Einstein became fully aware of this.

In a letter written to Sommerfeld in October 1912 we can already notice early signs of a shift in attitude, when Einstein wrote[15]:

> I occupy myself now exclusively with the problem of gravitation ... One thing is certain that I have never before had to toil anywhere near as much, and that I have been infused with great respect for mathematics, which I had up until now in my naivety looked upon as a pure luxury in its more subtle parts. Compared to this problem, the original theory of relativity is child's play.

Yet a total change was still years away, and along the way we find an additional significant station in the interchange between Einstein and Hilbert over the formulation of the equations of the GTR.

Hilbert and GTR

Beginning in October 1912, the German physicist Gustav Mie published a series of articles in which he developed in mathematical detail a theory of the structure of matter. Few physicists accorded a special significance to Mie's theory, and in particular his intended account of gravitational phenomena was strongly criticized by Einstein and by others. Hilbert, however, by mediation of his colleague Max Born, became keenly interested in the theory and eventually adopted it as a fundamental pillar for constructing a unified theory that would serve as foundation for all of physics[16].

In the summer of 1915 Einstein, then at the peak of his years-long efforts to formulate the field equations of GTR, was invited by Hilbert to lecture in Göttingen on the current state of his research. The months of October and November that year marked the culmination of the most dramatic period in Einstein's scientific career, leading to his publication of the correct equations[17] on 25 November. Hilbert was at that time immersed in the formulation of his own unified foundations of physics, based on combining ideas of Einstein into Mie's theory. During October and November, Hilbert and Einstein held an intensive correspondence where they kept each other informed about the current progress of their respective works. Then, on 20 November, Hilbert presented his theory to the Göttingen Scientific Society. Until recently, it was widely believed that at that opportunity Hilbert displayed the correct field equations of GTR five days before Einstein had done so, but recent historical research has proved that this was not the case[18]. At any rate, for a short while the relations between Einstein and Hilbert became somewhat stressed against the background of these events, but soon they returned to normality, namely, to a very high degree of mutual personal and scientific esteem.

But besides the personal issues at stake in this episode, it contains important aspects of the more basic question that occupies us here. Einstein's involvement with GTR implied by its very nature a much more mathematical, and perhaps less intuitively physical, approach than his work on STR had. And still, Hilbert's approach to the issue, which was typical of the Göttingen spirit and of Hilbert's own involvement with physical issues, was hard for Einstein to digest. Hilbert's derivation of the fundamental equations of physics (including the field equations of gravitation) was based on two axioms. The first was a variational argument based on the integral

$$\int H\sqrt{g}\,d\omega$$

where H is a Hamiltonian function taken from Mie's theory. The second axiom was the demand of general covariance, which Hilbert took from Einstein's theory[19].

Very much like in Minkowski's earlier work, Hilbert's derivation stressed the possibility of obtaining the central equations of the theory (that for Hilbert were indeed the basic equations of physics *in general*) starting from mathematical principles alone, and without recourse to any experiments. Einstein's negative reaction to this approach was explicitly stated in a letter[20] to Hermann Weyl (himself a leading Hilbert student) in November 1916:

> To me Hilbert's *Ansatz* about matter appears to be childish, just like an infant who is unaware of the pitfalls of the real world ... In any case, one cannot accept the mixture of well-founded considerations arising from the postulate of general relativity and unfounded, risky hypotheses about the structure of the electron ... I am the first to admit that the discovery of the proper hypothesis, or the Hamilton function, of the structure of the electron is one of the most important tasks of the current theory. The 'axiomatic method', however, can be of little use in this.

Einstein was not opposed to variational methods as such in physics, but he could not accept the far-reaching consequences that Hilbert intended to derive from such methods. In fact, in June of 1916 Einstein himself submitted to the Berlin Academy his own derivation of the field equations from a Hamiltonian principle.

An interesting offshoot of Hilbert's work was a debate of 1918 on the status of energy conservation theorems in GTR, involving Einstein, Klein and Emmy Noether[21]. The issue at stake was how to separate invariant equations having actual physical meaning from those with only a formal, mathematical content. Emmy Noether, one of the finest representatives of the Göttingen school and its approach, published important work bearing on these questions[22]. Einstein clearly appreciated the deep significance of Noether's work, as a brilliant instance of mathematical reasoning leading to the clarification of the basic underpinnings of a physical theory. One can assume that they provided additional stimulus to Einstein's increasing willingness to acknowledge, much more than in the past, the essential role of mathematics as a creative power in science.

Concluding remarks

In the Autobiographical Notes written in 1946, Einstein retrospectively analysed the main lesson he had learnt from his quest after GTR.

> A theory can be tested by experience, but there is no easy way from experience to the construction of a theory. Equations of such a complexity as are the equations of the gravitational field can be found only through the discovery of a logically simple mathematical condition that determines the equations completely or almost completely. Once one has obtained those sufficiently strong formal conditions, one requires only little knowledge of facts for the construction of the theory; in the case of gravitation it is the four-dimensionality and the symmetric tensor as an expression for the structure of space that, together with the invariance with respect to the continuous transformation group, determine the equations all but completely[23].

No formulation could have better encapsulated the spirit of the Göttingen approach to the question of the interrelation between physics and mathematics than Einstein's wording here. Einstein even cared to mention explicitly the specific contribution of Minkowski to his theory and the fundamental starting points of Hilbert's derivation of the field equations. The young Einstein would have certainly opposed these views in the most emphatic way!

References

1 Norton, J.D. (1995) in *The Creation of Ideas in Physics* (Leplin, J., ed.), pp. 59–64, Kluwer Academic Publishers

2 Einstein, A. (1954) *Ideas and Opinions*, p. 274, Bonanza Books

3 Pais, A. (1982) *'Subtle is the Lord ...' The Science and the Life of Albert Einstein*, p. 325, Clarendon Press

4 Norton, J.D. (1984) *Hist. Stu. Phys. Sci.* 14, 251–316

5 Renn, J. and Sauer, T. (1996) *Phys. Bl.* 52, 865–872

6 Corry, L. (1997) *Arch. Hist. Ex. Sci.* 51, 83–198

7 Pyenson, L. (1982) *Arch. Hist. Ex. Sci.* 27, 137–157

8 Corry, L. (1997) *Arch. Hist. Ex. Sci.* 51, 273–314

9 Pyenson, L. (1979) *Arch. Hist. Ex. Sci.* 21, 55–89

10 Sommerfeld, A. (1910) *Ann. Phys.* 32, 749–776; 33, 649–689

11 von Laue, M. (1911) *Das Relativitätsprinzip*, Vieweg

12 Pais, A. (1982) *'Subtle is the Lord ...' The Science and the Life of Albert Einstein*, p. 151, Clarendon Press

13 Seelig, C. (1954) *Albert Einstein*, p. 46, Europa Verlag

14 McCormmach, R. (1976) *Hist. Stu. Phys. Sci.* 7, xxvii

15 Hermann, A. (1968) *Albert Einstein – Arnold Sommerfeld. Briefwechsel*, p. 26, Schwabe

16 Corry, L. in *Alternatives to Einstein's General Theory of Relativity* (Renn, J., ed.), Birkhäuser Verlag (in press)

17 Einstein, A. (1915) in *The Collected Papers of Albert Einstein*, Vol. 6 (Kox, A.J., Klein, M.J. and Schulmann, R., eds), pp. 245–249, Princeton University Press

18 Corry, L., Renn, J. and Stachel, J. (1997) *Science* 278, 1270–1273

19 Hilbert, D. (1916) *Gött. Nach. – Math. Phys. Kl.* 395–407

20 Seelig, C. (1954) *Albert Einstein*, p. 200, Europa Verlag

21 Rowe, D.E. in *The Visual World: Geometry and Physics (1900–1930)* (Gray, J.J., ed.), Oxford University Press (in press)

22 Noether, E. (1918) *Gött. Nach. – Math. Phys. Kl.* 37–44, 235–257

23 Einstein, A. (1979) *Autobiographical Notes: A Centennial Edition*, p. 85, Open Court